Journal of the Korean Nuclear Society
Volume 16, Number 2, June, 1984

Development of a Method for Uncertainty Analysis in the Top Event Unavailability

Sang Hoon Han and Chang Hyun Chung
Seoul National University

Kun Joong Yoo
Korea Advanced Energy Research Institute
(Received April 19, 1984)

고장수목 정점사상 이용 불능도의 불확실성 분석용 방법 개발

한 상 훈 · 정 창 현
서울대학교

유 건 중
한국에너지연구소
(1984. 4. 19 접수)

Abstract

A method and computer code for the uncerainty analysis in the top event unavailability are developed and tested by combining Monte Carlo Method and Moments method with fault tree reduction technique. Using system fault trees and unavailability data selected in WASH-1400, the efficiency of the proposed method is tested and these results are compared with those obtained by Monte Carlo method. It is shown that the results are sufficiently good in accuracy and computation time is considerably reduced compared with those by Monte Carlo method.

요 약

고장수목 정점사상에 대한 이용불능도의 불확실성을 분석하기 위한 방법 및 전산코드를 개발하였으며 그 유용성을 검증하였다. 이 방법은 몬테카를로 방법과 모멘트 방법을 고장수목 축소 기법과 함께 조합하여 개발하였고 WASH-1400에 있는 고장수목과 신뢰도 자료를 이용하여 본 연구에서 개발된 코드의 효율성을 검증하였다. 몬테카를로 방법과의 비교결과 이 방법을 이용하면 계산시간을 상당히 줄일 수 있으며 충분히 정확한 결과를 얻을 수 있음을 입증하였다.

1. Introduction

In investigating possible failure mechanisms of systems, particularly complex multicomponent systems such as nuclear reactors, system reliability analysis which treats system failure in a probabilistic way has been widely used.

General procedures of system reliability or availability analysis are as follows: First, a system failure condition to be analyzed is identified and a fault tree which shows how the system fails is constructed. The system failure event considered is called the top event. Second, reliability data including components failures, which are called basic events in the fault tree, are gathered and analyzed. Next, minimal cut sets are obtained from the fault trees. Finally, quantitative analysis and uncertainty analysis are carried out.

In evaluating the probability of the top event occurence, wide uncertainty exists due to uncertainties in the basic events data. These uncertainties of data come from the following two reasons.[1] One is the random variability in some measurable quantities, since similar components in different systems may be in different conditions, for example, due to varying maintenance or different operational demands. The other is the lack of data due to relatively short operating experience of nuclear power plants. The uncertainty due to the lack of data can be reduced by increasing effort in data gathering, while random variability can not be reduced.

The realistic approach to quantify uncertainties of basic events is the use of probability distributions. In this study, only the lognormal distribution is considered due to following reasons.[2] The lognormal distribution is frequently used as a distribution for failure rates, especially when the failure rates typically encountered are enough low and sparse to make a logarithmic transformation attractive. This happens quite often for nuclear grade components, since it was used in WASH-1400[3] issued in 1975.

Given the fault tree and reliability data, the work to be done is to calculate the distribution of probability of the top event occurence. For uncertainty analysis, three kinds of methods such as Monte Carlo method, Moments method and Discrete probability distribution (DPD) method have been developed and widely used. DPD method[6] uses discrete probability distributions which are created either by numerical integration of continuous distributions or from the experimental data itself. The advantage of this method is that the exact calculation is possible without random sampling. However, it requires too much computation time to be useful for general cases. Moments method[7],[8],[9] calculates the moments (mean value and variance) for the top event from the moments of the probability distributions for basic events and analyzes uecertainty in the top event occurence from the moments. This method can give sufficiently accurate results with a few calculations for simple cases, but errors are large for complex cases. Monte Carlo method needs less computation time compared with other methods, when the same desired accuracy is required. Therefore, Monte Carlo method is generally preferred to Moments method and DPD method. However, it still requires a considerable amount of computatation time.

In this paper, a method for combining Monte Carlo method and Moments method together with fault tree reduction technique is conceived and computer-programmed. Given the fault tree, this method makes the tree simple by lumping parts of the tree. For this purpose, the fault tree is divided into independent subtrees which consist of many basic events. The fault tree is reconstructed with subtrees and the subtrees are treated like basic events in the reconstructed fault tree. After these procedures, probability distributions for subtrees are calculated by Moments method and the probability distribution for the top event occurence is computed by Monte Carlo method. From this distribution, uncertainty is analyzed.

The brief descriptions on the proposed method are given in Sec. 2 and the computer code developed is described in Sec. 3. Computational results and computation time by the proposed method and those by Monte Carlo method are presented in Sec. 4, for examples selected among system fault trees of WASH-1400. Finally, the conclusion is given in Sec. 5.

2. Method

The Proposed method consists of four steps; the reduction of fault tree, the calculation of probability distribution for subtree, the determination of minimal cut sets and the calculation of confidence limits for the top event occurence. The brief procedures are as follows:

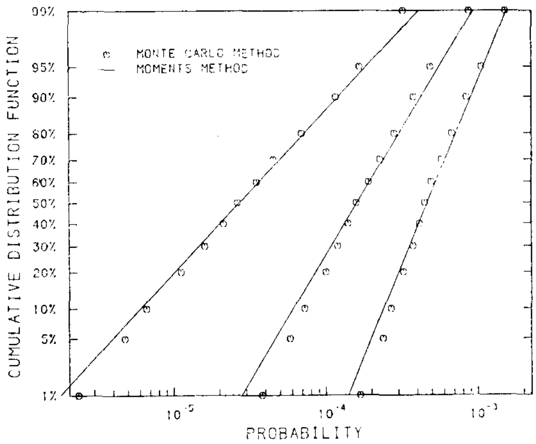
Step 1: The fault tree reduction

This step proposed by Rowsome[11] is to make the fault tree simple by lumping parts of the tree. The fault tree is divided into many subtrees which, in turn, consist of basic events appearing only in the subtree. Basic events lumped into any subtree cannot be related to the other subtrees. Then, the system fault tree is reconstructed with these subtrees which are treated like basic events in the reduced fault tree. Many basic events can be lumped into one subtree and subsequently the size of the fault tree can be significantly reduced as shown in Figs. 2 and 3.

Step 2: Calculation of the probability distributions for subtrees

Moments method gives good estimations and requires little computation time for simple cases such as subtrees. Especially, if probability distributions for basic events are lognormal, the distribution for a subtree, which is a product of basic events, is also lognormal. In the case of the summation, the resulted distribution is not precisely lognormal but similar to lognormal. In Fig. 1 is shown the comparison of Moments methed and Monte Carlo method for the summation of 2, 8 and 20 basic events. The scale is for lognormal. The straight line corresponds to the case that cumulative distribution function (cdf) is lognormal.

The density function of lognormal distribution



Fig, 1. Comparison of Moments and Monte Carlo method

is given by

$$f(t)=\frac{1}{\sqrt{2\pi}\alpha t}\exp\left[-\frac{1}{2}\left(\frac{\ln(t/\beta)}{\alpha}\right)^2\right]$$
$$\text{for } t>0 \tag{1}$$

where α is the shape parameter and β is the median value. The mean value m and variance σ^2 are defined as follows:

$$m=\beta\exp\left(\frac{1}{2}\alpha^2\right) \tag{2}$$

$$\sigma^2=m^2(\exp\alpha^2-1) \tag{3}$$

If the mean value and variance for each basic event are calculated using Eqs. 1 to 3, m and σ^2 for a subtree can be obtained as follows:

For an event of the product of events, the probability that the event occurs is

$$P(S)=\prod_{i=1}^{n} P(Xi) \tag{4}$$

where S is an event corresponding to the subree, Xi's are basic events and n is the number of basic events included in the subtree. The mean value and variance are

$$m=\mathrm{E}\{P(S)\}=\prod_{i=1}^{n} E\{P(Xi)\}=\prod_{i=1}^{n}\{m(Xi)\}$$

$$\sigma^2=E\{[P(S)]^2\}-[E\{P(S)\}]^2 \tag{5}$$

$$=\prod_{i=1}^{n}[\sigma^2(Xi)+m^2(Xi)]-m^2 \tag{6}$$

For an event of the summation of events.

$$P(S)=1-\prod_{i=1}^{n}[1-P(Xi)] \tag{7}$$

$$m=1-\prod_{i=1}^{n}[1-m(Xi)] \tag{8}$$

$$\rho^2=\prod_{i=1}^{n}[\sigma^2(Xi)+(1-m(Xi))^2]$$

$$-\prod_{i=1}^{n}[(1-m(Xi))^2] \tag{9}$$

If the mean value and variance are obtained, the parameters for the subtree are calculated using Eqs. 1 to 3.

Step 3: Determination of minimal cut sets

This procedure is to transform the fault tree into its logically equivalent form in terms of specific combinations of basic events sufficient to cause the undesired top event to occur. Each combination will be a "minimal cut set". A minimal cut set is a set of events, which cannot be reduced in number and whose occurence causes the top event.

In this study, trial and error test[10] is used to determine the minimal cut sets from the reduced fault tree. First, a combination of the failed basic events is selected. For the combination, it is tested whether failure of the top event occurs or not. Only when the top event fails, the selected combination is a cut set. From these cut sets obtained through the above procedure, minimal cut sets are determined.

Given the minimal cut sets, the probability of top event occurence can be calculated using upper bound approximation;

$$P(T)\cong 1-\prod_{i=1}^{n}[1-P(Mi)] \tag{10}$$

$$P(Mi)=\prod_{Xj\in Mi}P(Xj) \tag{11}$$

where $P(T)$, $P(Mi)$ and $P(Xj)$ are the probability of occurence of the top event, i-th minimal cut set and j-th basic event, respectively, and n is the number of minimal cut sets. If all $P(Mi)$'s are lower than 0.1, the approximation gives a sufficiently accurate result.

Step 4: Calculation of confidence limits for the top event occurence

The probability distribution for the top event is calculated by Monte Carlo simulation.[3),4),5)] First, by random sampling of values of $P(Xi)$'s from probability distributions of Xj's, $P(Mi)$'s are calculated by Eq. 11 and one $P(Tk)$ is obtained from Eq. 10. If this procedure is repeated N times,

$P(T):(P(Tk),k=1,N)$ is obtained.

Then, $P(Tk)$'s are sorted by magnitude and the cdf of $P(T)$ is determined. As shown in Figs. 4 to 6, confidence limits can be presented from the cdf.

3. Computer Program

The program REDCON is a general purpose computer program developed to perform uncertainty analysis by the method described in Sec. 2. The program is divided into two parts. In the first part, it takes a system fault tree and probability distributions for basic events as input data. Then, it performs the reduction of the fault tree and computation of probabilty distribution for each subtree obtained in the reduction process. In a subtree analysis, large error can be arisen due to the use of Moments method when the subtree contains a large numbur of basic events or different types of gates. Therefore, in the procedure of fault tree reduction, the number of basic events within a subtree is limited to 36 and different types of gates are not allowed in any subtree. Next, the first part of REDCON provides the second part with a FORTRAN logical equivalent of a reduced fault tree and the probability distribution for subtrees. In the second part, the code performs determination of minimal cut sets and calculation of confidence limits of the top event occurence by Monte Carlo simulation.

The program is written for CYBER-170 machine KAERI.

Table 1. Unavailability data for Example 1

INDEX	EVENT TYPE	MEDIAN VALUE	ERROR FACTOR	INDEX	EVENT TYPE	MEDIAN VALUE	ERROR FACTOR
1	HEVENTY	1.000E-05	3.00	44	XV1C21X	3.000E-05	3.00
2	JK00	1.100E-04	3.00	45	CV1C21C	1.000E-04	3.00
3	JD00	4.200E-04	3.00	46	CS4C43X	1.000E-03	3.00
4	JJ00	1.100E-04	3.00	47	CS04C3C	0.	1.00
5	JC00	4.200E-04	3.00	48	CN04C4C	0.	1.00
6	001000N	1.000E-03	3.00	49	CS5C43X	1.000E-03	3.00
7	TLUNISF	0.	1.00	50	CS05C3C	0.	1.00
8	PPLVLSP	0.	1.00	51	CN05C4C	0.	1.00
9	ST2H11F	3.700E-02	3.00	52	XV1D20X	3.000E-05	3.00
10	STXBPRF	1.300E-04	10.00	53	CV1D20C	1.000E-04	3.00
11	STXDPRF	1.300E-04	10.00	54	XV1D21X	3.000E-05	3.00
12	STTAPRF	1.300E-04	10.00	55	CV1D21C	1.000E-04	3.00
13	STTCPRF	1.300E-04	10.00	56	CS4D43X	1.000E-03	3.00
14	MVXB00K	1.000E-03	3.00	57	CS04D3C	0.	1.00
15	MVXD00K	1.000E-03	3.00	58	CN04D4C	0.	1.00
16	MVTA00K	1.000E-03	3.00	59	CS5D43X	1.000E-03	3.00
17	MVTC00K	1.000E-04	3.00	60	CS05D3C	0.	1.00
18	STXBCNF	3.500E-04	10.00	61	CN05D4C	0.	1.00
19	STXDCNF	3.500E-04	10.00	62	MV2A02C	1.000E-04	3.00
20	STTACNF	3.500E-04	10.00	63	ST2A02D	1.000E-06	3.00
21	STTCCNF	3.500E-04	10.00	64	PM2A03F	7.200E-04	10.00
22	XV1A20X	3.000E-05	3.00	65	PM2A03A	1.000E-03	3.00
23	CV1A20C	1.000E-04	3.00	66	ST2A03D	1.800E-03	3.00
24	XV1A21X	3.000E-05	3.00	67	PM2A03X	1.000E-03	3.00
25	CV1A21C	1.000E-04	3.00	68	MV2A01C	1.000E-04	3.00
26	CS4A43X	1.000E-03	3.00	69	ST2A01D	1.000E-06	3.00
27	CS04A3C	0.	1.00	70	CV2A01D	1.000E-04	3.00
28	CN04A4C	0.	1.00	71	PM1A01A	1.000E-03	3.00
29	CS5A43X	1.000E-03	3.00	72	ST1A01D	1.000E-06	3.00
30	CS05A3C	0.	1.00	73	PM1A01F	2.400E-02	10.00
31	CN05A4C	0.	1.00	74	MV2B02C	1.000E-04	3.00
32	XV1B20X	3.000E-05	3.00	75	ST2B02D	1.000E-06	3.00
33	CV1B20C	1.000E-04	3.00	76	PM2B03F	7.200E-04	10.00
34	XV1B21X	3.000E-05	3.00	77	PM2B03A	1.000E-03	3.00
35	CV1B21C	1.000E-04	3.00	78	ST2B03D	1.800E-03	3.00
36	CS4B43X	1.000E-03	3.00	79	PM2B03X	1.000E-03	3.00
37	CS04B3C	0.	1.00	80	MV2B01C	1.000E-04	3.00
38	CN04B4C	0.	1.00	81	ST2B01D	1.000E-06	3.00
39	CS5B43X	1.000E-03	3.00	82	CV2B01D	1.000E-04	3.00
40	CS05B3C	0.	1.00	83	PM1B01A	1.000E-03	3.00
41	CN05B4C	0.	1.00	84	ST1B01D	1.000E-06	3.00
42	XV1C20X	3.000E-05	3.00	85	PM1B01F	2.400E-02	10.00
43	CV1C20C	1.000E-04	3.00				

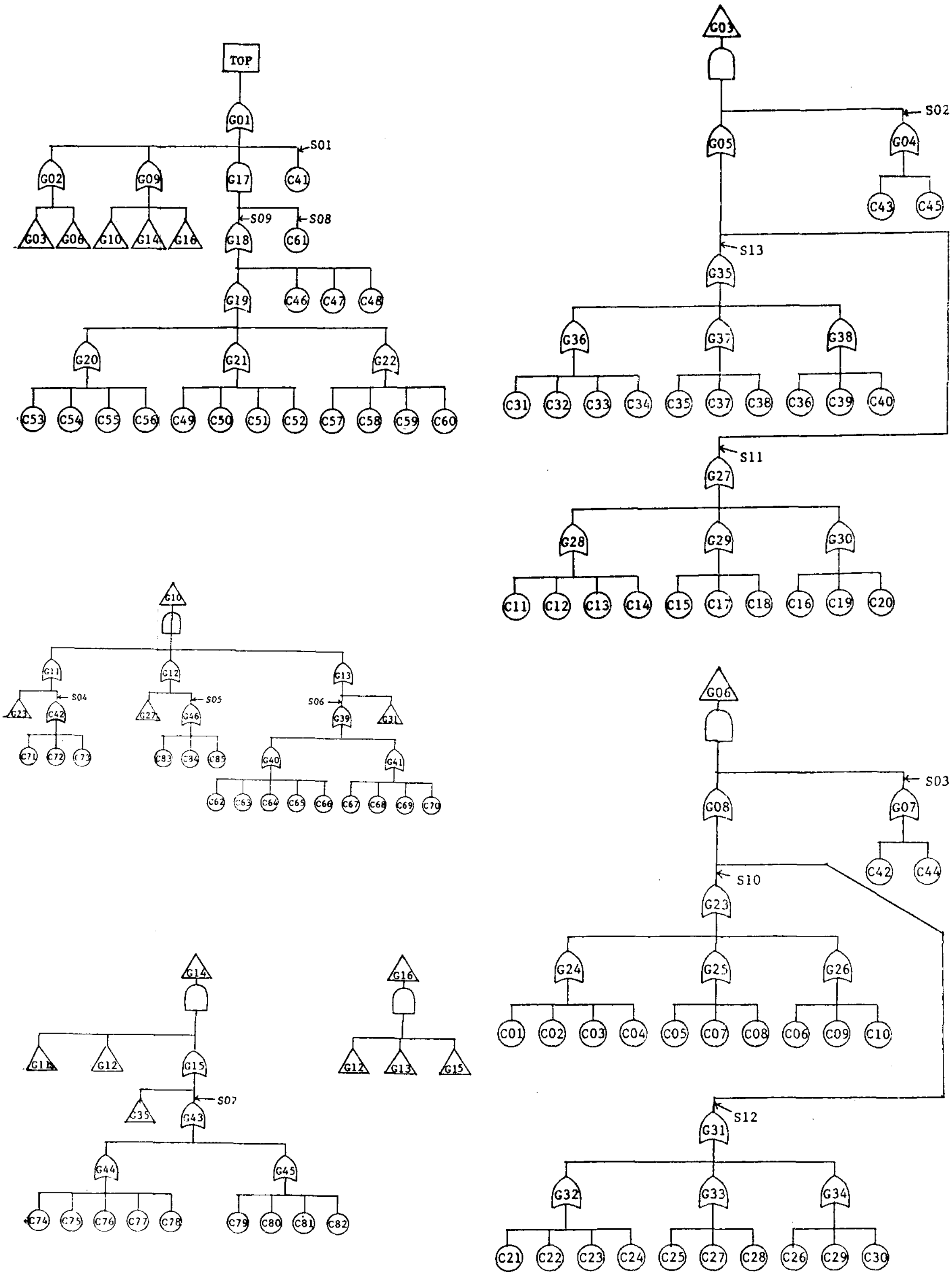


Fig. 2. Original fault tree for Example 1

4. Applications and Results

In order to compare the results of the proposed method with those obtained by exclusive use of Monte Carlo method, three examples are selected from WASH-1400 Appendix II. The Monte Carlo method is performed by the CONINT[5] which has been developed at KAERI. The CONINT code does essentially the same things as the SAMPLE code.[3]

Example 1: CHRS (Containment Heat Removal System)

The top event of the fault tree is "containment spray heat exchangers fail to sufficiently cool spray fluid". For this example, the system fault tree and reduced fault tree are presented in Figs. 2 and 3 and unavailability data are presented in Table 1. In Fig. 2, an arrow means that events below the segment indicated by the arrow are lumped into a subtree. The system fault tree with 85 basic events is reduced to a tree with 13 events. The proposed method calculates the median and 95% value for the top event unavailability to be 1.01×10^{-4} and 4.18×10^{-4}, respectively. The corresponding results of Monte Carlo method are 1.07×10^{-4} and 4.87×10^{-4}. The computation times of the proposed method and Monte Carlo method are 7.617 sec and 85.993 sec. The 50%, 80%, 90% and 95% values and computation time are presented in Table 2 together with the results of the next two examples. The cdf's are shown in Fig. 4 where the circles are the values of the proposed method and crosses are those of the Monte Carlo method.

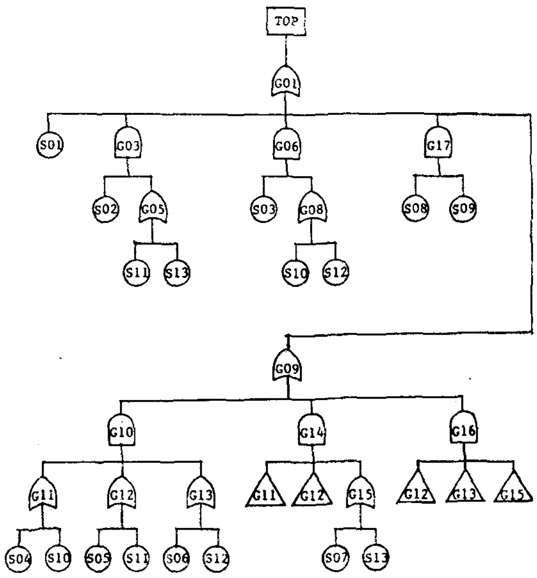


Fig. 3. Reduced fault tree for Example 1

Table 2. Summarized Results

	Method	Computation time	Results 50%	80%	90%	95%
Example 1	Monte Carlo	85.993[1)]sec	1.07×10^{-4}	2.07×10^{-4}	3.15×10^{-4}	4.87×10^{-4}
	Proposed method	7.617[2)]sec	1.01×10^{-4} (−5.6%)[3)]	1.88×10^{-4} (−9.2%)	2.88×10^{-4} (−8.0%)	4.18×10^{-4} (−14.2%)
Example 2	Monte Carlo	30.216 sec	3.78×10^{-3}	4.87×10^{-3}	5.67×10^{-3}	6.34×10^{-3}
	Proposed method	3.597 sec	3.79×10^{-3} (0.3%)	5.05×10^{-3} (3.7%)	5.81×10^{-3} (2.5%)	6.45×10^{-3} (1.7%)
Example 3	Monte Carlo	101.584 sec	2.04×10^{-2}	3.04×10^{-2}	3.87×10^{-2}	5.19×10^{-2}
	Proposed method	23.081 sec	2.00×10^{-2} (−2.0%)	3.02×10^{-2} (−0.7%)	3.67×10^{-2} (−5.2%)	4.29×10^{-2} (−5.2%)

1) excluding the calculation time of minimal cut sets
2) the calculation time needed for all steps
3) % difference of two methods

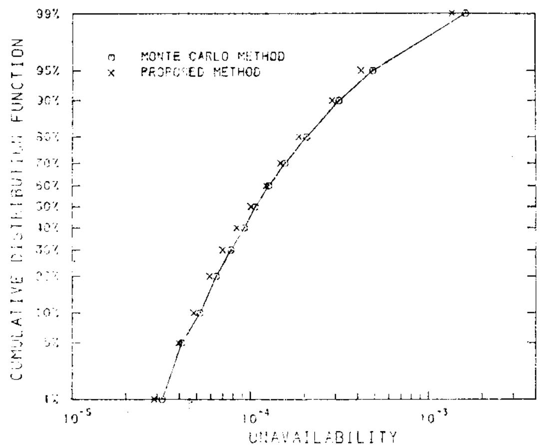


Fig. 4. cdf for Example 1

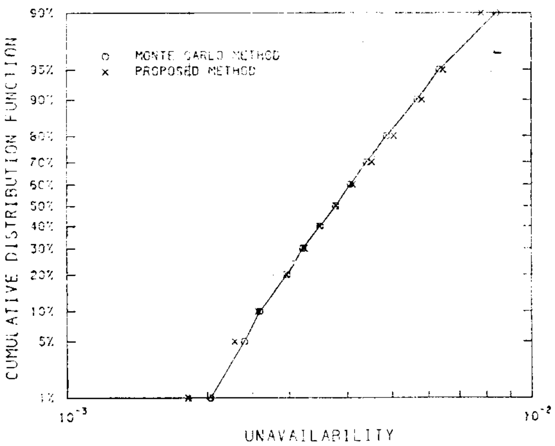


Fig. 5. cdf for Example 2

Example 2: LPIS(Low Pressure Injection System)

The top event is "insufficient LPIS coolant to a cold leg". The fault tree and data are not presented here, but they are given in Fig. II 5-32 and Table 5 II-16 of WASH-1400. The cdfs for the top event unavailability are presented in Fig. 5. As shown in Table 2 and Fig. 5, the median and 95% value of the proposed method are 3.78×10^{-3} and 6.34×10^{-3}, while those of the Monte Carlo method are 3.79×10^{-3} and 6.45×10^{-3}. The present method requires the computation time of 3.597 sec compared with 30.236 sec for the Monte Carlo method.

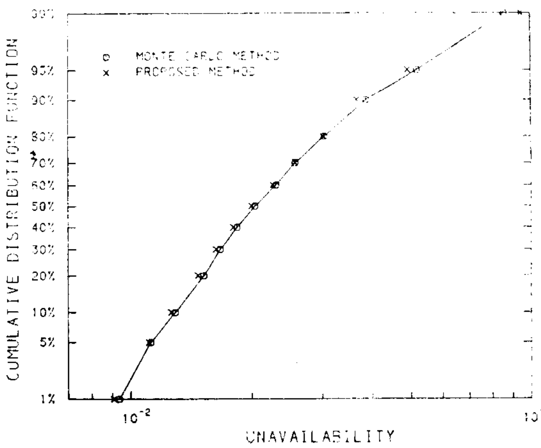


Fig. 6. cdf for Example 3

Example 3: HPIS (High Pressure Injection System)

The top event is "failure of HPIS to deliver sufficient borated water to the reactor coolant system when required". The fault tree and unavailability data are given in Fig. II 5-45 and Table II 5-18 of WASH-1400. The cdf's are shown in Fig. 6. The median values and 95% values of the proposed method and Monte Carlo method are 2.04×10^{-2}, 5.19×10^{-2}, 2.00×10^{-2} and 4.92×10^{-2}, respectively. The present method requires the computation time of less than the quarter of the time for the Monte Carlo method.

5. Conclusion

The method and computer code are developed in order to analyze the uncertainty in the top event unavailability by combining Monte Carlo method and Moments method together with fault tree reduction technique. At present, the program is limited to lognormal distribution for probability distributions for basic events. The differences between the results of the proposed method and those obtained by the Monte Carlo method are due to the use of Moments method, but no significant differences are found. The major contribution to the reduction of computation time comes from the use of fault tree reduction technique. The principal advantage of the proposed method is its ability to handle very large system fault trees which would require a great deal of computation time with existing computer codes.

References

1. USNRC, "PRA Procedures Guide," NUREG/CR-2300 (1983)
2. W.G. Hwang et al., "A Computer Code Development for Updating Reliability Data Using Bayes's Theorem and Its Application," *J. Kor. Nucl. Soc.*, 15, 41 (1983)
3. USNRC, "Reactor Safery Study," NUREG-75/014 (1975)
4. Y.T. Lee et al., "LIMIT-A Fast Monte Carlo Code for System Reliability," *ANS Trans.*, 28, 509 (1978)
5. W.G. Hwang et al., "A Study on the reliability and Risk Assessment of Nuclear Power Plants," KAERI/RR-362/82 (1983)
6. Shahid Ahmid, "A Method for Propagating Uncertainty in PRA," *Nucl. Tech.*, 59, 238 (1982)
7. G.E. Apostolakis et al., "Methods for the Estimations of Confidence Bounds for the Top-Event Unavailability of Fault Trees," *Nucl. Eng. Des.*, **41**, 411 (1977)
8. G.E. Apostolakis et al., "Confidence Bounds for the Top Event Unavailability of Fault Trees," *ANS Trans.*, **23**, 224 (1976)
9. M. Mazumdar, "An Approximate Method for Computation of Probability Intervals for the Top-Event Probability of Fault Trees," *Nucl. Eng. Des.*, **71**, 45 (1982)
10. W.E. Vesely et al., "PREP and KITT: Computer Codes for the Automatic Evaluation of a Fault Tree," In-1349 (1970)
11. F.H. Rowsome, "How Finely Faults Be Resolved in Fault Tree Analysis?," *ANS* Trans., **23**, 225 (1976)