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Design of Magnetic Systems for SNUT-79 Tokamak

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SNUT-79 토카막의 자장 계통 설계

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Abstract

A toroidal-field (TF) coil with a pure tension D-shape curve is designed for the confinement of high-temperature plasmas in the SNUT-79, which is a tokamak being built at Seoul National University. A toroidal assembly of 16 D-shape TF coils is designed to produce the magnetic field of up to 3T, of which ripples appear to be below 4% of the average toroidal field in the plasma region.

Exact positions and currents in six equilibrium coils distributed symmetrically in the $z=0$ plane are found by the solution of a set of linear equations which is transformed from a Fredholm integral equation of the first kind. The decay indices resulted from equilibrium field indicate that the stability condition for vertical and horizontal displacements is satisfied.

요 약

현재 서울대학교 원자핵공학과에서 제작중인 SNUT-79 토카막 장치에서의 고온 플라즈마의 구속을 위해서 순인장력 D형 곡선을 가진 토로이달 자장 코일을 수치 해석적 방법으로 설계하였다. 16개의 D형 토로이달 코일 뭉치는 플라즈마가 없는 경우 자장의 세기가 3T가 되도록 설계하였다. 토로이달 리플은 플라즈마영역에서 평균 토로이달 자장의 4%이하이다.

6개로 된 평형 코일의 위치와 전류 값을 Fredholm 제1종 적분 방정식을 선형 방정식으로 변환하여 얻었다. 평형 자장의 곡률도는 플라즈마 루프의 수직 수평 방향의 변위에 대한 안정화 조건을 만족시켰다.

1. Introduction

The SNUT-79 Tokamak in the Department of Nuclear Engineering at Seoul National University is scheduled to be completed for ohmic heating experiments in late 1984, which will be the first fusion device built in Korea. To date a toroidal vacuum vessel has been completed as a main frame of the plasma container. Presently, all efforts are concentrated on the completion of power and magnetic field systems for operating the tokamak. In the SNUT-79 Tokamak, properly-designed magnetic systems are firstly needed for the confinement of high-temperature plasmas and the equilibrium of the plasma column.

In fact, the design consideration for toroidal-field coils turns out to be dominated by the shape of coils that are subject to the large stresses due to Lorentz forces produced by coil currents and toroidal magnetic fields. Stresses placed on the toroidal-field coil cause the deformation of the toroidal-field coil. In order to minimize the tension of the coil material, a non-circular cross section of D-shape is employed in this work. The tension of the D-shape coil is uniform and less than that of a circular coil. Although more amount of material is needed for the D-shape coil than that for the circular coil, the supporter for preventing vertical deformation of the coil is not required for the D-shape coil. The Lorentz forces produced by the interaction between the poloidal current flowing in the toroidal-field coil and the magnetic field generated by the current result in the tension and the radial force exerted on the coil. The tension of coil material and the net radial force on the toroidal coil assembly can be determined by the calculation of Lorentz forces.

A toroidal plasma column tends to extend to the radial direction because the magnetic pressure in the inner region is greater than that in the outer region. In order to prevent the radial extension the equilibrium magnetic field is provided by vertical coils located external to the plasma. Details of the arrangement of equilibrium coils producing the equilibrium magnetic field are to be determined by solving the Fredholm integral equation of the first kind, which is complicated to solve by the analytical approach. Difficulties in finding solution to the integral equation can be overcome by transformation of the integral equation into a linear one which is easily dealt with by the numerical approach.

2. Toroidal-Field Coils

2.1. Design Basis

Toroidal-field coils produce the confinement field, which prevents the plasma column from contacting the wall of vacuum vessel and dropping the temperature of plasma. The high-temperature plasma is thus maintained in the torus during the fusion reaction. The basis to be considered for fabricating toroidal field coils for SNUT-79 Tokamak is as follows;

1) The strength of a toroidal magnetic field at the minor axis is 3 T in the absence of the plasma column.

2) The current density flowing in the coil conductor is limited to about 7×10^7 A/m^2 because of heating of coil conductor [1].

3) The tension on the coil produced by Lorentz force is less than the tensile stress of coil conductor.

4) Nonaxisymmetric toroidal-field ripples due to the discrete nature of toroidal-field coils have limited values for preventing enhanced plasma diffusion and heat losses by supperbanana orbits of trapped plasma particles.

2.2 D-shape Toroidal-Field Coils

If the shape of toroidal-field coil is circular, Lorentz forces may produce the tension on the coil cross section greater than the tensile stress of the coil material.. Thus, an appropriate shape of coil which produces the minimized tension on it has to be found. Immersion of a coil made of a flexible filamentary conductor in the toroidal field proportional to $1/R$ results in a shape over which the tension on the conductor is uniformly distributed. A coil designed to closely approximate this ideal shape with finite thickness and width has the following inner radii of curvatures of a pure-tension curve[2]:

$$\rho=\begin{cases}\dfrac{1}{d}\left(\dfrac{2\pi a^2b^2T}{\mu_0NI^2}-e\right), & (b\neq 0)\\ \dfrac{4\pi r_iT}{\mu_0NI^2}-\dfrac{a}{3}, & (b=0)\end{cases} \tag{1}$$

where N is a number of toroidal-field coils and I a current flowing in the coil. Constants a, b, c, d, and e are defined by

$$\begin{aligned} a&\equiv r_0-r_i,\\ b&\equiv\cos\Phi,\\ c&\equiv\ln\left(\frac{r_i+ab}{r_i}\right),\\ d&\equiv cr_1-a(1-c)b,\\ e&\equiv\frac{a^2b}{2}+a(1-c)r_i-\frac{c}{b}r_i^2,\end{aligned} \tag{2}$$

and tension T has the form

$$T=\frac{\mu_0NI^2}{8\pi a^2}\Big[(R_m+r_o)\ln\frac{R_m+r_o}{R_m-r_o}-(R_m^2+2r_ir_o-r_i^2)\cdot\ln\frac{R_m+r_i}{R_m-r_i}+2R_mr_o\ \ln\frac{R_m^2-r_o^2}{R_m^2-r_i^2}-2R_m(r_o-r_i)\Big]. \tag{3}$$

Here R_m is the distance from the major axis of the torus to the coil center, r_i a half of the coil bore, r_o a sum of r_i and coil thickness, and Φ an angle between a radius of curvature and a constant z plane as shown in Fig. 1.

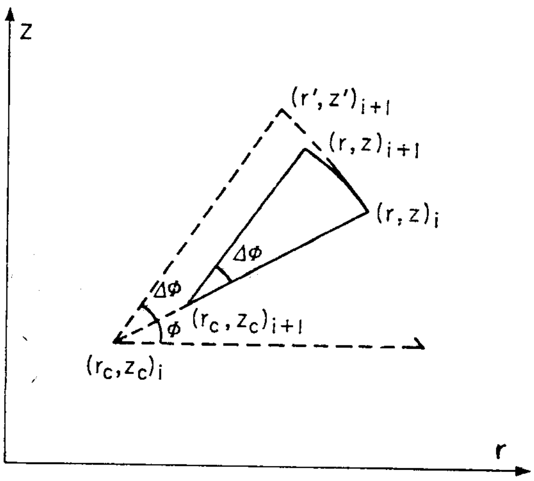

Fig. 1. Two-step Method for Generating a Series of Arcs.

The pure-tension curve is approximated as a series of circular arcs with coincident tangents. An algorithm for generating the pure-tension curve is as in the following. First of all, calculate a radius of curvature from Eq.(1) at some point $(r,z)_i$ on the pure-tension curve, and then locate a center of curvature $(r_c,z_c)_i$. Next, find the location of $(r',z')_{i+1}$ rotated by from $(r,z)_i$. However, $(r',z')_{i+1}$ is not a real point on the pure-tension curve because the increment of angle changes the radius of curvature. After calculating a radius of curvature from Eq.(1) again at $(r',z')_{i+1}$, take the average of the radii of curvatures at $(r,z)_i$ and $(r',z')_{i+1}$. This averaged value is used as a real radius of curvature at $(r,z)_i$, with which a center of curvature $(r_c,z_c)_{i+1}$ is located on the line connecting $(r,z)_i$ with $(r_c,z_c)_i$. Then. obtain the next real point $(r,z)_{i+1}$ on the pure-tension

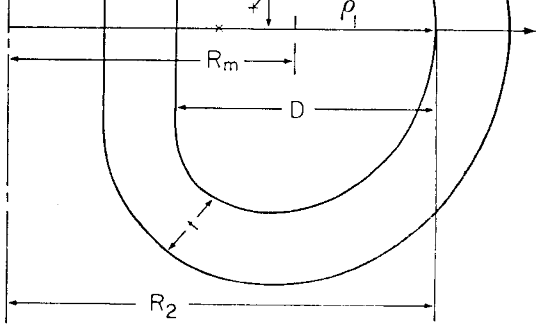

Fig. 2. D-shape Toroidal Field Coil (R_1=0.426m. R_2=0.916m, H=0.36m, R_m=0.671m, D=0.49m, t=0.14m, ρ_1=0.404m, ρ_2=0.392m, ρ_3=0.147m).

curve by rotating $(r, z)_i$ by $\Delta\Phi$. Every points on the approximated pure-tension curve are thus obtained successively by the infinitesimal increment $\Delta\Phi$. Fig. 2 shows a resultant D-shape coil which has been determined as a pure-tension curve by using Eqs. (1)-(3) and this algorithm.

2.3. Distribution of Toroidal Magnetic Field

In order to calculate the toroidal-field distributions, the Biot-Savart law has been used. A computer programming for calculating toroidal-field distributions according to this law has been developed[3].

When 16 D-shape coils depicted in Fig. 2 are placed along the circumference of the torus with equal spacing, and the current of 6×10^5 A is flowing in each coil, Fig. 3 shows the resultant toroidal-field distributions computed by this programming. In Fig. 3, a solid line indicates the magnitudes of toroidal field over the major radius in the cross section plane of the coil, and a broken line shows those in the midplane between adjacent coils.

The maximum magnetic field occurs at the inner surface R_1 of the coil conductor. The field strength is approximately proportional to $1/R$ in the plasma region, and linearly varies in the coil conductor region. The maximum toroidal ripple produced in the plasma region turns out to be less than 4% of the average toroidal field. The ripple at the center of the plasma column is about 0.5%.

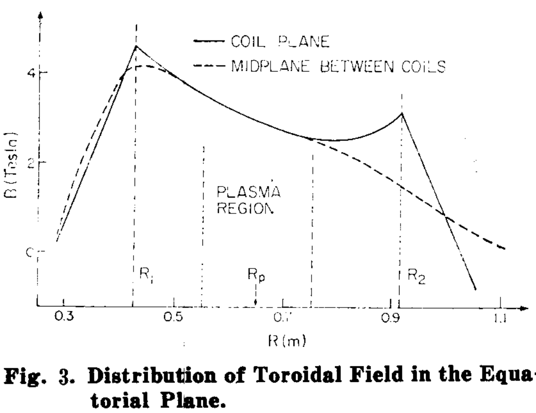

Fig. 3. Distribution of Toroidal Field in the Equatorial Plane.

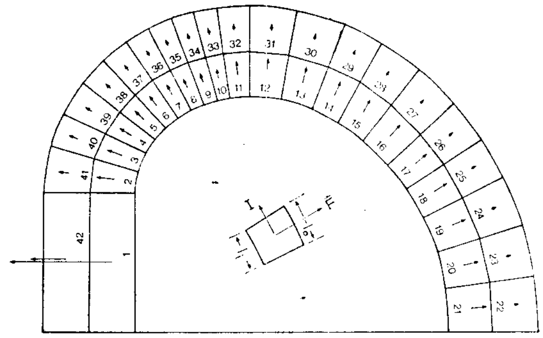

Fig. 4. Electromagnetic Forces Exerted on the Segments of the D-shape Toroidal Coil.

2.4. Stress on Toroidal-Field Coils

In order to calculate the stress, the upper half of coil is devided into 42 trapezoidal segments as shown in Fig. 4. The Lorentz force exerting on each trapezoidal segment has the form

$$F=\frac{1}{4}\left(\frac{2l_i+l_0}{3}\right)BI, \tag{4}$$

where l_i and l_0 are an inner and an outer lengths of the trapezoid, respectively. B is a magnetic flux density at the center of each segment, and I is a total current flowing in the coil. Magnitudes of Lorentz force F are illustrated in Fig. 4 as arrows proportional to F. A half sum of vertical components of Lorentz force acts as a tension on the conductor cross section perpendicular to the current flow. When I is 6×10^5 A, the calculation shows that a vertical force of 2.30×10^5 N(23.5 ton·w) is applied on the cross section area of 0.014m^2. Accordingly, the tension per unit area becomes 2,400 psi, which makes no problem in using, as a coil conductor, mild copper with a tensile stress of 28,000 psi. An inward radial force to the torus center by one toroidal-field coil is related to a sum of horizontal components, of which value is 4.92×10^5 N(50.2 ton·w). A vertical supporting cylinder which is contact with the straight vertical sections of 16 toroidal-field coils, is sufficiently strong to resist the inward radial compressive force. When the vertical cylinder of 0.572m in diameter is used as a supporting column, a compressive pressure of 2,900 psi is acted on the supporting cylinder.

3. Equilibrium-Field Coils

3.1. Equilibrium Magnetic Field

A toroidal plasma column tends to extend to radial direction because the magnetic pressure in the inner region is greater than that in the outer region. This extending force is balanced by the force of interaction between the toroidal plasma current and equilibrium magnetic field produced by the external coil. The z-component of external field for equilibrium, i.e., the vertical magnetic field has the form[4],

$$B_v = B_z \approx -\frac{\mu_0 I_p}{4\pi R_p}\left(\ln\frac{8R_p}{a} + \Lambda - \frac{1}{2}\right), \quad (4)$$

where R_p and a are major and minor radii of the plasma column, respectively, and I_p the plasma current. Λ is a coefficient of asymmetry of the poloidal field[5],

$$\Lambda = \beta_p + \frac{1}{2}l_i - 1. \quad (5)$$

where β_p is a poloidal beta and l_i is an internal inductance per unit length of the plasma column.

If the vertical magnetic field is homogeneous, the equilibrium of the plasma column will be broken to the vertical displacement. If the lines of the equilibrium magnetic field are concave, the position of the plasma loop is stable relative to the vertical displacement. The barrel shape of the vertical magnetic field is characterized by the decay index[4, 6],

$$n \equiv -\frac{r}{B_v}\frac{\partial B_v}{\partial r}. \quad (6)$$

The stability condition of the equilibrium position of the plasma loop relative to the vertical displacement along the axis of symmetry can be written as $n > 0$, and the stability condition relative to the horizontal displacement as $n < 1.5$[6].

The determination of the current distribution to produce a given field is very important for the design of equilibrium field coils. One of the method for calculating such current distribution is known as the Zakharov method[7]. But this analytical method for solving a Fredholm equation of the first kind is limited and difficult in its use for a practical design of equilibrium field coils.

First of all, it is necessary to consider the magnetic field $b^*(r, z; r', z')$ which is the magnetic field at the observation point (r, z) in a coil carrying a unit current and located at point (r', z') on the coil boundary l'. The expressions for b^* are well known[8]:

$$b_r^*(r, z; r', z') = \frac{\mu_0}{2\pi}\frac{k}{2\sqrt{rr'}}\left[K(k) + \frac{\{r'^2 - r^2 - (z-z')^2\}k^2}{4rr'}E(k)\right],$$

$$b_z^*(r, z; r'z') = \frac{\mu_0}{2\pi}\frac{k(z-z')}{2r\sqrt{rr'}}\left[-K(k) + \frac{\{r'^2 + r^2 + (z-z')^2\}k^2}{4rr'}E(k)\right] \quad (7)$$

where $k^2 = 4rr'/\{(r+r')^2 + (z-z')^2\}$, and $E(k)$ and $K(k)$ are complete elliptic integrals.

The magnetic field produced by the current distribution $i'(r', z')$ flowing on a coil boundary l' has the form

$$B(r, z) = \int_{l'} b^*(r, z; r', z')\, i(r', z')\, dr'dz', \quad (8)$$

and as a matter of convenience in the toroidal coordinates (ρ, θ, ϕ) using the relations, $r = R_p + \rho\cos\theta$, $z = \rho\sin\theta$, $r' = R_0 + \rho'\cos\theta'$, $z' = \rho'\sin\theta'$ (R_0 is a major radius of the coil),

$$B(\rho, \theta) = \int_{l'} b^*(\rho, \theta; \rho,'\theta')\, i(\rho', \theta')\, J d\rho' d\theta', \quad (9)$$

where J is Jacobian of the transformation.

Here, the expression $B(r, z)$ should be derived for solving this integral equation. In order to obtain the magnetic field $B(r, z)$, the flux function is firstly obtained from the homogeneous equation $L\Psi = 0\left(L = \frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}\right)$. The polynomials r^2, $r^4-4r^2z^2$, $r^6-12r^4z^2+8r^2z^4$, etc., satisfy this homogeneous equation[7]. The homogeneous solution at $r=R_0$ is expressed as

$$\Psi(r,z)=C_0+\frac{C_1}{2R_o^2}(R_o^2-r^2)+C_2\left[\frac{1}{4R_o^4}(R_o^2-r^2)^2+\frac{1}{R_o^4}(R_o^2-r^2)^2z^2-\frac{z^2}{R_o^2}\right], \tag{10}$$

where C_0, C_1 and C_2 are constants. The z-component of the magnetic field which is the equilibrium field wanted can be derived.

$$B_z(r,z)=\frac{1}{r}\frac{\partial\Psi}{\partial r}=-\frac{C_1}{R_o^2}-\frac{C_2}{R_o^4}(R_o^2-r^2+2z^2). \tag{11}$$

The first term with C_1 is a homogeneous vertical field, and the second term with C_2 is a control field which makes a barrel shape to the homogeneous field. The homogeneous field in expression Eq.(4) is equal to the first term of Eq.(11), then the constant C_1 has the form

$$C_1=-\frac{\mu_0 I_p R_o^2}{4\pi R_p}\left(\ln\frac{8R_p}{a}+\beta_p+\frac{1}{2}l_i-1.5\right). \tag{12}$$

The constant C_2 is obtained from the stability condition at $z=0$ plane. Decay index n at $z=0$ plane is

$$n=-\frac{r}{B_z}\frac{\partial B_z}{\partial r}=\frac{2\alpha(r/R_o)^2}{1+\alpha\{1-(r/R_0)^2\}}, \tag{13}$$

where $\alpha=C_2/C_1$. Once the proper value of α is chosen from the stability condition $0<n<1.5$ in the plasma column, the constant C_2 is, then, obtained from this relationship.

3.2. Arrangement of Equilibrium Field Coils

The following integral equation has to be solved to obtain the current distribution, which flows in the arbitrary coil boundary,

$$B_\tau(\rho,\theta)=\int_{l'} b_\tau^*(\rho,\theta;\rho',\theta')i(\rho',\theta')Jd\rho'd\theta', \tag{14}$$

where

$$b_\tau^*(\rho,\theta;\rho',\theta')=\vec{b}^*(\rho,\theta;\rho',\theta')\cdot(\hat{n}\times\hat{\phi}). \tag{15}$$

This equation is the Fredholm integral equation of the first kind. The solution of this equation is not unique. Thus, the solution of the superconducting current i_s, which does not create a magnetic field inside the coil boundary l' must be added. This superconducting current ohmically heats the plasma column. This problem does not discuss in the present work.

When positions of equilibrium coils are (ρ_1,θ_1), (ρ_2,θ_2), $\cdots$, (ρ_N,θ_N), where N is the number of equilibrium coils, the current function $i(\rho,\theta)$ is expressed as

$$i(\rho,\theta)=\frac{I_1}{\rho_1}\delta(\rho-\rho_1)\delta(\theta-\theta_1)+\frac{I_2}{\rho_2}\delta(\rho-\rho_2)\delta(\theta-\theta_2)+\cdots+\frac{I_i}{\rho_i}\delta(\rho-\rho_i)\delta(\theta-\theta_i)+\cdots+\frac{I_N}{\rho_N}\delta(\rho-\rho_N)\delta(\theta-\theta_N), \tag{16}$$

where I_i is the current flowing in the ith equilibrium coil at the position (ρ_i,θ_i). Therefore, the integral equation can be easily transformed to a set of linear equations

$$\begin{aligned}
&a_{11}x_1+a_{12}x_2+\cdots\cdots+a_{1N}x_N=b_1,\\
&a_{21}x_1+a_{22}x_2+\cdots\cdots+a_{2N}x_N=b_2,\\
&\quad\vdots\qquad\vdots\qquad\qquad\vdots\qquad\vdots\\
&a_{i1}x_1+\cdots\cdots+a_{ij}x_j+\cdots\cdots+a_{iN}x_N=b_i,\\
&\quad\vdots\qquad\qquad\vdots\qquad\qquad\vdots\qquad\vdots\\
&a_{m1}x_1+a_{m2}x_2+\cdots\cdots\cdots+a_{mN}x_N=b_m,
\end{aligned} \tag{17}$$

where $a_{ij}=b_\tau^*(\rho_i,\theta_i;\rho_j',\theta_j')\rho_j'$ and $b_i=B_\tau(\rho_i,\theta_i)$, $x_j=i(\rho_j',\theta_j')$. m is the number of equations, which is greater than or equal to N. These linear equations are written as following,

$$AX=b. \tag{18}$$

Here, the matrix A is an overdetermined matrix, since the system has more equations than unknowns. In such a case, the equations are satisfied only approximately. An important method for the treatment of the overdetermined linear equation system is the method of least-squares [9]. A least-squares solution to the overdetermined system of equations $AX=b$ is a vector X which minimizes the Euclidean length of the residual vector r, i.e., minimizes

$$\|r\|_2=(r^Tr)^{1/2},\quad r=b-AX, \tag{19}$$

where a superscript T designates the transpose of r. The least-squares solution satisfies the normal equation

$$(A^TA)X = A^Tb. \qquad (20)$$

In order to obtain solution of Eq. (18), Eq. (20) has to be solved.

For the SNUT-79 Tokamak which is designed with a toroidal magnetic field $B_T=3T$, a safety factor $q=3.5$, a poloidal $\beta_p=1$, an inductance of plasma loop per unit length $l_i=1$, a major radius $R_p=0.65$m and a minor radius $a=0.1$m of the plasma loop, and the maximum plasma current $I_p=-65.9$KA, a homogeneous magnetic field B_v of 400 Gauss is needed by Eq.(4). In this case, $C_1=-0.016934$, and if $\alpha=0.4$ is chosen from the stability condition $0<n<1.5$ by using Eg.(13), then C_2 becomes -0.006774.

Numerical computation for the solutions of Eq.(17) results in the arrangement of six equilibrium coils as shown in Fig. 5, and spatial position and current flowing in each coil are listed in Table 1. The resultant arrangement of equilibrium coils produces the distribution

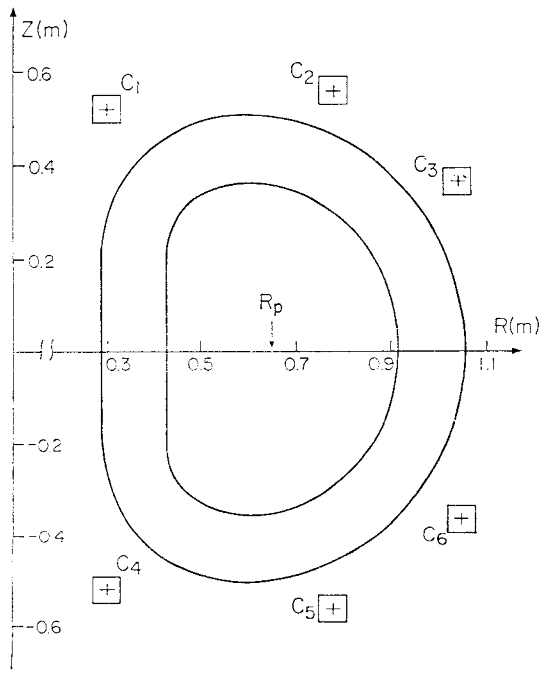

Fig. 5. Arrangement of Equilibrium Coils

Table 1. Positions and Currents of Equilibrium Coils.

Coil number	Position(r,z)	Current(Ampere)
1	(0.30, 0.52)	3.56×10^4
2	(0.78, 0.56)	1.29×10^5
3	(1.04, 0.34)	-8.69×10^4
4	(0.30, −0.52)	3.56×10^4
5	(0.78, −0.56)	1.29×10^5
6	(1.04, −0.34)	-8.69×10^4

of equilibrium magnetic field at $z=0$ plane, which are plotted in Fig. 6, The equilibrium magnetic field produced by equilibrium coils corresponds to desired equilibrium field. Fig. 7 shows decay indices along the radial direction at $z=0$ plane. The maximum value of decay index in the plasma region is 1.2, which occurs at $r=0.75$m. In the plasma region the stability condition for the vertical and horizontal displacements is then satisfied.

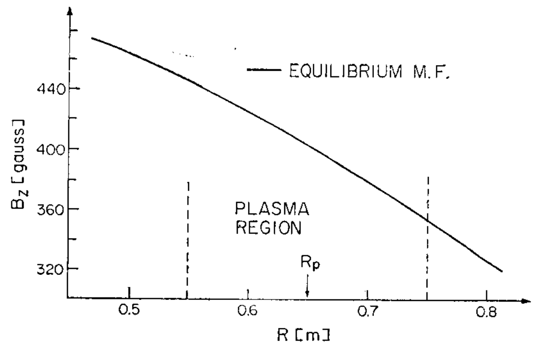

Fig. 6. Distribution of the Equilibrium Field at z=0 Plane.

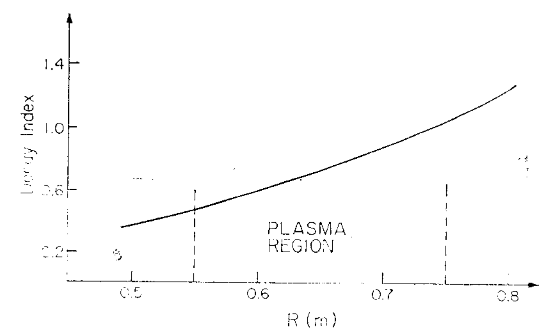

Fig. 7. Decay Index along the Radial Direction at z=0 Plane.

4. Conclusion

A toroidal-field coil with pure tension D-shape curve is designed by the numerical method. We obtain the D-shape coil of which dimensions are D=0.49m. 2H=0.72m, t=0.14m, and w=0.1m.

Magnetic field distributions generated by an assembly of 16 designed coils are obtained by the computer programming for calculating the Biot-Savart law. The field distribution is approximately proportional to $1/R$ in the plasma region. It agrees well with the theoretical variation in the plasma region. When the current flowing in each coil is 6×10^5 A, the toroidal magnetic field strength in the absence of the plasma column reaches up to 3 T. The problem of cooling of coil material is not severe since the temperature rise of coil material is maintained below 6°C relative to the surrounding temperature. The magnetic fiield varies linearly in the coil conductor. The toroidal-field ripple at the center of the plasma column is about 0.5% of the average toroidal field, and the maximum ripple occuring at the outer boundary of the plasma region is about 4%. This value is below the limited values, for which 1% and 5% are usually used at the centers of small and large tokamaks, respectively.

In order to calculate the stress distributions over the coil cross section, the inward radial force to the torus center, and the tension along D-shape curve, the coil cross section is devided into 42 trapezoidal segments. When the current of each coil as 6×10^5 A, a tension applied on the transverse cross section is 2,400 psi, and the radial force is 50.2 ton·w. If mild copper is used for coil material, there are no material problems to withstand these electromagnetic forces.

From the MHD equilibrium theory the homogeneous vertical magnetic field of 400 Gauss is needed for preventing plasmas from outward drifts. The arrangement of equilibrium coils can be obtained from solving the Fredholm integral equation of the first kind. In order to solve the integral equation, the integral equation is transformed to overdetermined linear equations. The arrangement of six equilibrium coils with their exact positions and currents is obtained from the solution of the linear equations. Since the values of decay indices vary from 0.47 to 1.2 in the plasma region, the stability condition for the vertical and horizontal displacements appears to be satisfied.

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