Journal of the Korean Nuclear Society
Volume 20, Number 2, June, 1988

Approximations for Array of Point Sources in Groundwater Contaminant Transport Modeling

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(Received May 10, 1988)

지하수 오염물질 이동모형에 있어서 배열된 점원의 근사방법 연구

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(1988. 5. 10 접수)

Abstract

A strategic question in groundwater contaminant transport modeling is whether we need to treat waste packages or drums as individual, discrete sources or as approximately lumped sources. In this paper we present analyses of array sources in porous media.

We analyze a planar array of sources in porous media with groundwater flow. We compare the concentration field predicted by a detailed model of individual point sources to concentration fields predicted by an infinite plane source and a single point source, all of the same equivalent strength.

From this study we identified three regions: (1) a region close to the sources where the effects of adjacent sources are significant and individual source models should be used, (2) a region extending from a few meters to hundreds to thousands of meters downstream, where an equivalent source of infinite extent gives accurate results, and (3) a far-field region, wherein an equivalent source of finite extent gives accurate results.

요　　약

지하수 오염물질의 이동모형 개발에 있어서 중요한 문제의 하나가 각각의 폐기물 저장용기나 드럼을 독립된 선원으로 볼 것이냐 아니면 상응하는 합쳐진 선원으로 볼 것이냐 하는 문제이다. 본 논문에서는 포화된 다공성 매질에서의 분산된 선원에 대한 연구를 소개한다.

지하수의 유동이 있는 다공성 매질에서 평면상에 분산된 점 형태의 선원들을 가정하여 얻어진 지하수내의 오염물질의 농도분포를 상응하는 크기를 가진 무한한 평면선원이나 하나의 점 선원을 농도분포들과 비교하였다.

본 연구의 결과로 다음의 세 영역을 찾아낼 수 있었다. 즉, (1) 선원에 가까운 관계로 인근 선원의 영향이 커서 각각의 선원들을 따로 고려한 모형을 사용하여야 하는 영역, (2) 선원에서 몇 미터 떨어진 지점으로부터 수백 내지 수천미터 떨어진 지점까지의 각각의 선원 대신 상응하는 무한한 평면 선원이 정확한 결과를 얻게하는 영역, (3) 선원으로부터 충분히 멀리 떨어져 있어서 상응하는 유한한 평면 선원만이 정확한 결과를 얻게하는 영역이다.

1. Introduction

Nuclear waste in geologic repositories and hazardous materials in disposal sites will be emplaced in thousands of individual containers. In evaluating contaminant transport from such facilities, one would like to know whether it is necessary to consider each individual waste source? In this paper we compare the space-time-dependent concentration field predicted for an array of discrete sources with the concentration field predicted for an infinite plane source. We also compare the concentration field predicted for an array of discrete sources with the concentration field predicted for an equivalent single point source. We develop solutions for an array source by superpositioning solutions for individual point sources.

2. Analytic Solutions-Single Point Source

For a single point source in an infinite porous medium, the governing equation for the hydrodynamic dispersion of a radioactive species is

$$K\frac{\partial C}{\partial t} + \check{u}_1 \frac{\partial C}{\partial x_1} + \check{u}_2 \frac{\partial C}{\partial x_2} + \check{u}_3 \frac{\partial C}{\partial x_3} = \frac{\partial}{\partial x_1}\check{D}_1\left(\frac{\partial C}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\check{D}_2\left(\frac{\partial C}{\partial x_2}\right) + \frac{\partial}{\partial x_3}\check{D}_3\left(\frac{\partial C}{\partial x_3}\right) - \lambda KC$$

where $x_i \in D_\infty$, $i = 1,2,3$; $t>0$ (1)

where the Cartesian coordinates were labeled x_1, x_2, x_3, $C(x_1,x_2,x_3,t)$ is the species concentration in the groundwater, $\check{D}_i$ is the dispersion coefficient in the i-th direction, K is the retardation coefficient of the contaminant, and D_∞ is the unbounded space. The strength of the nuclide source, located at the origin of the axes is $M(\tau)d\tau$ and measures the mass of material released at time τ during the time span $d\tau$. The release gives rise to the concentration $C(x_1,x_2,x_3,t)$ at position x_1,x_2,x_3 at time $t > \tau$. This concentration is initially zero throughout D_∞ and satisfies suitable boundedness condition at an infinite distance from the source position. Some solutions to this problem, without limit on the velocity field $\check{u}_1$, $\check{u}_2$, $\check{u}_3$, and for radioactive decay chains were obtained by P.L. Chambré *et al.*[1]

The solution for an uniform flow along the x_1 axis with a pore velocity of u is

$$C(x_1,x_2,x_3,t) = \int_0^t \frac{M(\tau)\exp\{-\lambda(t-\tau)\}}{\epsilon K\{4\pi D(t-\tau)\}^{3/2}} \exp\left\{\frac{-1}{4(t-\tau)}\left[\frac{(x_1-u(t-\tau))^2}{D_1} + \frac{x_2^2}{D_2} + \frac{x_3^2}{D_3}\right]\right\} d\tau \qquad (2)$$

where $M(\tau)$ = Radioactive species input rate at time τ [M/T],

D_i = Normalized dispersion coefficient in the x_i direction,

$D_i = \check{D}_i/K$ [L^2/T],

u = Normalized groundwater pore velocity in the x_1 direction,

$u = \check{u}_1/K$ [L/T],

λ = Radioactive decay constant [T^{-1}],

ϵ = Porosity of the medium,

and $D = (D_1 \cdot D_2 \cdot D_3)^{1/3}$.

The dispersion coefficients and the groundwater velocity are normalized by dividing them by the retardation coefficient of the contaminant.

If the rate of waste species input is temporally constant, the solution in eq.(2) can be simplified by the substitution of a constant input rate M, and integrated to give

$$C_{pt}(x_1,x_2,x_3,t) = \frac{Me^{x_1u/2D_1}}{8\pi\epsilon D^{3/2}K(x_1^2/D_1+x_2^2/D_2+x_3^2/D_3)^{1/2}} \cdot \left[\exp\left(-\sqrt{\left(\frac{x_1^2}{D_1}+\frac{x_2^2}{D_2}+\frac{x_3^2}{D_3}\right)\left(\lambda+\frac{u^2}{4D_1}\right)}\right)\right.$$

$$\cdot\left\{2-\mathrm{erfc}\left(\sqrt{(\lambda+\frac{u^2}{4D_1})t}-\sqrt{(\frac{x_1^2}{D_1}+\frac{x_2^2}{D_2}+\frac{x_3^2}{D_3})\cdot\frac{1}{4t}}\right)\right\}$$

$$+\exp\sqrt{(\frac{x_1^2}{D_1}+\frac{x_2^2}{D_2}+\frac{x_3^2}{D_3})(\lambda+\frac{u^2}{4D_1})}\cdot\mathrm{erfc}\left(\sqrt{(\lambda+\frac{u^2}{4D_1})t}+\sqrt{(\frac{x_1^2}{D_1}+\frac{x_2^2}{D_2}+\frac{x_3^2}{D_3})\cdot\frac{1}{4t}}\right)\Bigg\} \quad (3)$$

3. Analytic Solutions-Array Sources

For an array of WxYxZ point sources, the contaminant concentration field resulting from the array is given by superpositioning solutions for individual point sources:

$$C^a(x_1,x_2,x_3,t)=\frac{M}{8\pi\ \varepsilon KD^{3/2}}\sum_{w=1}\sum_{y=1}\sum_{z=1}\frac{\exp((x_1-x_1^w)u/2D_1)}{\left[\frac{(x_1-x_1^w)^2}{D_1}+\frac{(x_2-x_2^y)^2}{D_2}+\frac{(x^3-x_3^z)^2}{D_3}\right]^{1/2}}$$

$$\cdot\left[\exp\left(-\sqrt{\left(\frac{(x_1-x_1^w)^2}{D_1}+\frac{(x_2-x_2^y)^2}{D_2}+\frac{(x_3-x_3^z)^2}{D_3}\right)}\sqrt{\lambda+\frac{u^2}{4D_1}}\right)\cdot\left\{2-\mathrm{erfc}\left(\sqrt{\lambda+\frac{u^2}{4D_1})t}-\sqrt{\left(\frac{(x_1-x_1^w)^2}{D_1}+\frac{(x_2-x_2^y)^2}{D_2}+\frac{(x_3-x_3^z)^2}{D_3}\right)\frac{1}{4t}}\right)\right\}\right.$$

$$+\exp\left(\sqrt{\left(\frac{(x_1-x_1^w)^2}{D_1}+\frac{(x_2-x_2^y)^2}{D_2}+\frac{(x_3-x_3^z)^2}{D_3}\right)}\sqrt{\lambda+\frac{u^2}{4D_1}}\right)\cdot\mathrm{erfc}\left(\sqrt{(\lambda+\frac{u^2}{4D_1})t}\left.+\sqrt{\left(\frac{(x_1-x_1^w)^2}{D_1}+\frac{(x_2-x_2^y)^2}{D_2}+\frac{(x_3-x_3^z)^2}{D_3}\right)\frac{1}{4t}}\right)\right] \quad (4)$$

where C^a = concentration from the array source. The location of the individual point sources in the array is given by

$$x_1^w=d_1(w-\frac{W+1}{2}),\ w=1,\ldots,W$$

$$x_2^y=d_2(y-\frac{Y+1}{2}),\ y=1,\ldots,Y$$

$$x_3^z=d_3(z-\frac{Z+1}{2}),\ z=1,\ldots,Z$$

where d_i are the separation or pitches along the i-th coordinate.

4. Analytic Solution-Infinite Plane Source

For an infinite plane source the governing equation for the dispersion of a radioactive species is

$$\frac{D_1}{K}\frac{\partial^2 C_p(x_1,t)}{\partial x_1^2}-\frac{u}{K}\frac{\partial C_p(x_1,t)}{\partial x_1}-\frac{\partial C_p(x_1,t)}{\partial t}-\lambda C_p(x_1,t)=-\frac{M(t)}{\varepsilon KA}\delta(x_1) \quad (5)$$

The solution for the concentration field is given by[2,3]

$$C_p(x_1,t)=\int_0^t\frac{M(t-\tau)}{\varepsilon KA}\frac{e^{-\lambda\tau}}{\sqrt{4\pi D_1\tau}}\exp(-(x_1-u\tau)^2/4D_1\tau)d\tau \quad (6)$$

where $A=d_2d_3$.

5. Numerical Illustrations

We present numerical illustrations from a 3x3 planar array of point sources, perpendicular to the flow of groundwater, as shown in Figure 1. In these calculations we assumed

—Each point source is of equal and time-invarient strength.

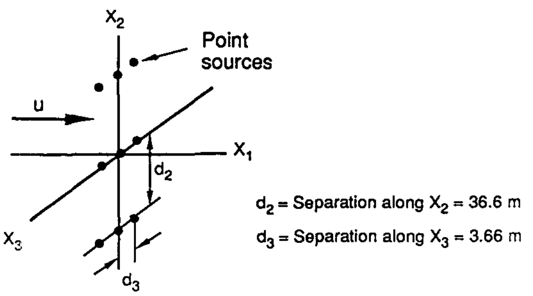

Fig. 1. Array of Point Sources in Porous Media

—The center of the array is at the origin of the axes.

—The constant source strength was derived from a separate model based on solubility-limited dissolution of the contaminant from the source.[2]

—Groundwater velocity is along x_1 axis only and is constant.

—The dispersion coefficients are constant in space and time. This is not a requirement of the solutions. Actually, in the solutions the dispersion coefficients can be functions of velocity, although not of position.

The separations between sources have been chosen arbitrarily[4] as

d_2 = 36.6 meters

d_3 = 3.66 meters

The other parameter values used in the calculation are[1]

D = 50m^2/yr or 5 m^2/yr, as stated

u = 1 m/yr

M = 0.48 g/yr

λ = 0

K = 1

ε = 0.05

Figure 2 shows a contaminant plume from the array source of Figure 1, after local steady state has been reached. This is of the order of a thousand years after the start of contaminant dispersion. In this case the transverse dispersion coefficients are one-tenth the longitudinal dispersion coefficient. It can be seen that the plumes from all nine point sources have merged into an overall plume, and that this plume has moved downstream.

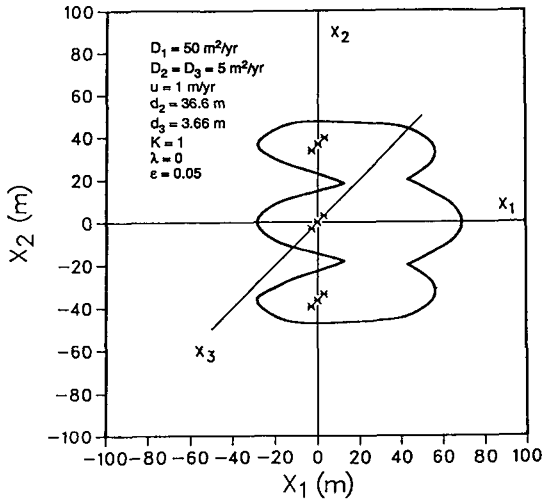

Fig. 2. Isopleth of a Contaminant at Concentration 0.01 g/m^3 for Steady-state, Anisotropic Dispersion for the Array of Point Sources in Figure 1.

The important question is when simpler mathematical models of array sources can give equally valid predictions? Figure 3 shows the comparison between two sets of predictions. The ordinate is the steady-state contaminant concentration along the x_1 axis predicted for the discrete array sources, normalized to the concentration predicted for an infinite plane source of the same areal dissolution rate, plotted as a function of a distance parameter $\theta = \sqrt{x_1 D_3/u}$. When the ratio is unity, the two models give identical predictions. For a value of the distance parameter of approximately ten meters, the ratio is above unity and the detailed array-source model should be used. Beyond this region, the infinite plane source model either gives identical predictions, or it overestimates conservatively. Where the concentration ratio becomes less than unity with increasing θ, the concentration field can be accurately predicted by replacing the discrete-source array by an equivalent finite plane source.

We next compare the predictions of the detailed array source model with a single point at the origin. This single point source has the strength of all the point sources in the array combined. The re-

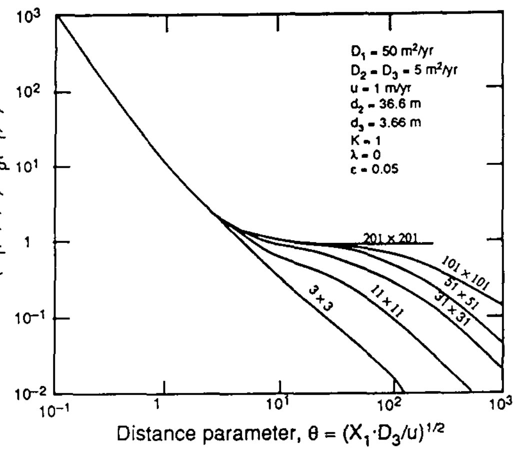

Fig. 3. Comparison of Concentration from Array Model with Concentration from Infinite-plane Model(at Steady State, Along X_1 Axis, Anisotropic Dispersion)

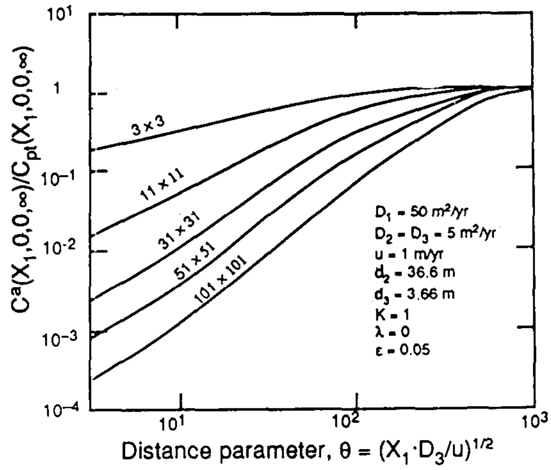

Fig. 4. Comparison of Concentration from Array Model with Concentration from a Single Equivalent Point Source (at Steady State, Along X_1 Axis)

sults are shown in Figure 4. Close to the center of the array plane and thus near the single equivalent point source, the strength of the single point source overwhelms the predictions for the array of sources. At a distance parameter of about 100 meters and greater, transverse dispersion has reduced the prediction for the single equivalent point source to that for the array of sources. For the values of velocity and dispersion coefficients used, a distance parameter of 100 corresponds to a downstream distance of 2,000 meters. Thus, for predicting contaminant concentrations at large distances, a single point source can replace the array of sources.

6. Conclusion

Analytic solutions for contaminant dispersion in groundwater from an array of point sources in a porous medium are given and illustrated. The numerical illustrations indicate that for distances tens of meters downstream from the sources simpler plane-source models might give equally satisfactory predictions.

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