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Analysis of Anisotropic Turbulent Heat Transfer in Nuclear Fuel Bundles

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핵연료 집합체내의 비등방성 난류 열전달에 관한 해석적 연구

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Abstract

The prediction of clad surface temperatures is important to the design and the safety anlaysis of nuclear reactor. The accurate prediction requires the detailed knowledge of the flow structure and heat transfer, which is complicate due to anisotropic turbulent phenomena. A two-equation model including anisotropic eddy viscosity model is applied to forecast the velocity distribution. And the temperature field is calculated with uniform wall heat flux. The Galerkin's weighted residual finite element method has been used to calculate the turbulent quantities right up to the wall. The numerical results show good agreement with available data and that turbulence anisotropy strongly affects on the mean flow and thus the temperature field. And Nu-P/D correlation is established for sodium coolant in close-packed equilateral triangular bundle in the P/D range of 1.05 to 1.30.

요 약

원자로의 설계나 안전성 분석을 위해서는 핵연료 집합체 내의 유동 구조와 열전달에 대한 지식이 매우 중요하다. 따라서 핵연료 집합체 내의 유체 온도 분포를 정확히 계산하기 위해서는 냉각재 유로 내에서의 속도분포를 정확히 알아야 하는데 이것은 복잡한 난류 현상 때문에 예측하기가 매우 어렵다. 본 연구는 비등방성을 고려한 2-방정식 모형을 사용하여 속도분포를 구하고 핵연료 표면에서의 균일열속을 가정하므로써 유로내에서의 속도 분포를 예측하였다. 수치해는 Galerkin유한 요소법에 의해 핵연료봉 표면까지 구하여졌다. 수치 결과는 알려진 실험치 및 계산치와 비교되어 잘 일치하고 있고, 또한 난류 비등방성이 유로 내의 평균 속도와 온도분포에 영향을 미치고 있음을 보았다. 그리고 조밀한 삼각 배열 핵연료 집합체(P/D=1.05-1.3) 내에서 나트륨 냉각재를 사용한 경우의 Nu-P/D 관계식을 수립하였다.

1. Introduction

It is very important to predict the clad surface temperature for the design and the safety analysis of nuclear reactor. The accurate claculation of the temperature field in subchannels bases on the precise analysis of the flow and heat transfer, which is difficult to describe quantitatively due to the complexity of the turbulence phenomena and the flow geometries. And the anisotropic turbulent phenomena greatly influence the flow and heat transfer structure in the complex geometry such as nuclear subchannels.

To analyze such a complex turbulence phenomena, the two-equation model is adopted as turbulence model, which is known to be k- ϵ model with turbulent kinetic energy equation and turbulent energy dissipation rate equation. And the results of two-equation model are compared with those of one-equation model which uses only the turbulent kinetic energy equation. The effect of anisotropy can be considered by introducing different length scales for the eddy viscosities normal and parallel to the rod surface for the purpose of predicting hydro-dynamic behaviors accurately.

The purpose of this work is to predict the velocity and the temperature profiles for steady, fully developed turbulent flow in subchannels of bare rod bundle. The secondary flow is neglected. At rod surface, the constant heat flux boundary condition is used, which is more reasonable than the constant temperature assumption. To solve the nonlinear governing equations, the finite element method has been introduced using the Galerkin's weighted residual method (WRM). This numerical technique is known to be suitable to provide an accurate description of such complex geometries as the nuclear fuel bundle. The numerical results are compared with available experimental data and shown to be in good agreement. Finally, with this model, the Nusselt numbers have been calculated according to the various P/D ratios in close-packed equilateral triangular array having the Na coolant. The Nu-P/D correlation is constructed in the P/D range of 1.05 to 1.3.

2. Modeling

2.1. Turbulence model

In order to model the turbulent phenomena, various models have been suggested.[1],[2] Among them, the mixing length theory has the fatal shortcoming that the characteristic velocity is zero whenever the velocity gradient is zero. Thus, the mixing length model is not compatible with accurate calculations of the complex turbulent flow field. In one-equation model,[3] Reynolds stress can be expressed in terms of axial velocity gradient and eddy viscosity which is based on turbulent kinetic energy by Kolmogorov-Prandtl turbulent kinetic energy hypothesis. In this model, local turbulent phenomeon is dependent on length scale and turbulent kinetic energy due to fluctuations of the turbulent flow. Thus the transport equation for the turbulent kinetic energy equation constructs the one-equation where the length scale can be obtained algebraically. In practical point of view, the length scale can be determined more generally by transport equation than by algebraic equation. Thus, the second equation, i.e. turbulent energy dissipation rate equation, of two-equation model[4] is formulated to determine the dissipation length scale in the turbulent kinetic energy equation.

In this study, two-equation model is applied through the boundary layer right up to the wall. The inclusion of the wall region in the vicinity of walls is made because of the following reasons;[3]

1) The no slip boundary condtion can be applied to solid walls, which is exact.

2) Due to anisotropy, the flow might become more complex, even in the vicinity of the wall.

3) The wall shear stress is directly computed from the turbulent velocity predictions.

4) Extension of the finite element mesh to discretize the wall surface region itself can be possible to take account of the heat transport by conduction in cases of temperature predictions.

Eddy viscosities are very important parameters to describe the turbulent phenomena in the flow domain of such a complex geometry as nuclear fuel bundle.[5] Even though the isotropic eddy viscosity model is widely used, experiments have shown that the effects of the anisotropic eddy viscosities play an important role in the flow and heat transfer mechanism. In this study, the effect of anisotropy is introduted into the computational procedure by regarding it as a function of the distance to the wal! and the direction to the wall.

According to Wolfshtein's description[2] for the length scale, l_μ and l_d are expressed as,

$$l_\mu = x_l\ [1 - \exp(-A_\mu R)] \tag{1}$$

and

$$l_d = x_l [1 - \exp(-A_d R)] \tag{2}$$

wher A_μ and A_d are constants and x_l is the distance from the wall. R is local turbulent Reynolds number which is defined as,

$$R = \rho x_l k^{1/2} \mu \tag{3}$$

Slagter[3] has proposed anisotropic length scale model which depended on locus as well as direction from the wall and was deduced from the experimental correlation of Carajilescov-Todreas.[5] The length scale of turbulent viscosity nomral to the wall:

$$l_{\mu,1} = \begin{cases} x_l[1-\exp(-A_\mu R0] & \text{for } x_l < 0.25\hat{x}_l \\ \hat{x}_l\ \{0.25 + 0.066\sin[(\pi/0.55)(x_l/\hat{x}_l) - 0.25)]\} & \text{otherwise} \end{cases} \tag{4}$$

and for that parallel to the wall,

$$l_{\mu,2} = x_l[1 - \exp(-A_\mu R)] \tag{5}$$

where $\hat{x}_l$ is the profile length denoting the normal distance from the wall to the position of the maximum velocity.

To evaluate the anisotropic eddy viscosities, the eddy viscosity in the direction normal to the wall was computed as,

$$\mu_{11} = C_1 \rho k^{1/2} l_{\mu,1} \tag{6}$$

and for that parallel to the wall,

$$\mu_{22} = C_2 \rho k^{1/2} l_{\mu,2} \tag{7}$$

where C_1 and C_2 are empirical constants.

2.2. Governing equations

The governing equations are constructed for steady state, fully developed turbulent flow of an incompressible fluid with constant properties. And the secondary flow is neglected. The Reynolds stress is stated in terms of anisotropic eddy viscosities.

The time averaged momentum equation can be written as,

$$\frac{\partial}{\partial x_i}\left(\mu \frac{\partial U_3}{\partial x_i} - \rho \overline{u_i u_3}\right) - \frac{\partial \bar{p}}{\partial x_3} = 0 \tag{8}$$

where the usual summation convention is used. $\partial \bar{p}/\partial x_3$ is independent of x_1 and x_2, and a constant in the cross-sectional area. The quantities $\overline{\rho\, \overline{u_i u_3}}$ are Reynolds stresses and can be represented $\overline{\text{as}}$

$$-\rho \overline{u_i u_3} = \mu_{ij} \frac{\partial U_3}{\partial x_i} \tag{9}$$

where μ_{ij} is anisotropic eddy viscosity. Since the main axes of anisotropy are perpendicular and parallel to the wall, the quantities μ_{ij} are zero if $i \neq j$.

Substituting Eq. (9) into Eq. (8) yields,

$$\frac{\partial}{\partial x_i}\left[(\mu \delta_{ij} + \mu_{ij}) \frac{\partial U_3}{\partial x_j}\right] - \frac{\partial \bar{p}}{\partial x_3} = 0 \tag{10}$$

To evaluate the eddy viscosity, the Kolomogorov-Prandtl turbulent energy hypothesis[2] is introduced. In this hypothesis, the eddy viscosity is related to the local values of the turbulence length scale l_μ and the turbulent kinetic energy k by the formula

$$\mu = C \rho k^{1/2} l_\mu \tag{11}$$

where C is constant.

The time-averaged turbulent kinetic energy is defined as,

$$k = u_i u_j / 2 \tag{12}$$

From the transport equation of turbulent kinetic energy for fully developed turbulent flow, the turbulent kinetic energy equation becomes

$$\frac{\partial}{\partial x_i}\left[(\mu \delta_{ij} + \mu_{ij}/\sigma_k) \frac{\partial k}{\partial x_j}\right] - C_d \rho k^{3/2}/l_d$$

$$-\overline{u_i u_3}\frac{\partial U_3}{\partial x_i}=0 \tag{13}$$

where σ_k is the turbulent Prandtl number for kinetic energy transport. The quantity C_d is a constant and l_d a length scale. Substituting Eq. (9) into Eq. (13) yields the one-equation model as,

$$\frac{\partial}{\partial x_i}[(\mu\,\delta_{ij}+\mu_{ij}/\sigma_k)\frac{\partial k}{\partial x_j}]-C_d\rho k^{3/2}/l_d$$

$$+\mu_{ij}\frac{\partial U_3}{\partial x_j}\frac{\partial U_3}{\partial x_i}=0 \tag{14}$$

However, for the purpose of the generality in the turbulence calculation, l_d in Eq. (14) is determined by another transport equation relating to l_d rather than by such algebraic equation as Eq. (2). Thus the energy dissipation rate and its transport equation are introduced. For two-equation model, by the definition of the energy dissipation rate is defined as,

$$\epsilon=C_d k^{3/2}/l_d \tag{15}$$

Then, Eq. (14) becomes

$$\frac{\partial}{\partial x_i}[(\mu\,\delta_{ij}/\gamma_k+\mu_{ij}/\sigma_k)\frac{\partial k}{\partial x_j}]-\rho\,\epsilon$$

$$+\mu_{ij}\frac{\partial U_3}{\partial x_j}\frac{\partial U_3}{\partial x_i}=0 \tag{16}$$

where γ_k is correction factor for the compensation of the wall effect.

In the same way, the transport equation for the dissipation rate is expressed as,

$$\frac{\partial}{\partial x_i}[(\mu\,\delta_{ij}/\gamma_\epsilon+\mu_{ij}/\sigma_\epsilon)\frac{\partial \epsilon}{\partial x_j}] \tag{17}$$

$$-C_{2\epsilon}\frac{\rho\,\epsilon^2}{k}+C_{1\epsilon}\frac{\epsilon}{k}\mu_{ij}\frac{\partial U_3}{\partial x_j}\frac{\partial U_3}{\partial x_i}=0$$

where γ_ϵ is correction factor for the compensation of the wall effect.

If the internal heat generation does not exist and viscous dissipation, kinetic and potential energy are negligible, the energy equation becomes

$$\frac{\partial}{\partial x_i}(\lambda\frac{\partial U_3}{\partial x_i}-\rho\,C_p\overline{u_i T'})$$

$$-\rho\,C_p U_3\frac{\partial T}{\partial x_3}=0 \tag{18}$$

The turbulent heat flux, $\rho\,C_p\overline{u_i T'}$, can be also stated as the form of the Reynolds stress.

$$-\rho\,\overline{u_i T'}=\frac{\mu_{ij}}{\sigma_T}\frac{\partial T}{\partial x_j} \tag{19}$$

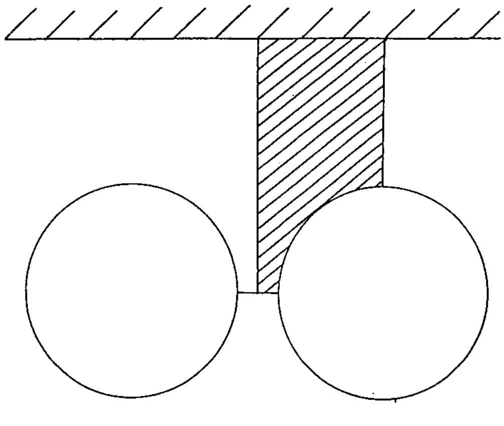


Fig. 1. Cross sectional area of the Case 1.

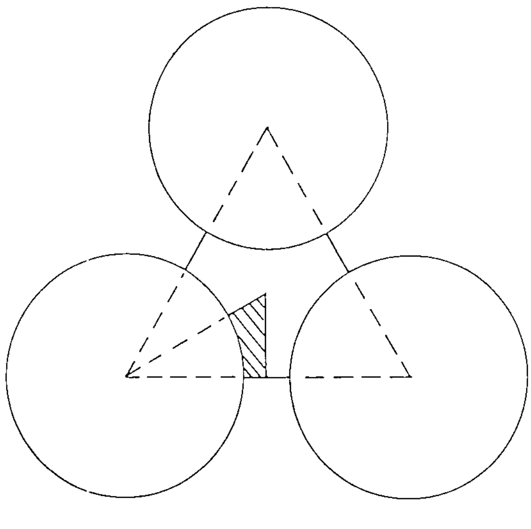


Fig. 2. Cross sectional area of the Case 2.

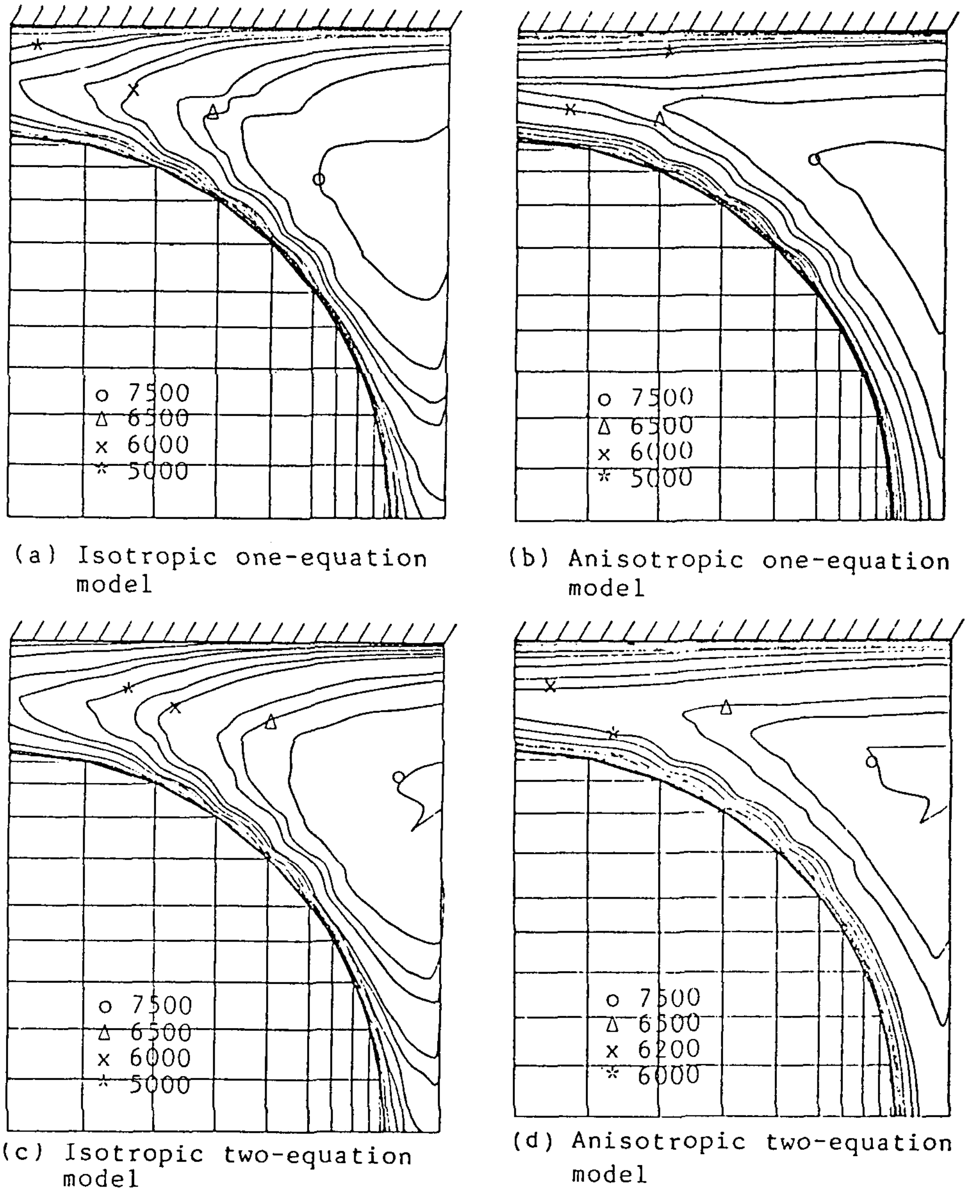


Fig. 3. Velocity contour U(cm/sec)

where σ_T is turbulent Prandtl number.

From Eq. (19) and the definition of Prandtl number, Eq. (18) will be

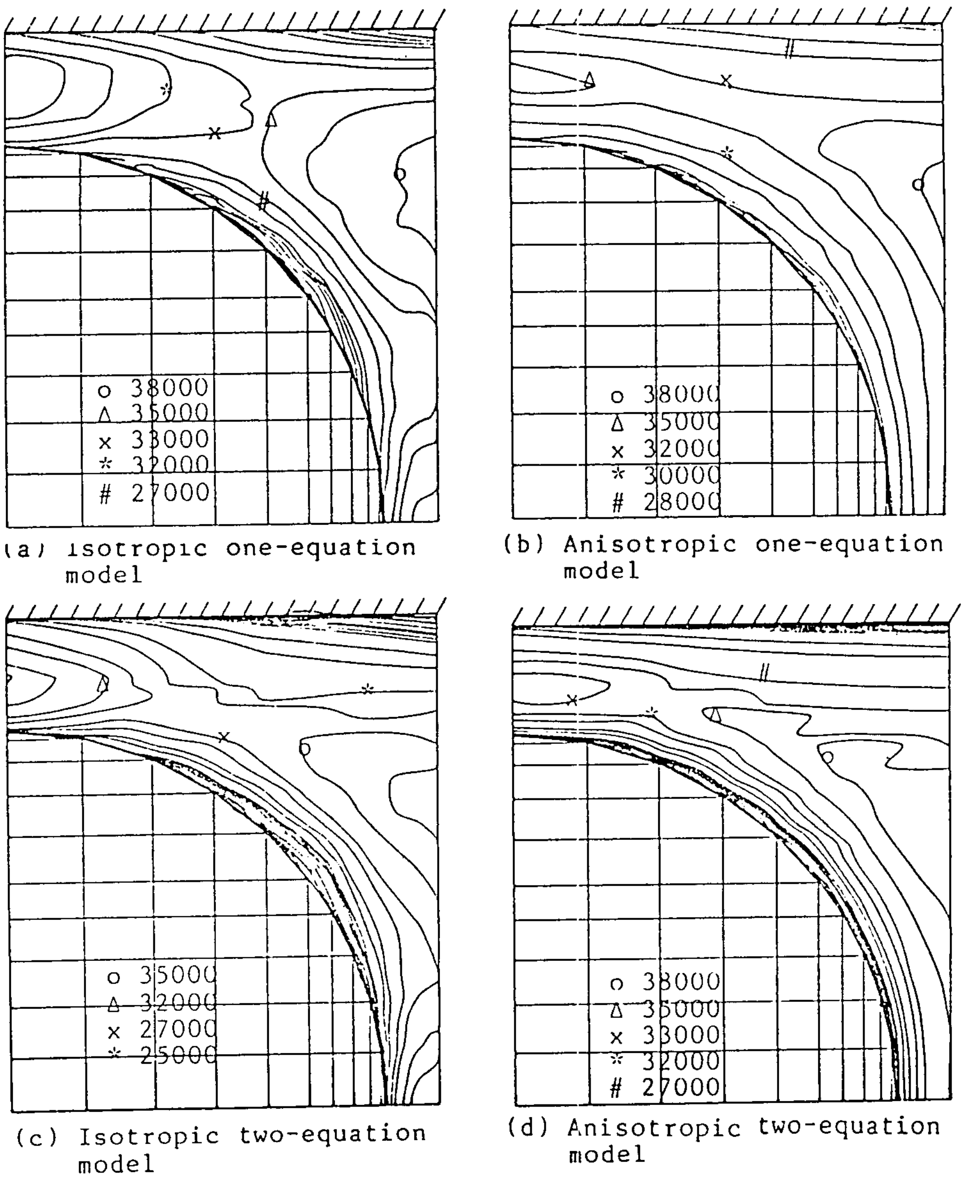
$$\frac{\partial}{\partial x_i}[(\mu\delta_{ij}/\sigma+\mu_{ij}/\sigma_T)\frac{\partial T}{\partial x_j}]$$

$$-\mu U_3\frac{\partial T}{\partial x_3}=0 \tag{20}$$

And because of the uniform heat flux from the rod surface, the axial temperature gradient is determined by the heat flux q'' and axial average velocity $\bar{U}_3$. Thus, the final form of the energy equation is

$$\frac{\partial}{\partial x_i}[(\mu\delta_{ij}/\sigma+\mu_{ij}/\sigma_T)\frac{\partial T}{\partial x_j}]$$

$$-\frac{\lambda q'' P_H}{\rho C_p A_{x\,s}}\frac{U_3}{\bar{U}_3}=0 \tag{21}$$



(a) Isotropic one-equation model

(b) Anisotropic one-equation model

(c) Isotropic two-equation model

(d) Anisotropic two-equation model

Fig. 4. Turbulent kinetic energy contour k(cm^2/sec^2)

The boundary condtions to solve the above governing equations are

1) No slip boundary condtion at the solid wall

2) Symmetry boundary condition at the external symmetry boundary

3) Uniform wall heat flux at the rod surface.

The heat transfer calculation of the close-packed rod array is difficult because circumferential variation of the rod surface temperature can not be neglected when P/D $<$ 1.3 for a triangular spacing. And LMFBRs will most likely be fueled with close-packed rod bundle with P/D $<$ 1.3 and circumferential variations in heat transfer coefficients will be appreciable. Thus, in this study, Nu-P/D correlation is established in P/D range of 1.05 to 1.3 and has the similar form of Lyon's correlation[6] which is for P/D $>$ 1.3.

2.3. Numerical Methods

To find the solution of the above equations, finite element method is introduced and integral

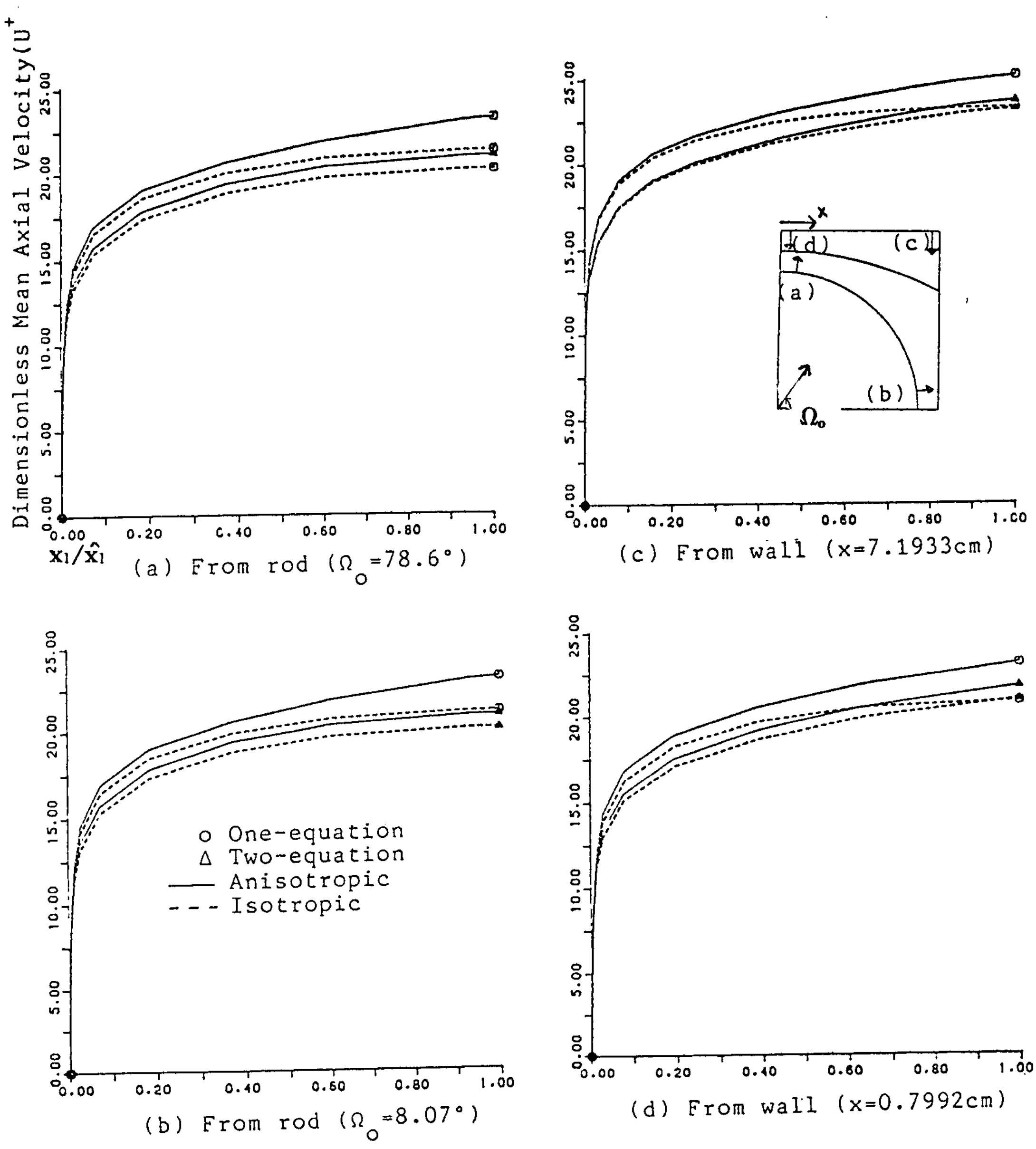


Fig. 5. Mean axial velocity normal to rod or wall.

formulations are constructed by Galerkin's WRM.[7] The two dimensional flow region is discretized into triangular finite elements. $C^{(0)}$ continuity on the element boundary is used.

Finally, the successive-substitution iteration technique[8] is used to solve the non-linear governing equations. Three non-linear equations, Eqs. (10), (16), and (17), are solved first. Next, the energy equation, Eq. (22), is solved independently with obtained velocity data for uniform wall heat flux assumption.

3. Results

For verification of this model, we analyze turbulent flow and heat transfer in rod bundle geometries such as Figure 1 and 2. Case 1 shown in Figure 1 (wall-channel region) is adopted because

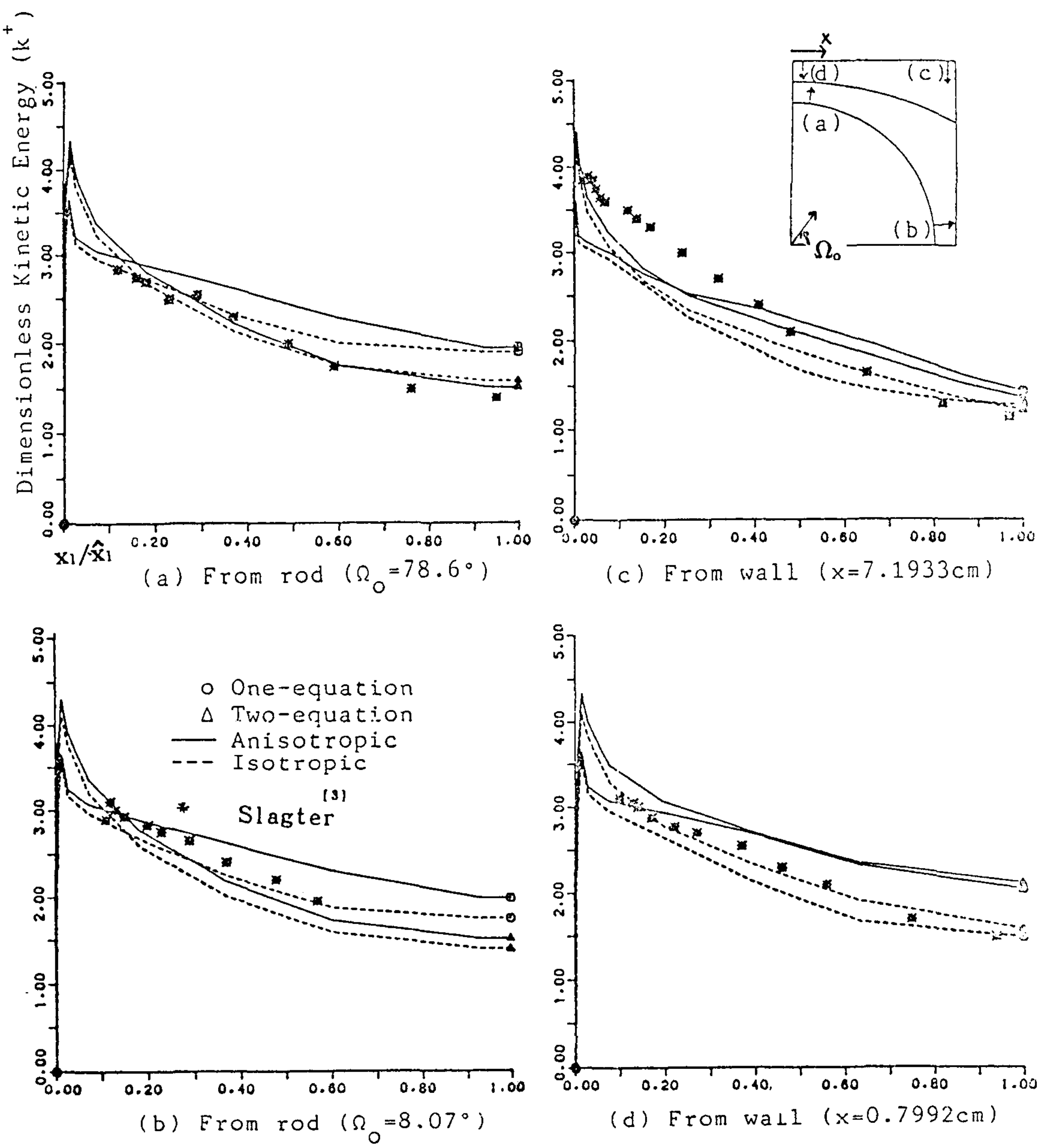


Fig. 6. Turbulent kinetic energy normal to rod or wall.

there may be dominant anisotropic effect. Case 2 shown in Figure 2 (close-packed equilateral triangular array which is common in LMFBR) is considered for prediction of temperature profile in the subchannel and the effects of P/D.

3.1. Velocity profile in the wall channel region (Case 1)[3]

In case 1, P/D = 1.15 and W/D=1.15 when D =139 cm. Air is used for working fluid with Re= 120,000. The results of two-equation model are compared with those of one equation model and also anisotropy model is compared with isotropy model for both turbulent models.

As shown in Figure 3 and 4, the distributions of the velocity and the turbulent kinetic energy, which are calculated by anisotropy model, are more uniform along the solid wall than by isotropy model. Figure 5 shows that the maximum

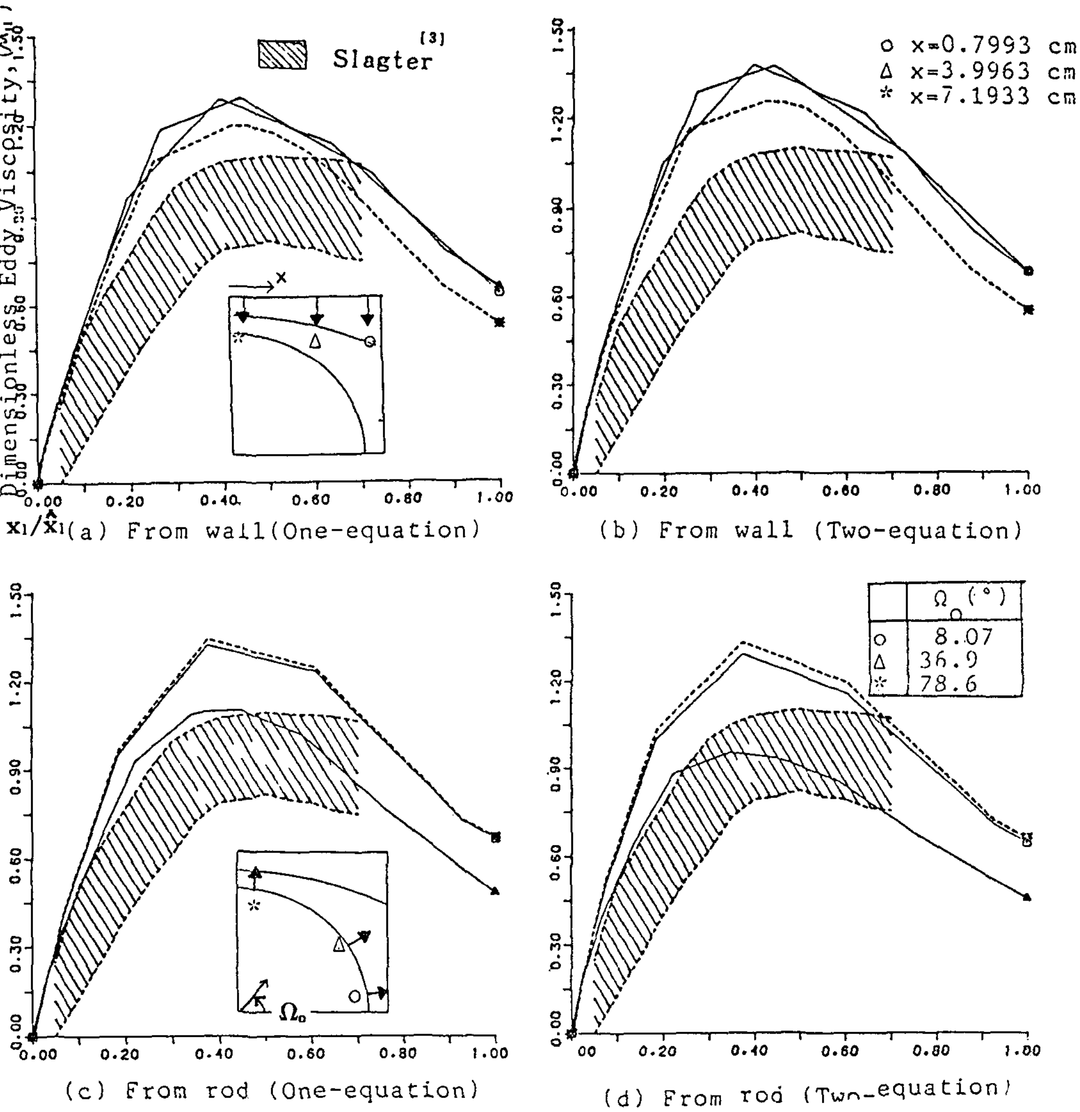


Fig. 7. Eddy viscosity normal to rod or wall.

velocity predicted by the one-equation model is higer than that by the two-equation model. Thus, it has been concluded from Figure 6. that two--equation model is more consistent with experimental data. The effects of the eddy viscosities normal to the wall and parallel to te wall are shown in Figure 7 and 8. The normal eddy viscosities have its highest value near the half position between wall and maximum velocity line while the parallel eddy viscosities are proportional to the distance from the wall. And both eddy viscosities have their minimum at the maximum profile length point.

3.2. Temperature profile in the equilateral triangula array (Case 2)

In the case 2, with uniform wall heat flux, the temperature profile calculation is performed varying P/D from 1.05 to 1.3 for Pe > 100, where the axial conduction can be neglected. And a ratio of heat diffusivity to eddy viscosity is

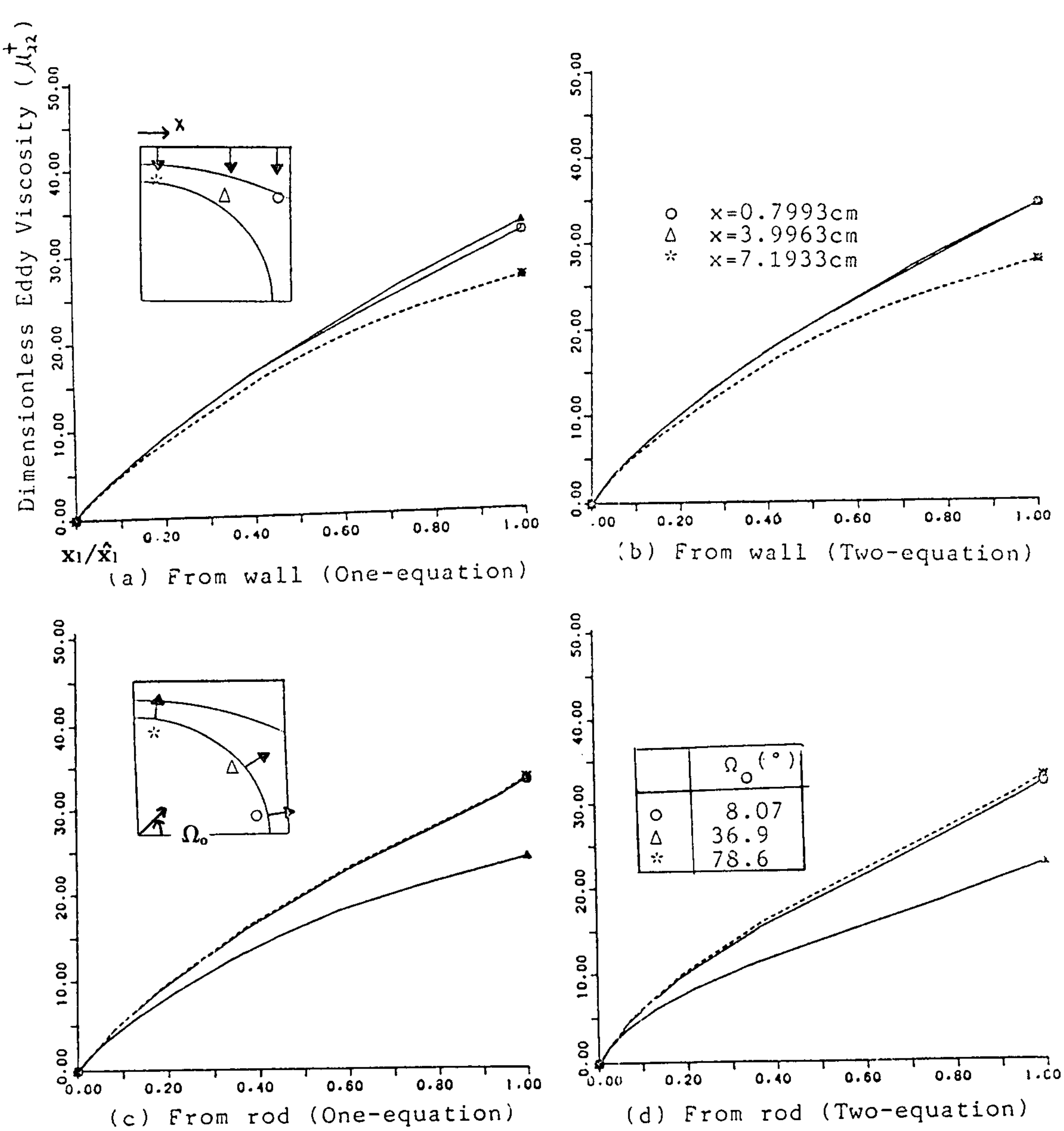


Fig. 8. Eddy viscosity parallel to rod or wall.

assumed to be one for both radial and tranential direction. Figure 9 represents the temperature at the rod surface and comparisons with calcuated values by Bartzis-Todreas[4] when P/D is 1.1, working fluid is sodium, rod diameter is 0.635 cm, wall heat flux is 2.460 $\times$ 10^9 erg/cm/sec, and Re is about 90,000. Results show that, at the wall surface, maximum temperature difference of anisotropic model is smaller than that of isotropic model, because parallel eddy viscosities have a greater influence on heat transfer than normal eddy viscosities.

Table 1 shows the variation of Nusselt number at the rod surface for P/D = 1.1. Nusselt numbers are calculated for P/D =1.05−1.3 and, from the results, Nu-P/D correlation with similar form of Lyon's is obtained as

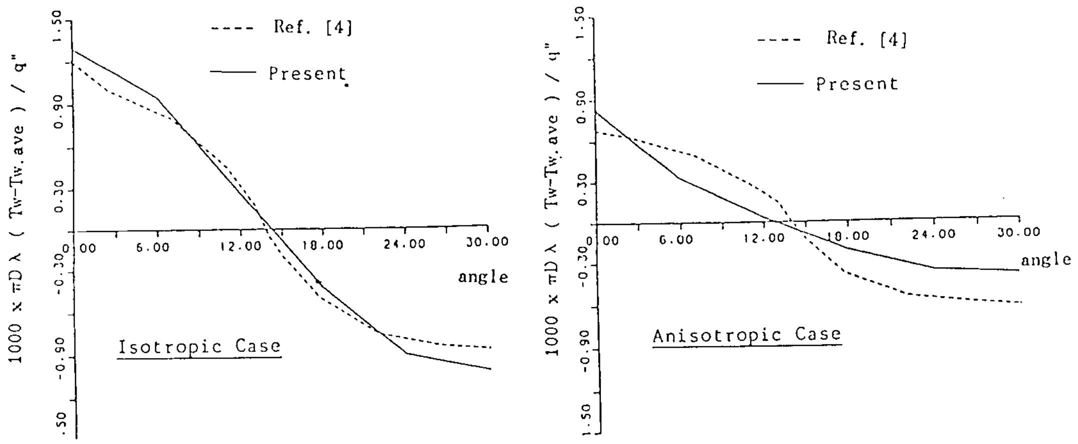


Fig. 9. Temperature profile at the rod surface (P/D=1.1).

Table 1. Nusselt number at the rod surface (P/D=1.1)

angle(degree)	(isotropy)	(anisotropy)
0	10.8	11.8
6	11.2	12.1
12	12.6	12.9
18	14.4	13.8
24	14.6	14.5
30	14.8	14.7

Table 2. Nusselt number to P/D

P/D	Nu	Pe
1.05	14.5	993
1.10	19.6	1,318
1.15	22.8	1,644
1.20	25.5	1,962
1.25	25.7	1,899
1.30	29.6	2,525

$$Nu = \alpha + 0.0155(\Psi Pe)^{0.86} \quad (22)$$

where,

$$\alpha = 90.1 - 159(P/D) + 79.5(P/D)^2$$

Table 2 shows the calculated Nusselt number according to P/D.

4. Discussion and Conclusions

In this study, the thermal-hydraulic properties, such as axial velocity, turbulent kinetic energy, turbulent energy dissipation rate, eddy viscosity and temperature, are calculated using k-ε model in the nuclear fuel bundles where complex turbulence phenomena exist. Anisotropic eddy viscosity is used to produce more realistic results. The results of one-queation model are compared with those of two-equation model and, moreover, anisotropic effects are made comparisons with isotropic effects of each model. For the verification of this model, calculated thermal-hydraulic quantities are compared with available experimental data and show good agreements.

From the results of this study, it is convinced that anisotropy has a great effect on the turbulence phenomena and, especially, the rod surface temperature variation strongly depends upon the choice of model and anisotropy.

Therefore, anisotropic model should be recommended to precise analysis of the turbulence phenomena in nuclear fuel bundles. Moreover, the Nu-P/D correlation is constructed for the close-packed equilateral triangular array in the P/D range of 1.05 to 1.3, having the Na coolant. And this correlation can be used for the heat transfer calculation of LMFBR.

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Nomenclatures

A_d, A_μ	emperical constants
A_{x-s}	cross-sectional area
$C, C_d, C_1, C_2, C_{1\epsilon}, C_{2\epsilon}$	empirical costants
C_p	specific heat
D	diameter of the fuel element
k	turbulent kinetic energy
$k^+ = k/U^{*2}$	dimesionless turbulent kinetic energy
l_d	length scale for dissipation
l_μ	length scale for eddy viscosity
$l_{\mu,1}, l_{\mu,2}$	length scales for anistropic eddy viscosities
Nu	Nusselt number
p	pressure
P	pitch
Pe	Peclet number
P_H	heated perimeter
q''	heat flux from the fuel rod surface
R	local turbulence Reynolds number
Re	Reynolds number
T	temperature
T'	fluctuation component of temperature
Tw	wall temperature
Tw.ave	averaged wall temperature
U_3	mean axial velocity
U^*	friction velocity
$U^+ = U_3/U^*$	dimensionless mean axial velocity
$u_i u_3$	turbulent shear stress
$u_i T'$	term relating to turbulent eddy diffusivity
W	wall pitch
x_i	for $i=1$; coordinate normal to the wall for $i=2$; coordinate parallel to the wall for $i=3$; axial coordinate
x_l	distrance from the wall
$\hat{x}_l$	profile length
$\gamma_k, \gamma_\epsilon$	correction factor in $k-\epsilon$ model
δ_{ij}	Kronecker delta
ϵ	turbulent energy dissipation rate
λ	thermal conductivity of the coolant
μ	laminar viscosity
μ_{ij}	tensorial eddy viscosity
μ_{11}	eddy viscosity normal to the wall.
μ_{22}	eddy viscosity parallel ot the wall
μ^+	dimensionless eddy viscosity
ρ	density
σ	laminar Prandtl number
σ_k	Prandtl number for turbulent kinetic energy
σ_T	turbulent Prandtl number
σ_ϵ	Prandtl number for turbulent kinetic energy dissipation rate
Ψ	ratio of heat diffusivity to eddy diffusivity