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Analytic Prediction of Friction Factors for Turbulent Flow in Longitudinally Finned Rod Bundles

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길이 방향 핀이 달린 봉 다발에서의 난류 마찰계수 산출을 위한 해석적 방법

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Abstract

This work is concerned with the development of an analytical model to predict the friction in longitudinally finned rod bundles. Such bundles are currently considered in KMRR design. The present model assumes the validity of the Law of the Wall over entire flow area. The flow channel area is divided into the interfin region and a number of element channels, and the algebraic form of the Law of the Wall is integrated over each element channel and interfin region to yield an analytic expression for the pressure drop. The model reasonably predicts the 6 fin KMRR data, and overpredicts the 8 fin data about 15 percent.

요 약

본 연구에서는 길이 방향의 핀이 달린 봉 다발의 난류 마찰계수를 예측할 수 있는 해석적 모델이 개발 되었다. 현재 KMRR의 연료봉으로 핀이 달린 봉다발이 고려되고 있다. 본 모델에서는 "Law of the Wall"이 전체 수로에 적용 가능하다고 가정하였다. 전체 수로를 작은 요소 수로와 핀 간 수로로 나누고 각 요소 수로마다 "Law of the Wall" 속도 분포 곡선을 적분하여 난류 마찰계수를 예측할 수 있는 해석적 방정식을 얻는다. 이 방법을 KMRR 연료봉에 적용한 결과 6핀 실험결과는 잘 예측하고 8핀 실험 결과는 15 % 정도 높게 예측하였다.

1.0 Introduction

Nuclear reactor fuels usually consist of rod bundles wherein the coolant flows parallel to the rods. Knowledge of the pressure drop of rod bundles is essential to assess the performance of the nuclear fuels. While considerable amount of data are accumulated on rod bundles with bare rods, very little information is available on rod bundles consisting of rods with longitudinal fins. Finned rod bundles have been used in a couple of research reactors [1], and are currently considered in KMRR (Korea Multi-purpose Research Reactor) fuel design. Longitudinally finned rod bundles are illustrated in Figure 1.

The hydraulic diameter Blasius friction correlation [equation (1)] which reasonably predicts bare rod bundle friction factors [2] tends to overpredict those of finned rod bundles [1, 3, 4].

$$f=0.079\ Re^{-0.25} \tag{1}$$

This trend was also observed by Carnavos [5] for internally finned tubes. Scott and Webb [6] attributed the inaccuracy of the hydraulic diameter

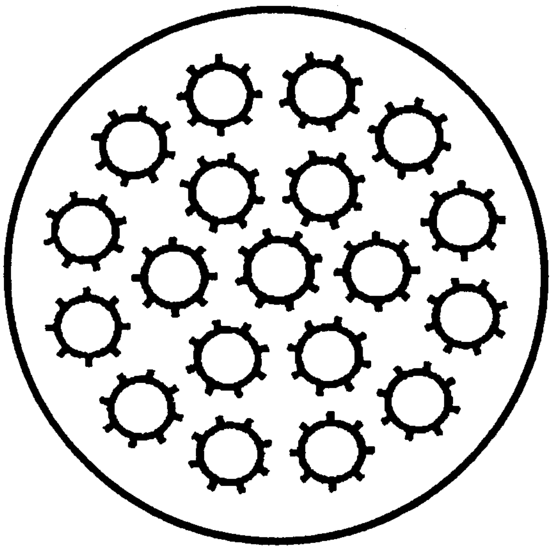


Fig. 1. Longitudinally finned rod bundle

correlation to the reduced velocity in the inter-fin region relative to that in a smooth tube.

Literature shows no analytic model for prediction of the friction characteristics in finned rod bundles. This work seeks to develop an analytic method assuming the validity of Law of the Wall across the flow area.

2.0 Theoretical Development

The theoretical model is based on the Law of the Wall [7]. The Law of the Wall assumes that the velocity distribution in the viscous influenced wall region is independent of the channel shape or pipe diameter. For turbulent dominated region ($y^+>26$), the algebraic form of the Law of the Wall is

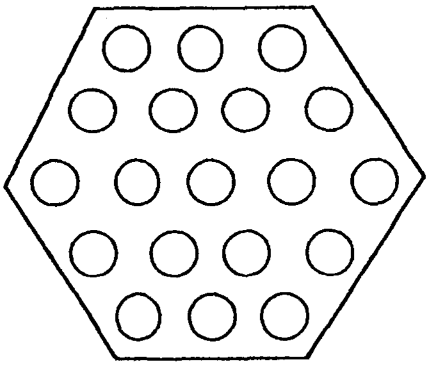
$$u^+=2.5\ \ln y^+ +5.5 \tag{2}$$

The friction factor in circular tubes is well-predicted assuming equation (2) applies over the entire pipe radius. Equation (2) may be integrated over the pipe radius with identity $\bar{u}/u^*=\sqrt{2/f}$, which results in the friction factor equation

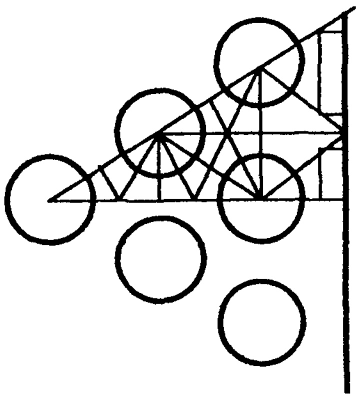
$$\sqrt{\frac{2}{f}}=2.5\ln\left(Re\sqrt{\frac{f}{2}}\right)+1.75 \tag{3}$$

Several investigators have applied this idea to channels with more complex geometries – smooth rectangular channels [8, 9], internally finned channels [6] and rod bundles [10].

Kim et al. [10] applied the concept to predict the friction in rod bundles. The cross sectional flow area was divided into small element channels, and equation (2) was integrated over each element channels to obtain an expression for the friction factor. Figure 2 shows sketches of a typical rod bundle and the corresponding element channels. They applied the concept to predict the friction factors of several rod bundle geometries (square, hexagonal and circular), and the results were reasonsably good. Figure 3 reproduces one



a) typical rod bundle



b) Corresponding element channels

Fig. 2. Sketch of a typical rod bundle and element channels

of their results.

Scott and Webb [6] have predicted the friction in internally finned tubes. Figure 4 shows a internally finned tube. The cross–sectional flow area of a circular finned tube was divided into two flow areas : 1) the inter–fin region and 2) core region. Then, integrating equation (2) over each flow area, they could predict the available data within ± 10 percent.

Due to the prior success in applying the Law of the Wall to complex flow geometries, the concept is applied to finned rod bundles in this work. Figure 5 shows that the cross–sectional flow area (A) is divided into the interfin region (A_f) and a

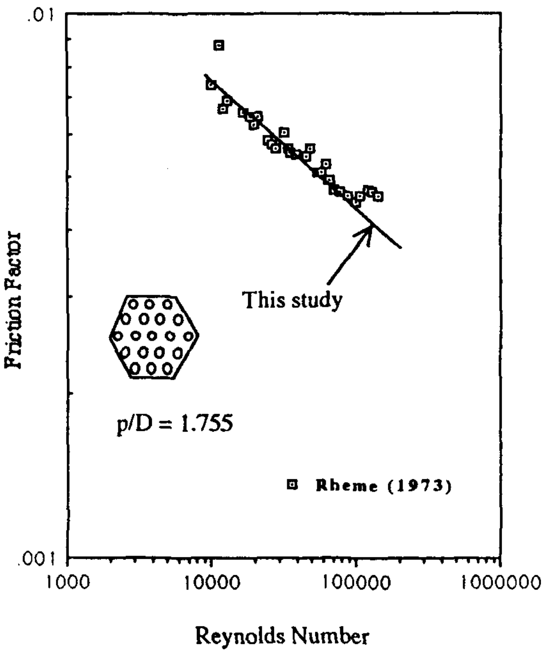


Fig. 3. Predicted friction factors [10]

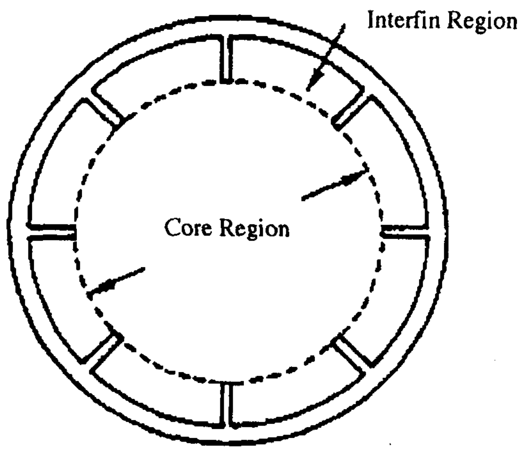


Fig. 4. Sketch of a internally finned tube

number of element channels (A_1, A_2, ···, A_n). Then,

$$A = A_f + A_1 + A_2 + \cdots + A_n \tag{4}$$

Writing the continuity equation for each region yields

$$\bar{u}A = \bar{u}_f A_f + \bar{u}_1 A_1 + \bar{u}_2 A_2 + \cdots + \bar{u}_n A_n \tag{5}$$

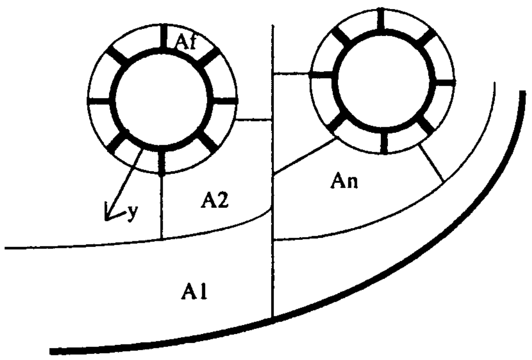


Fig. 5. Element area division of a finned channel

Dividing by u^* and using $\bar{u}/u^*=\sqrt{2/f}$, equation (5) becomes

$$\sqrt{\frac{2}{f}}=\frac{\bar{u}_f}{u^*}\frac{A_f}{A}+\frac{\bar{u}_1}{u^*}\frac{A_1}{A}+\frac{\bar{u}_2}{u^*}\frac{A_2}{A}+\cdots+\frac{\bar{u}_n}{u^*}\frac{A_n}{A} \quad (6)$$

The expressions for $\bar{u}_i/u^*$ for each region can be obtained by integrating equation (2) over each region.

Interfin region

Figure 6 shows the regions over which equation (2) is integrated. The interfin region is assumed an open rectangular channel. To simplify the integration, symmetry is employed. The integrations to be performed is

$$\frac{\bar{u}_f}{u^*}=\frac{1}{eb}\Big[\int_0^e\int_0^{\frac{b}{e}y}(2.5\ln\frac{zu^*}{\nu}+5.5)dzdy$$

$$+\int_0^b\int_0^{\frac{e}{b}z}(2.5\ln\frac{yu^*}{\nu}+5.5)dydz\Big] \quad (7)$$

where

$$b=0.5\,(p\text{-}t)$$

$$p=\frac{\pi(D_i+D)}{2n}$$

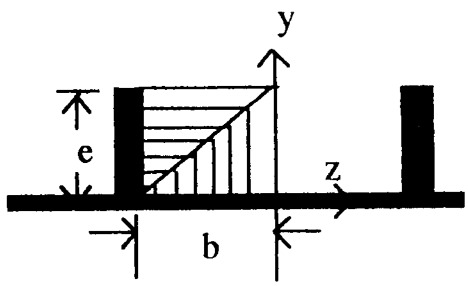


Fig. 6. Inter–fin region

In the above equations, n is the number of fins, D_i is the diameter to the base of fins, D is the diameter to the fin tip, and e and b are defined in Figure 6. The result of the integration is

$$\frac{\bar{u}_f}{u^*}=1.25[\ln(\frac{b}{D_h}Re\sqrt{\frac{f}{2}})$$

$$+\ln(\frac{e}{D_h}Re\sqrt{\frac{f}{2}})]+1.75 \quad (8)$$

where Re and the hydraulic diameter (D_h) are defined in the nomenclature.

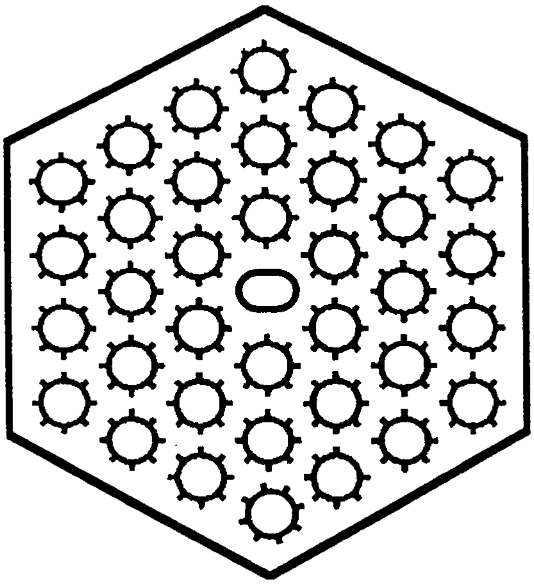
Element channel region

Equation (2) is also applied to the element channel region. Typical element channels were shown in Figure 5. The average element channel velocity is obtained by integrating equation (2) over each element channel.

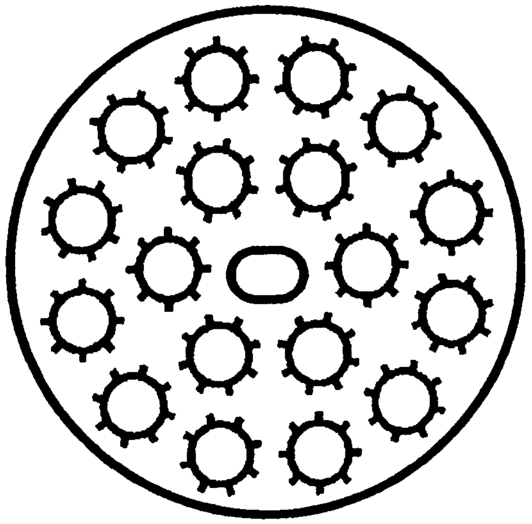
$$\frac{\bar{u}_i}{u^*}=\frac{1}{A_i}\iint_{A_i}(2.5\ln\frac{yu^*}{\nu}+5.5)dA \quad (9)$$

The friction veloctity u^* is defined as $\sqrt{\bar{\tau}_w/\rho}$. For a smooth tube, local wall shear stress τ_w is uniform peripherally, and is the same as average wall shear stress $\bar{\tau}_w$. For a rod bundle, however, τ_w may vary along the rod periphery. The measurement by Subbotin et al. [11] shows that τ_w varies significantly along the rod periphery when the pitch to diameter ratio p/D is close to 1. For p/D>1.2, however, τ_w is almost uniform peripherally.

In this study, the local wall shear stress τ_w is assumed to be the same as the bundle average wall shear stress $\overline{\tau}_w$. This may be a reasonable assumption for the rod bundles with $p/D>1.2$. For $p/D<1.2$, the assumption may cause some error. However, the error may not friction factor



a) Hexagonal rod bundle



b) Circular rod bundle

Fig. 7. KMRR rod bundle geometry

by Equation (6) may smooth out the effects by local shear stress variation. Then, the expressions for $\overline{u}_i/u^*$ for each region may be obtained by integrating Equation (2) over each region. The friction factor is obtained by substituting equations (8) and (9) into equation (6).

One should notice that y coordinate in Figure 5 starts from the base of the fin, not from tip of the fin. This means that the non-dimensionalized velocity (u^+) at the fin tip ($y=e$) is assumed to be the same as that which would occur at the same wall distance in a smooth tube.

3.0 Application to KMRR fuel geometry

The current method is applied to predict the friction in KMRR fuel bundle. Two types of rod bundle geometry are considered as shown in Figure 7. The hexagonal rod bundle consists of 36 fuel rods, and circular rod array consists of 18 fuel rods. The figure also shows the cross-section of the tie rod at the center of the rod bundle, which

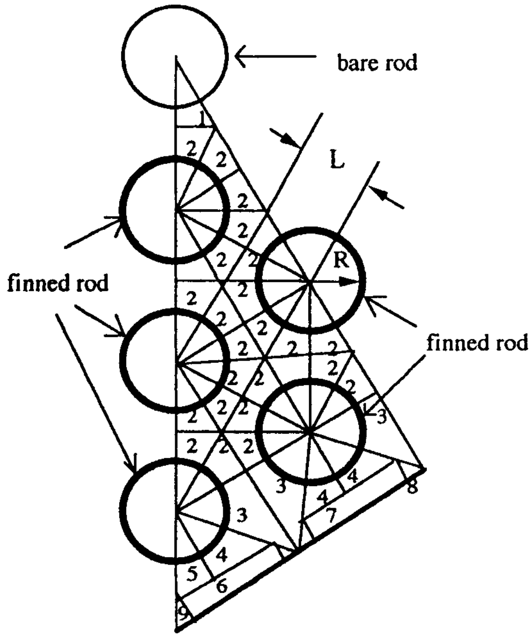


Fig. 8. Element channels for the hexagonal rod bundle

supports the rod bundle during operation or maintenance. The tie rod has a non–circular cross–sectional shape.

Figure 8 shows the flow element channels for the 36 element hexagonal rod bundle. The non–circular central tie–rod is assumed to be a circular rod whose cross–sectional area is the same as that of the non–circular tie–rod. The calculated diameter of the central rod is approximately equal to the fuel rod fin–tip diameter (D). Equation (2) is integrated over each element channel and interfin region, and several of them are listed below as examples (for element channels 1, 2 and interfin region).

$$\frac{\overline{u}_1}{u^*} = \frac{1}{A_1}\int_0^{\pi/6}\frac{L\sec\theta + R}{2}\int_0^{L\sec\theta - R}\left(2.5\ln\frac{yu^*}{\nu} + 5.5\right)dy\,d\theta \qquad (10)$$

$$\frac{\overline{u}_2}{u^*} = \frac{1}{A_2}\int_0^{\pi/6}\frac{L\sec\theta + R}{2}\int_e^{L\sec\theta - R}\left(2.5\ln\frac{yu^*}{\nu} + 5.5\right)dy\,d\theta \qquad (11)$$

$$\frac{\overline{u}_f}{u^*} = \frac{1}{A_f}\left[1.25\left(\ln\frac{bu^*}{\nu} + \ln\frac{eu^*}{\nu}\right) + 1.75\right] \qquad (12)$$

In the above equations, R is the rod radius to the fin tip and L is half the rod pitch. The friction factor is obtained by substituting above equations into equation (6).

The results are compared with the data in Figures 9 and 10. The 6 fin results are shown in Figure 9, and 8 fin results are shown in Figure 10. The figures show that the current method reasonably predicts the 6 fin data at high Reynolds number range ($Re > 4 \times 10^4$). Most data are predicted within $\pm 5\%$. At lower Reynolds numbers, the model overpredicts the data approximately 15%.

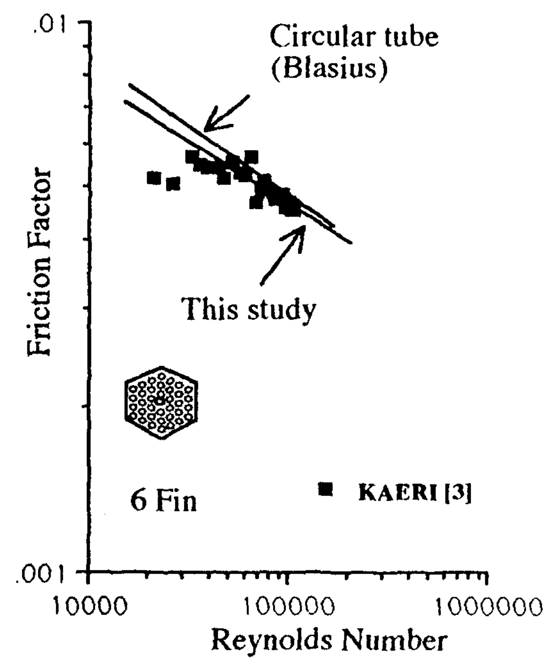


Fig 9. Friction prediction, 6 fin, hexagonal shape

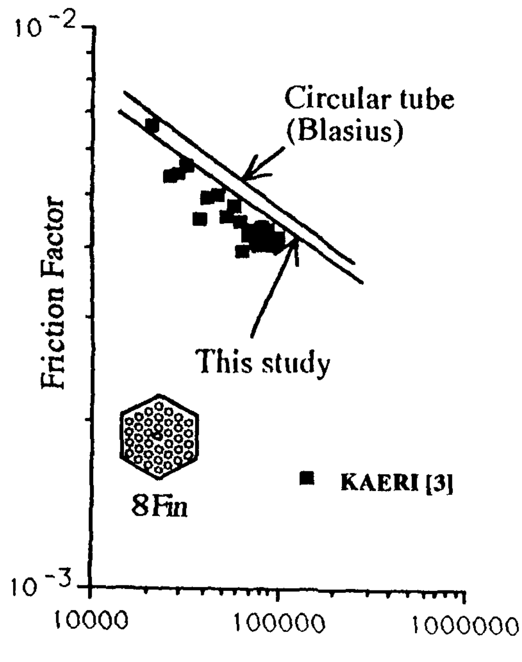


Fig. 10. Friction prediction, 8 fin, hexagonal shape

Figure 10 shows current model overpredicts the 8 fin data about 10 percent. The Blasius friction correlation is also compared with the data in the figures. The figures show that the current method predicts the data better than Blasius correlation does.

Figure 11 shows the flow element channels for the 18 element circular rod bundle. Equation (2) is integrated over each element channel and the interfin region. The integration procedure is similar to the previous one, and thus omitted for clarity. The results are compared with the data in Figures 12 and 13. The figures show a similar trend to the hexagonal rod bundle results–reasonable prediction (±5%) for the 6 fin data and about 15 percent over–prediction for the 8 fin data. The figures also show that the current method predicts the data better than the Blasius correlation does.

Current model clearly shows the possibility of applying the Law of the Wall to predict the friction in finned rod bundles. Literature survey shows very little information on finned rod bun-

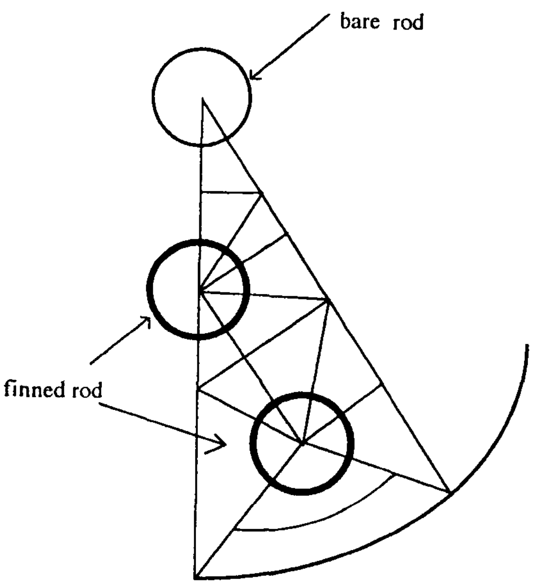


Fig. 11. Element channels for the circular rod bundle

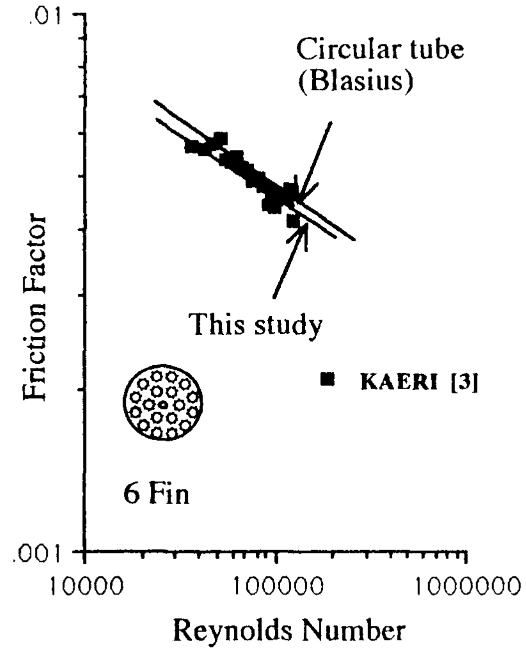


Fig. 12. Friction prediction, 6 fin, circular shape

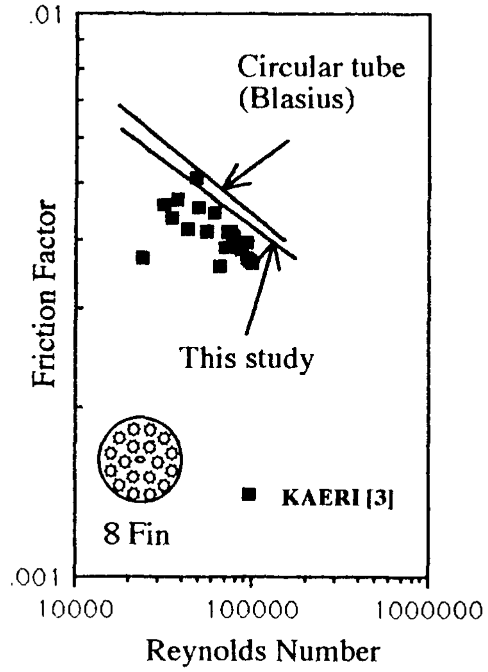


Fig. 13. Friction prediction, 8 fin, circular shape

dles, and only KMRR data were available to the authors. Comparison with more experimental data may add credibility to the model.

4.0 Conclusions

An analytical model was developed to predict the friction factor for turbulent flow in finned rod bundles. The model applies the Law of the Wall to the interfin and element channels. The flow channel area is divided into the interfin region and a number of element channels, and the algebraic form of the Law of the Wall is integrated over each element channel and interfin region to yield an analytic expression for the pressure drop. The model reasonably predicts the 6 fin KMRR data, and overpredicts the 8 fin data about 15 percent.

Nomenclature

A_i	Element channel flow area
b	half the fin spacing
D	Rod diameter to the fin tip
D_i	Rod diameter to the base of the fin
D_h	Hydraulic diameter
e	fin height
f	Fanning friction factor
K	Geometry factor for laminar flow
L	half the rod pitch
n	number of fins per rod
p	Rod pitch
R	Rod diameter
Re	Reynolds Number $(\overline{u}D_h/\nu)$
t	fin thickness
u	Flow velocity
$\overline{u}_i$	Mean flow velocity for sub–channel flow area A_i
u^*	Friction velocity$(\sqrt{\tau_w/\rho})$
u^+	Nondimensional velocity (u/u^*)
y	Coordinate normal to the wall
z	Coordinate parallel to the wall
y^+	Nondimensional distance (yu^*/ν)
ρ	Density
ν	Kinematic viscosity
τ_w	Wall shear stress

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