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Application of the Fuzzy Set Theory to Analysis of Accident Progression Event Trees with Phenomenological Uncertainty Issues

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현상학적 불확실성 인자를 가진 사고진행사건수목의 분석을 위한 퍼지 집합이론의 응용

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Abstract

An example application of the fuzzy set theory is first made to a simple portion of a given accident progression event tree with typical qualitative fuzzy input data, and thereby computational algorithms suitable for application of the fuzzy set theory to the accident progression event tree analysis are identified and illustrated with example applications. Then the procedure used in the simple example is extended to extremely complex accident progression event trees with a number of phenomenological uncertainty issues, i.e., a typical plant damage state 'SEC' of the Zion Nuclear Power Plant risk assessment. The results show that the fuzzy averages of the fuzzy outcomes are very close to the mean values obtained by current methods. The main purpose of this paper is to provide a formal procedure for application of the fuzzy set theory to accident progression event trees with imprecise and qualitative branch probabilities and/or with a number of phenomenological uncertainty issues.

요 약

전형적인 정성적 퍼지형태의 입력데이타를 가진, 주어진 사고진행사건수목의 일부분에 대하여 퍼지집합이론(fuzzy set theory)의 응용 예를 먼저 보여주고, 이 예를 통해서 퍼지집합이론을 사고진행사건수목에 적용하기 위해 적절한 계산알고리즘을 찾아내고 또 예를들어 설명하였다. 그리고, 간단한 예제에 사용한 계산절차를 많은 현상학적 불확실성 인자를 포함한 아주 복잡한 사고진행사건수목 즉, 최근 Zion 발전소 위험도평가(PRA)에 사용된 전형적인 발전소 손상군의 하나인 'SEC'에 응용해서 적용하였다. 퍼지집합이론으로 평가한 계산값들의 퍼지평균치들은 최근 통계적 PRA 평가 방법론으로 얻은 값들의 평균치와 거의 같은 결과를 보여주고 있다.

본 논문의 주요목적은 부정확하고 또 정성적인 분기점확률이나 또는 많은 현상학적 불확실성인자들을 가진 사고진행사건수목들에 이 퍼지집합이론을 적용하기 위한 공식적 계산절차를 제공하는데 있다.

1. Introduction

An important issue faced by contemporary risk analysists of nuclear power plants is how to deal with uncertainties that arise in each phase of risk assessments. In general, assessment of risk from the operation of nuclear power plants is comprised of five major principal steps: (1) accident frequency(system) analysis ; (2) accident progression, containment loadings, and structural response analysis, (3) radioactive material transport(source tem) analysis, (4) offsite consequence analyses, and (5) risk calculations.[1] There are multiple sources and types of uncertainty in these processes of risk assessment. The major uncertainty addressed here is the one that arises in the second part of the risk analysis which treats the physical processes affecting the core after an initiating event occurs. The type of phenomenon that contributes the most to uncertainty in risk analysis is the phenomenon that is poorly understood so that there may be several competing models, each incomplete with respect to various aspects of the problem. For example, there is no single accepted scenario for both the high pressure melt ejection and the subsequent effects leading to direct containment heating(DCH). The physico–chemical processes for these phenomena are extremely complex and varied. Major uncertainties involve the impact of DCH on early containment failure, and the impact of core–concrete interactions on both early and late containment failure.

In the most recent risk assessment,[1] an expert opinion polling process was used to assess the uncertainty related to physical phenomena, in conjuction with the limited Latin hypercube(LLH) sampling approach(a modified Monte Carlo method) for the propagation of uncertainty. Though it seems that the expert opinion polling as a means of identifying issues and estimating uncertainty is an acceptable part of the current PRA process, a clear disadvantage of this approach is that the final results can not be more robust than the information upon which the experts base their judgement. More efforts are certainly needed to make it effective and it is desirable to develop and explore some alternative methods or procedures.

Recent advances in the theory of fuzzy sets make it possible to study the complex and ill–defined concepts where uncertainty is due to fuzziness, or degree of vagueness. To date, however, the use of fuzzy set theory in risk and reliability analyses has been very limited. In an effort to explore the full potential of fuzzy set theory as a methodology for dealing with phenomena that are too complex or too ill–defined to be susceptible to analysis by conventional means, example applications of the fuzzy set theory are made first to the portion of a simple accident progression event tree(APET) with imprecise and qualitative branch probabilities and then to extremely complex accident progression event trees of Zion[2] with a number of phenomenological uncertainty issues.

The main purpose of this work is to (1) show how the fuzzy set theory can be used to represent the imprecision which surrounds the probabilities under certain circumstances, while retaining the structures of a given event tree and consistency which the probability theory provides, and (2) provide a formal procedure for application of the fuzzy set theory to APETs with imprecise and qualitative branch probabilities and/or with a number of phenomenological uncertainty issues.

2. Example Application of Fuzzy Set Theory to An APET

In conventional event tree analysis, the branch point probabilities have been treated as exact values. As already mentioned, however, for many top event questions of the APETs regarding the phenomena encountered during severe accidents, it is often difficult to assign exact branch probabilities(e.g., 'probabilities concerning the location of induced failure of the reactor coolant system pressure boundary') or parameters(e.g., 'the magnitude of pressure loading at vessel breach due to DCH and steam spike' and 'the containment failure pressure') from the current state of knowledge. To examine the applicability of the fuzzy set approach to this type of problem, first, a simple portion of a given APET with typical qualitative fuzzy input data has been analyzed by fuzzy set approach in the following.

2.1 Formulation of a Fuzzy APET Analysis Problem

To illustrate how the fuzzy set theory can be applied to APETs with qualitative and imprecise input data, suppose one is given with a portion of APET as shown in Fig.1. A typical portion of the given APET has three questions A_1,A_2, and A_3(as summarized in Table I), and each question is assumed to have only two branches(success or failure of a system ; and/or occurrence of nonoccurrence of a phenomenon) for simplicity in presentation. Suppose, because of the nature of the top events considered, it is not possible to assign unique numerical probability values between 0 and 1 to the branches of three top events shown in Fig.1, and assume, therefore, that they are specified with imprecise and qualitative fuzzy variables as summarized in Table I.

The major difficulty of the given problem is that a unique numerical value between 0 and 1 is not assigned to each branch probability. To overcome this difficulty, the concept of fuzzy probability can be introduced in the analysis of the event tree and the branch probability can be defined as a fuzzy set on [0,1]. That is, the fuzzy probability can be described as a fuzzy set defined in probability space and its functional form can be represented by the combination of a range of the potential probability(instead of the unique probability) and the degree of possibility of a probability value within the probability range. This fuzzy probability is often used as a linguistic representation of the probability such as 'highly probable'. More specifically, the 'possibility' of occurrence of a phenomenon(or success of a system) defined in a certain range on [0,1] is used instead of a unique

Table I. Three Top Event Questions and Specified Fuzzy Variables.

Top Event Number	Top Event Questions	Specified Fuzzy Variables $\tilde{P}_i$
A_1	Is the reactor cavity dry when a small LOCA has occurred?	'approximately between 0.2 and 0.4' = $\tilde{P}_1$
A_2	What is the probability of occurrence of direct containment heating (DCH) and steam spike(SS) events?	'highly probable' = $\tilde{P}_2$
A_3	What is the probability of occurrence of hydrogen burn(HB) at vessel breach?	'improbable' = $\tilde{P}_3$

value of probability.

The remaining problem is then to calculate the 'possibility' of occurrence of accident pathways(i.e., combination of accident sequences and containment events) as a fuzzy set, given the 'possibility' of occurrence(or success) of top events as shown in Fig.1. In essence, one is dealing with a fuzzy number on [0,1], viz. 'fuzzy probability' or 'possibility' of occurrence, instead of a specific value of probability. A detailed procedure to handle this problem is given successively in the following.

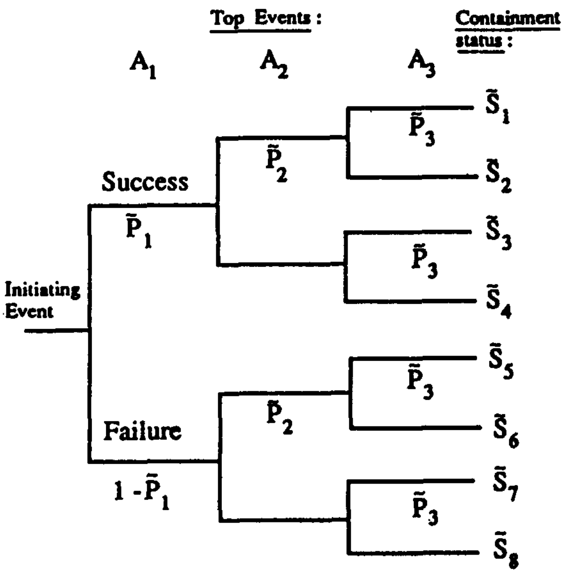

Fig.1. Sample Event Tree and Outcomes.

2.2 Conversion of Qualitative Fuzzy Variables into Quantitative Fuzzy Probabilities by Membership Functions

In order to quantify the event tree shown in Fig.1 using the fuzzy set theory, the qualitative fuzzy variables must be first converted into quantitative fuzzy probabilities. This can be done by introducing the membership function of the fuzzy set theory. The membership function is the central concept of Zadeh's fuzzy set theory and it represents numerically the degree to which an element belongs to a set.[3] This function takes on values between 0 and 1. The membership function is assessed subjectively in any instance, small values representing a low degree of membership and high values representing a high degree of membership. The assignment of membership to elements in a fuzzy set is very difficult and is still unsolved. It is a matter of subjective opinion, but the membership of an element is not a statistical quantity as expounded by some authors.[4] Since the question of how to assess the degree of membership is not the major concern here, it is assumed throughout this example that membership functions are given.

Following are the linear membership functions assumed for the three fuzzy qualitative variables $\tilde{P}_i(i=1,2,3)$. Here $\mu_{\tilde{P}_i}(z_i)$ indicates the membership function corresponding to the potential probability z_i of fuzzy variable $\tilde{P}_i$:

$\tilde{P}_1$: "Approximately between 0.2 and 0.4"

$$\mu_{\tilde{P}_1}(z_1)=\begin{cases}5z_1, & \text{if } 0\le z_1\le 0.2\\ 1.0, & \text{if } 0.2\le z_1\le 0.4\\ 3.0-5z_1, & \text{if } 0.4\le z_1\le 0.6\\ 0, & \text{otherwise.}\end{cases} \quad (1)$$

$\tilde{P}_2$: "highly probable"

$$\mu_{\tilde{P}_2}(z_2)=\begin{cases}5z_2-4, & \text{if } 0.8\le z_2\le 1.0\\ 0, & \text{otherwise.}\end{cases} \quad (2)$$

$$\mu_{\tilde{P}_3}(z_3)=\begin{cases}1.0, & \text{if } 0\le z_3\le 0.1\\ 1.5-5z_3, & \text{if } 0.1\le z_3\le 0.3\\ 0 & \text{otherwise.}\end{cases} \quad (3)$$

The linear membership functions given by Eqs. (1), (2), and (3) are depicted in Fig.2.

2.3 Calculation of Fuzzy Probabilities for Each Pathway

For the given event tree in Fig.1 there are eight possible pathways(or outcomes) : The fuzzy out-

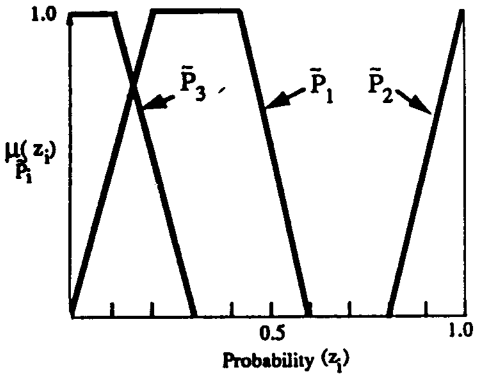

Fig.2. Membership Functions Specified for Three Qualitative Fuzzy Variables($\tilde{P}_i$).

comes $\tilde{S}_j(j=1,\cdots,8)$ are comprised of the product or subtraction arithmatics of three fuzzy variables. The pathway $\tilde{S}_2$, for example, is given by

$$\tilde{S}_2=\tilde{P}_1\times\tilde{P}_2\times(1-\tilde{P}_3) \tag{4}$$

The calculus of fuzzy sets, on the other hand, is based on three specific operators of set complement, union, and intersection.[3,4] A basic principle that allows the generalization of crisp mathematical concepts to the fuzzy framework is known as the 'extension principle' of Zadeh.[4] That is, to calculate the fuzzy probability for each pathway $\tilde{S}_j$ via the input membership functions $\mu_{\tilde{P}_i}(z_i)$ $(i=1,2,3)$, these relationships have been used in the following way.[5]

Let $\tilde{S}_j$ be a real function of three variables($\tilde{P}_1$, $\tilde{P}_2$, and $\tilde{P}_3$) and let $\tilde{P}_1,\tilde{P}_2$, and $\tilde{P}_3$ be three fuzzy $\tilde{S}_j$ sets of R. The 'extension principle' allows one to define the image of $\tilde{P}_1,\tilde{P}_2$, and $\tilde{P}_3$ through $\tilde{S}_j=f_j(\tilde{P}_1, \tilde{P}_2,\tilde{P}_3)$ whose membership function is:

$$\mu_{\tilde{S}_j}(z)=\sup_{z_1,z_2,z_3\in f_j^{-1}(z)}\{\min[\mu_{\tilde{P}_1}(z_1),\mu_{\tilde{P}_2}(z_2),\mu_{\tilde{P}_3}(z_3)]\} \tag{5}$$

where $f_j^{-1}(z)=\{(z_1,z_2,z_3)\in R \mid f_j(z_1,z_2,z_3)=z\}$.

This principle can be interpreted as follows: the possibility for the quantity($\tilde{P}_1,\tilde{P}_2,\tilde{P}_3$) to be represented by (z_1,z_2,z_3) is

$$\mu_{\tilde{P}_1\times\tilde{P}_2\times\tilde{P}_3}(z_1,z_2,z_3)=\min[\mu_{\tilde{P}_1}(z_1),\mu_{\tilde{P}_2}(z_2),\mu_{\tilde{P}_3}(z_3)] \tag{6}$$

The product of three fuzzy variables $\tilde{P}_1\times\tilde{P}_2\times\tilde{P}_3$ is the Cartesian product of $\tilde{P}_1,\tilde{P}_2$, and $\tilde{P}_3$. The possibility for $\tilde{S}_j=f_j(\tilde{P}_1,\tilde{P}_2,\tilde{P}_3)$ to be represented by z is the greatest possibility value for the quantity (z_1,z_2,z_3) in the converse image of z, $f_j^{-1}(z)$, to be in $\tilde{P}_1\times\tilde{P}_2\times\tilde{P}_3$. Note that whenever $f_j^{-1}(z)=0$, $\mu_{\tilde{S}_j}(z)=0$. Thus, fuzzy numbers can be processed in this manner similar to the non-fuzzy case, and the operations are sometimes called the 'extended operations'(extended addition, extended subtraction, etc.).[6]

In practice, however, the implementation of the solution procedure is not trivial, although the solution of the various extended operations is defined by the extension principle of Eq.(5). The reason is that the solution procedure corresponds to a nonlinear programming problem which is very complex except for the simplest mapping functions. In the present work, the extension principle of Eq.(5) has been implemented by the 'fuzzy weighted average'(FWA) algorithm proposed by Dong and Wong.[6] The computational algorithm of the FWA operation is based on ideas from the α-level representation of fuzzy sets which indicates any membership grade, nonlinear programming implementation of the extension principle, and interval analysis. The method provides a discrete but exact solution to extended algebraic operations in a very efficient and simple manner.[6]

For example, to obtain the simple product of three fuzzy probabilities $\tilde{S}_1=\tilde{P}_1\times\tilde{P}_2\times\tilde{P}_3$ by means of the FWA computational algorithm, it requires the following steps:

(1) Select a particular α-level value(shown on the $\mu_{\tilde{S}_j}(z)$-coordinate in Fig.3) where $0\leq\alpha\leq1$.

(2) Find the interval(s) in fuzzy probabilities of $\tilde{P}_1,\tilde{P}_2$, and $\tilde{P}_3$ which correspond to α (these

are the α-levels of $\tilde{P}_1$,$\tilde{P}_2$, and $\tilde{P}_3$).

(3) Using interval operations, compute the interval(s) in $\tilde{S}_1$ which correspond to those of $\tilde{P}_1 \times \tilde{P}_2 \times \tilde{P}_3$ (the results are the α-levels of $\tilde{S}_1$).

The above steps are repeated for as many values of α as needed to refine the solution.

The eight fuzzy outcomes $\tilde{S}_j(j=1,\cdots,8)$, i.e., possibilities of occurrence of accident pathways as a fuzzy set, given the possibilities of occurrence of top events $\tilde{P}_i$ (i=1,2,3) shown in Table I have been obtained following the procedures outlined above and they are shown in Fig.3. The output fuzzy probability distributions $\tilde{S}_j(j=1,\cdots,8)$ shown in Fig.3 are the result of repeating the above steps for 100 values of α.

2.4 Comparison of Output Fuzzy Probability Distributions

The next question is how to compare the output functions(i.e., the eight fuzzy outcomes) $\tilde{S}_j(j=1,\cdots,8)$ obtained in the previous step and shown in Fig.3. This question is equivalent to the problem of ranking n fuzzy subsets of the unit interval. A number of methods for comparing fuzzy subsets of the unit interval have been suggested and tested in the literature.[7] For the purpose of present work, only two methods are selected and used to compare the output functions $\tilde{S}_j$: One is the 'ranking function' proposed by Yager[7,8] and the other is the 'interval of confidence' suggested by Freeling.[9]

A simple method to order the $\tilde{S}_j(j=1,\cdots,8)$ consists in the definition of a 'ranking function' F mapping each fuzzy set into the real line, where a natural order exists. This approach has been followed by Yager[8] and the ranking can be obtained from the index proposed by Yager. The ranking function used here is the following[8]:

If $\tilde{S}^{\alpha}_j$ is the α-level set of $\tilde{S}_j$ and if $M(\tilde{S}^{\alpha}_j)$ is the mean value of the elements of $\tilde{S}^{\alpha}_j$, then

$$F(\tilde{S}_j) = \int_0^{\alpha \max} M(\tilde{S}^{\alpha}_j) d\alpha \qquad (7)$$

where $\alpha = [0,1]$

Consider, for example, a fuzzy subset $\tilde{S}_1$ with the membership grade shown in Fig.3, where for each grade of membership the dashed line represents the average value of the elements having at least that grade of membership. Then $F(\tilde{S}_1)$ is equal to the area between the dashed line and the membership axis. The advantage of this method is its ability to compare crisp members, discrete fuzzy subsets, and continuous fuzzy subsets of the unit interval. It doesn't require convexity, nor does it require normality of the sets compared.[8] Therefore, this method has been selected here to compare output fuzzy probability distributions $\tilde{S}_j(j=1,\cdots,8)$, and the results obtained by Eq.(7) are shown on the first row in Table II.

Freeling,[9] on the other hand, has made an initial attempt at providing an axiomatic basis where the 'possibilities' are to be interpreted as 'degrees of confidence'. An 'interval of confidence' in R is an ordinary subset of R which represents a type of uncertainty. The symbolic representations of an 'interval of confidence' is usually written as[10]

$$\tilde{S}^{\alpha}_j = [z_a, a_b] \qquad (8)$$

Freeling[9] shows that if the 'intervals of confidence' of α-level in the membership functions are intervals (as in Fig.2), then the 'interval of confidence' at α-level in the fuzzy outcome is an interval. The endpoints of this interval are defined by the extremes of the 'intervals of confidence' at α-level of the membership functions. Therefore, if the 'intervals of confidence' of the inputs are interpreted as defining the range within which the input membership functions lie at degree of confidence α, the range for the output at that level of confidence is simply the 'interval of confidence' at α-level in the output. Figure 4 shows the 'inter-

Table II. Ordered Rank of $F(\tilde{S}_j)$ Values and Intervals of Confidence for 8 Fuzzy Outcomes $\tilde{S}_j$.

Outputs	$\tilde{S}_6$	$\tilde{S}_2$	$\tilde{S}_5$	$\tilde{S}_1$	$\tilde{S}_8$	$\tilde{S}_4$	$\tilde{S}_7$	$\tilde{S}_3$
$F(\tilde{S}_j)+$	0.6401	0.2920	0.0926	0.0522	0.0472	0.0270	0.0112	0.0064
$F(\tilde{S}_j)*$	0.5485	0.2539	0.0783	0.0439	0.0389	0.0222	0.0091	0.0052
$F(\tilde{S}_j)++$	0.5002–0.8200	0.1552–0.4200	0.0–0.0984	0.0–0.0504	0.0–0.0164	00–0.0084	0.0–0.0020	0.0–0.0010
$F(\tilde{S}_j)**$	0.4828–0.7915	0.1498–0.4054	0.0–0.0950	0.0–0.0487	0.0–0.0158	0.0–0.0081	0.0–0.0019	0.0–0.0010

\+ : Fuzzy Averages of Unnormalized Fuzzy Outcomes
* : Fuzzy Averages of Normalized Fuzzy Outcomes
\+ + : Intervals of Confidence at 0.9–level of Unnormalized Fuzzy Outcomes
* * : Intervals of Confidence at 0.9–level of Normalized Fuzzy Outcomes

vals of confidence' at $\alpha=0.9$ level in the output fuzzy $\tilde{S}_j$ values. The numerical values of $\tilde{S}_1^{\alpha=0.9}$ $\tilde{S}_8^{\alpha=0.9}$ that correspond to Fig.4 can be obtained directly from Fig.3 and they are also listed on the third row of Table II for direct comparison with those values calculated by Eq(7). In Fig.3, in particular, notice that the range of $\tilde{S}_6$ values that corresponds to $\alpha=0.9$ level is indicated by dashed lines.

In general, a fuzzy set is called 'normalized' when at least one of its elements attains the maximum possible membership grade. However, the term 'normalized' in the present work is used to denote that fuzzy probabilities satisfy $\sum_{j=1}^{n} F(\tilde{S}_j)=1$. More specifically, if the eight fuzzy probabilities $\tilde{S}_j$(j $=1,\cdots,8$) satisfy $\sum_{j=1}^{8} F(\tilde{S}_j)=1$, where $\tilde{S}_j$ values are such that they satisfy the following equations, then

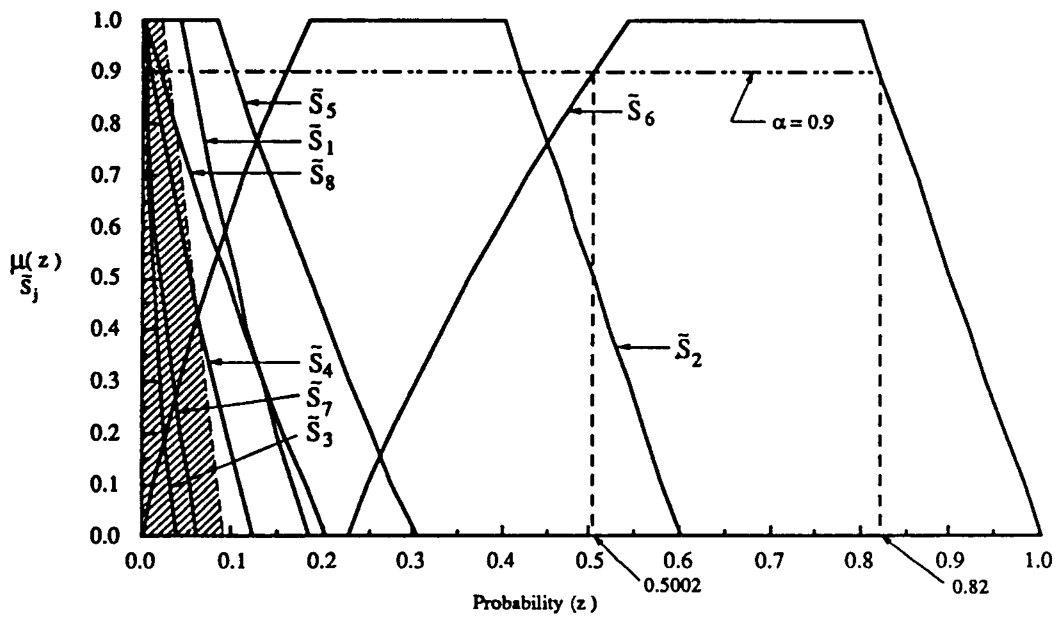

Fig.3. Output Fuzzy Probability Distributions($\tilde{S}_j$) obtained by the FWA Algorthm.

the fuzzy outcomes $\tilde{S}_j$ are normalized in the context of present definition:

$$\tilde{S}_j^* = \int_0^1 \tilde{S}_j^\alpha / N^\alpha \, d\alpha \qquad (9)$$

$$N^\alpha = \sum_{j=1}^{n} [(\tilde{S}_j^\alpha)^L + (\tilde{S}_j^\alpha)^R]/2 \qquad (10)$$

where

$\tilde{S}_j^*$ = normalized fuzzy probability for pathway j,

n = number of fuzzy outcomes(8 in the present work),

$(\tilde{S}_j^\alpha)^L$ = left extreme value of $\tilde{S}_j$ at α-level,

$(\tilde{S}_j^\alpha)^R$ = right extreme value of $\tilde{S}_j$ at α-level,

N^α = algebraic mean value of $(\tilde{S}_j^\alpha)^L$ and $(\tilde{S}_j^\alpha)^R$ corresponding to α-level given by Eq.(10).

The normalized values of fuzzy probabilities $\tilde{S}_j^*$ obtained by this approach are shown on the second and fourth rows of Table II for direct comparison with unnormalized quantities.

3. Application of Fuzzy Set Theory to Zion Accident Progression Analysis

To further examine the applicability of fuzzy set theory to actual evaluation of extremely complex APETs with a number of phenomenological uncertainties, an application of the procedure used in the foregoing example is made to a typical plant damage state(PDS) 'SEC' of the Zion. The representative accident sequence for the PDS 'SECS' is a small LOCA, ECCS failure on injection, and with operational containment sprays but without operational fan coolers.[2] That is, for the purpose of comparison between the two approaches, one by the NUREG-1150 methodology[1] and the other by fuzzy set theory approach, Zion APET for 'SEC' has been evaluated by the two methods.

For convenience in discussion, a brief summary of (1) the methodology of Zion accident progression analysis, (2) Zion APET analysis of 'SEC' by statistical methods, and (3) Zion APET analysis of 'SEC' by fuzzy set theory is presented successively in the following.

3.1 Methodology of Zion Accident Progression Analysis

The risk from a nuclear power plant can be defined by

$$R^k = \sum_i f_i \sum_j C_{ij} \, r_j^k(S_{ij}) \qquad (11)$$

where

R^k = risk of type k (associated with consequence k),

f_i = frequency of PDS i,

C_{ij} = conditional probability of containment release category j given PDS i (i.e., containment matrix),

S_{ij} = fission product source term for containment release category j of PDS i,

r_j^k = consequence of type k, given fission product source term S_{ij}, for release category j.

The first part of the risk analysis('accident frequencies' f_i) represents the estimation of the frequencies of accident sequences leading to core damage. In this part of the analysis, combinations of potential accident initiating events and system failure frequencies are calculated.

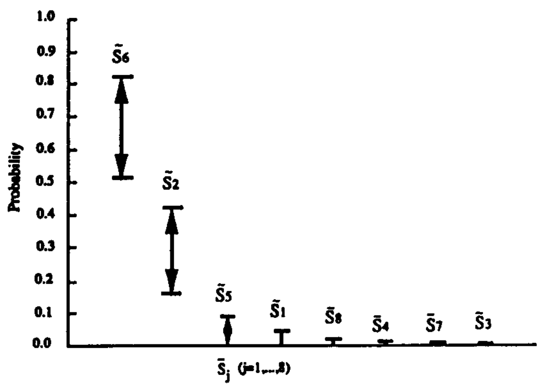

Fig.4. Interval of Confidence at α = 0.9-level in the Fuzzy Outcomes.

The major concern in the present work is the second part of the risk analysis(accident progression, containment loading, and structural response analysis to obtain C_{ij}) which deals with the progression of the accident after the core has begun to degrade. For each general type of accident, defined by the PDSs, the analysis considers the important characteristics of the core melting process, the challenges to the containment building, and the response of the building to those challenges. Event trees were used to organize and quantify the large amounts of information used in this analysis. The event trees combined information from many sources, e.g., detailed computer accident simulations and panels of experts providing interpretations of available data.[1]

The principal steps of the 'accident progression analysis' are : (1) development of APETs, (2) probabilistic quantification of event trees, and (3) grouping of event tree outcomes into a smaller set of'accidecnt progression bins'. In the Zion study,[2] the APET in the form of computer codes(such as EVNTRE and EVNTREISS) provided the necessary framework for quantification of the likelihood of various failure modes. The structure of the Zion APET is based on 59 top events, many of which have multiple outcomes or branches. The list of top event questions for the Zion APET can be found in Ref. 2. Depending on the type of input, there are six different types of top events as shown in Table III.

3.2 Zion APET Analysis of 'SEC' by Statistical Methods

The uncertainty analysis in Zion APET relies on the selection of key uncertainty issues that can have a significant impact on the estimated risk at Zion. The approach used in the selection and evaluation of key uncertainty issues for Zion APET is essentially the same as that used for Surry[11] : Uncertainties in the estimates of containment loading and performance were treated through a stratified Monte Carlo sampling procedure called LLH. The elicitation of expert judgements was necessary to develop the probability distributions for some individual parameters in this uncertainty analysis. For certain key issues in the uncertainty analysis, panels of experts were convened to discuss and help to develop the needed probability distributions.[1]

For statistical quantification of the Zion APET, the EVNTREISS code[2] is used with 'issue' and 'sample' data along with the three input data required in the EVNTRE code(i.e., data for 'binning', 'branch-point probability', and 'dependency').

The product of the accident progression and containment loading analysis is a set of accident progression bins. Each bin consists of a group of postulated accidents(with associated probabilities for each PDS) that have similar outcomes with

Table III. Six Types of APET TOP Events.

Type of Input	Dependency Upon Prior Events	
	Independent	Dependent
(1) Branch Point Probabilities Only	Type 1	Type 2
(2) Branch Point Probabilities and Parameter Values	Type 3	Type 4
(3) A Set of Parameters to be Summed and Compared to Reference Parameters to be Obtain Branch Point Probabilities	Type 5	Type 6

respect to the subsequent portion of the risk analysis(i.e., analysis of radioactive material transport). Quantitatively, the product consists of a matrix of conditional probabilities, with rows and columns defined by the sets of PDSs and accident progression bins, respectively.[1]

The results of statistical APET analysis for the plant damage state 'SEC'(selected from 14 PDSs of Zion) is shown in Table IV to provide a direct comparison with the results obtained by fuzzy set theory approach. The numbers shown on the second column(in Table IV) are the mean conditional probabilities obtained by the LLH approach. These LLH results were obtained by application of the EVNTREISS computer code that has incorporated the LLH sampling technique for the key uncertainty issues of the containment loading and performance. The number of LLH samples was limited to one hundred.

3.3 Zion APET Analysis of 'SEC' by Fuzzy Set Theory Approach

In order to analyze the Zion APET for the plant damage state 'SEC' by fuzzy set theory and compare directly with the results obtained by the LLH procedure, the following steps are taken : Except for the eight containment loading and performance issues, all other input data for the Zion APET remain unchanged(sample input data for the Zion APET can be found in Ref. 2).

For the key eight issues included in the Zion APET uncertainty analysis by the LLH procedure,[2] the 'issue data' for the EVNTREISS code are replaced by fuzzy inputs prepared for each issue. In addition, the EVNTREISS code has been modified to treat the fuzzy set theory procedure used in the example application.

In the Zion analysis,[2] the different sets of values for branches of each issue are called 'level'. The levels may represent different opinions about the severities of a certain physical phenomenon. However, the probability of occurrence of each 'level' can be different from each other. The 'weighting factors' are used for this subjective probability of occurrence of each level. To prepare fuzzy inputs, the weighting factors and branch point probabilities for each level used for each issue in the uncertainty analysis of the Zion APET have been transformed into 'triangular fuzzy numbers(TFN)[10] as shown in IN Fig.5 : A 'TFN' can be defined by a triplet($z_{j,min}, z_{j,m}, z_{j,max}$). For a given dependency case in a given issue, the 'minimum' and the 'maximum' values shown in Fig.5 correspond to the minimum and the maximum probabilities of all levels given for a branch point in the given dependency case. The 'mode' of a branch point shown in Fig.5, on the other hand, is the summation of the products of the weighting factors and the branch point probabilities for each level. These quantities can be more concisely expressed in mathematical forms. Suppose that Table V is the given LLH issue data, then the minimum, maximum, and modes of triangular fuzzy numbers that correspond to the values shown in Table V can be expressed as :

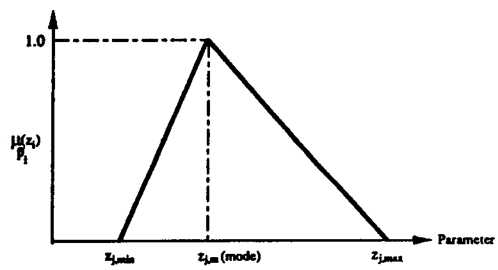

Fig.5. Fuzzy Input Constructed from LLH Issue Data.

$$\text{Modes}: \; z_{j,m} = \sum_{i=1}^{4} z_{ij} w_i, \quad j = 1,2,3,$$

$$\text{Minimum value}: \; z_{j,min} = \min\left[z_{1j}, z_{2j}, z_{3j}, z_{4j}\right], \quad (12)$$

$$\text{Maximum value}: \; z_{j,max} = \max\left[z_{1j}, z_{2j}, z_{3j}, z_{4j}\right].$$

These fuzzy inputs are obtained based on the

Table IV. Mean Conditional Probabilities Obtained by LLH Approach and Fuzzy Average Outcomes for PDS 'SEC'.

Bin No.	Mean Values of LLH Results	Fuzzy Average Outcomes
Bin1	0.0	0.0
Bin2	0.0	0.0
Bin3	3.908–3	4.308–3
Bin4	1.240–4	1.405–4
Bin5	0.0	0.0
Bin6	0.0	0.0
Bin7	2.302–3	2.263–3
Bin8	6.908–2	2.349–2
Bin9	3.150–2	3.257–2
Bin10	8.507–3	2.187–2
Bin11	0.0	0.0
Bin12	0.0	0.0
Bin13	0.0	0.0
Bin14	0.0	0.0
Bin15	8.664–1	9.088–1
Bin16	0.0	0.0
Bin17	0.0	0.0
Bin18	4.447–3	1.845–3
Bin19	1.373–2	4.681–3

assumption that the mode($z_{j,m}$) is the most possible value(possibility approaches one) in a given range and the extremes are the least possible values(possibility approaches zero). Fuzzy numbers of this type are very simple to manipulate. The membership function is then obtained from these triangular fuzzy numbers. The rest of the procedure to obtain the final fuzzy probabilities for each pathway on the APET is essentially the same as the steps used in the example application.

The results of the Zion APET analysis for 'SEC' obtained by the fuzzy set theory approach are shown on the third column in Table IV. To allow for meaningful comparisons with the mean values of the LLH results, only the fuzzy averages(that is, the values calculated from ranking function F) of the fuzzy outcomes are given in this table.

Table V. Representation of a Typical LLH Issue Data.

Branch Point / Level No.	b_1 b_2 $\overline{b_3}$	Weighting Factors
level 1	z_{11} z_{12} z_{13}	w_1
level 2	z_{21} z_{22} z_{23}	w_2
level 3	z_{31} z_{32} z_{33}	w_3
level 4	z_{41} z_{42} z_{43}	w_4

Constraints : $\sum_{i=1}^{4} w_i = \sum_{j=1}^{3} z_{ij} = 1$,

where : b_j = branch point for a given issue(top event)

z_{ij} = probability for bronch point j and for level i

w_i = weighting factor for level i

4. Results and Discussion

In the preceding sections, an effort has been made to establish a formal procedure of application of the fuzzy set theory to APETs with qualitative and imprecise input data. In addition, the computational algorithms suitable for application of the fuzzy set theory to the analysis of APETs, in particular, are selected and illustrated with example applications.

In the example application, it is shown that the major computational steps of the fuzzy set theory application to the analysis of APETs with imprecise input data are :

1. Conversion of qualitative fuzzy variables into quantitative fuzzy probabilities by means of membership functions.
2. Calculation of fuzzy probabilities for each pathway using the computational algorthm of FWA operation.[6]
3. For meaningful comparisons between the output fuzzy probability distributions, the 'ranking function' proposed by Yager[7,8] or the 'interval of confidence' suggested by Freeling[9] can be used. In addition, normal-

ized values of fuzzy probabilities can be obtained by Eqs.(9) and (10).

The applicability and validity of the above procedure have been re-examined by extending the same procedure to the more complex and representative problem, i.e., the plant damage state 'SEC' of the Zion. From the final results shown in Table IV, it can be observed that the fuzzy averages of the fuzzy outcomes are very close to the mean values obtained by the LLH approach shown in the same table. It should be recalled, however, that the fuzzy inputs used in the form of TFNs are based on the same weighting factors and branch point probabilities used in the LLH analysis of the Zion APET. This fact implies that when the membership functions are constructed in the form of TFNs based on the same input data and judgements of experts, then the LLH procedure of the APET uncertainty analysis can be effectively replaced by the fuzzy set theory approach suggested in the present work without considerably changing the final outcomes.

From the applications made here, it can be inferred that there are some advantages and disadvantages of the fuzzy set theory for practical applications. The main advantages of the fuzzy-set calculus are that it is well suited for APET analyses when the phenomenon and evidence is itself fuzzy in nature and that it is very flexible. The major disadvantage, on the other hand, is that it is not always clear how to construct reasonable membership functions. Various methods have been proposed including the use of statistical data and the composition of simpler functions, but no completely general approach seems to exist yet. However, this problem is beyond the scope of this paper.

5. Concluding Remarks

In conclusion, fuzzy set theory is proposed to be used in the analysis of the APET with imprecise fuzzy branch probabilities as an alternative to the methods currently used in the risk assessment of nuclear power plants. This paper provides a formal procedure for applying the fuzzy set theory to APETs with imprecise and qualitative branch probabilities and/or with a number of physical uncertainty issues. In addition, the computational algorithms suitable for application of the fuzzy set theory to the APET analysis for each step are identified and illustraited with example applications.

However, a considerable further research is necessary to find the most reasonable methods to construct proper membership functions, in particular, for the application of the fuzzy set theory to the APET. More comparative studies of fuzzy and probabilistic approaches to the APET analysis are also needed. The results presented in this paper should be viewed as a first attempt at constructing a formal procedure for practical applications of the fuzzy set theory to the analysis of APETs with imprecise and phenomenological uncertainties.

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Nomenclature

A_i : top event numbers in the event tree

APET : accident progression event tree

b_j	: barnch point for a given issue(top event)
C_{ij}	: containment matrix
DCH	: direct containment heating
ECCS	: emergency core cooling system
F	: ranking function suggested by Yager
f_i	: frequency of PDS i
f_j	: fuzzy output function for pathway j
f_j^{-1}	: converse image of z
$F(\tilde{S}_j)$	: fuzzy average of fuzzy outcome $\tilde{S}_j$
FWA	: fuzzy weighted average
LLH	: limited Latin hypercube
LOCA	: loss of coolant accident
min	: minimum
$M(\tilde{S}_j^{\alpha})$	: mean value of the elements of $\tilde{S}_j^{\alpha}$
n	: number of fuzzy outcomes
N^{α}	: algebraic mean value of $(\tilde{S}_j^{\alpha})^L$ and $(\tilde{S}_j^{\alpha})^R$
PDS	: plant damage state
$\tilde{P}_i$	: fuzzy input probability(qualitative variables)
PRA	: probabilistic risk assessment
R	: real line
r_j^k	: consequence of type k for containment release category j
R^k	: risk of consequence type k
S_{ij}	: fission product source term for release category j of PDS i
$\tilde{S}_j$	: fuzzy outcome for pathway j(fuzzy probability)
$\tilde{S}_i^{\alpha}$	: α-level set of $\tilde{S}_j$(or interval of confidence of α-level in the membership function)
$(\tilde{S}_j^{\alpha})^L$	: left extreme value of the α-level set of $\tilde{S}_j$
$(\tilde{S}_j^{\alpha})^R$	: right extreme value of the α-level set of $\tilde{S}_j$
$\tilde{S}_j^*$	: normalized fuzzy outcome of $\tilde{S}_j$
Sup	: supremum
TFN	: triangular fuzzy number
z	: potential probability for fuzzy outcomes
z_a, z_b	: left and right extreme values of α-level set in a fuzzy set, respectively
z_i	: potential probability for fuzzy variables
z_{ij}	: probability for branch point j and for level i
$z_{j,m}$	: mode of the triangular membership function
$z_{j,max}$	: maximum value of the triangular membership function
$z_{j,min}$	: minimum value of the triangular membership function
w_i	: weighting factor for level i for LLH issue data
α	: α-level of membership function
$\mu_{\tilde{P}_i}(z_i)$	: membership function of fuzzy probability $\tilde{P}_i$
$\mu_{\tilde{S}_i}(z)$	: membership function of fuzzy outcome $\tilde{S}_i$

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