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Two–Dimensional Approach for Stress Intensity Factor Solution of a Semi–Elliptical Crack

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2차원적 해석을 통한 반타원 결함의 응력세기계수 산출

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Abstract

An engineering approach for estimating the stress intensity factors of a semi-elliptical crack is presented. An approximate 2–dimensional approach solution for semi-elliptical crack is derived in terms of simple equation, through weight function technique, by reflecting on the physical character of cracks.

요 약

본 연구에서는 반타원 표면결함의 응력세기계수를 계산하는 공학적–일반식을 유도하였다. 가중치함수를 이용하여 임의의 응력이 작용할 시에 평판 및 실린더에 존재하는 반타원결함에 대한 응력세기계수 산출에 유한요소해석이나 탄성파괴 핸드북을 사용하지 않고 2차원적 해석방법을 이용한 단일화된 식으로서 쉽게 계산할 수 있는 방법을 제시하였다. 본 방법에 의한 계산결과는 기존에 발표된 연구물에 제시된 값과 잘 일치하였으며, ASME의 산출방법보다도 현실적이며 실제 경우에 사용하기 용이함을 보여준다.

1. Introduction

When assessing the structural integrity of nuclear power plants components, it is often required to employ failure assessments.

Depending on whether a ductile or brittle fracture behavior is expected under service conditions, stress intensity factor or J-integral is most widely used for failure assessments. Particularly, the evaluation of the possibility of brittle failure is based on the stress intensity factor, K_I. A crack will propagate when the applied stress intensity factor

*Formerly With Korea Power Engineering Company exceeds the material fracture toughness, K_{IC}, at the current temperature.

In general, the analysis of stress intensity factor is specialized for through-thickness or semi-elliptical cracks in an infinite plate or a cylinder. Since Reactor pressure vessel operate around 300℃, brittle fracture does not look like a potential mode of fracture. However brittle fracture behavior should be considered in the viewpoint of the neutron-irradiation-induced reduction in fracture thoughness of vessel materials in core beltline region, the pressurized thermal shock(PTS) loads, etc..

Various methods[1,2] of calculation for stress intensity factor have been developed for 2–dimensional crack problems. Even though there are a few methods[3,4] of analysis for 3–dimensional crack or semi-elliptical crack, these methods mostly relied on finite element method (FEM) for the magnification factor calculation. Morever there was significant scatter among these data. And we should consider the relatively large magnitude of safety factors applied in actual flaw assessments[5]. Therefore it would be very useful to derive a simple, approximate equation for a calculation of stress intensity factors.

In the present work, the mode I analysis problem of a semi-elliptical crack in a semi-infinite body is treated through weight function technique including the effects of the geometrical crack character.

2. Analytical Study

2.1. Background

Labbens *et al.*[2] introduced weight functions to calculate stress intensity factor for the 20–dimensional crack analysis.

Bueckner[3] and Rice[6] demonstrated that a particular function, normally termed the Bueckner weight function, is a property of a cracked geometry and is independent of the loading. The weight function may be employed in the derivation of stress intensity factor solutions provided details of crack-line loading are available. A weight function may be thought of as a form of the Green's function for a cracked body.

Bueckner[3] proposed the weight function for a finite width, edge-cracked, infinite strip, as shown in Fig.1. This weight function is given as:

$$M(x,a)=\left[\frac{1}{2\pi(a-x)}\right]^{1/2}\left[1+m_1\left[\frac{a-x}{a}\right]+m_2\left[\frac{a-x}{a}\right]^2\right] \quad (1)$$

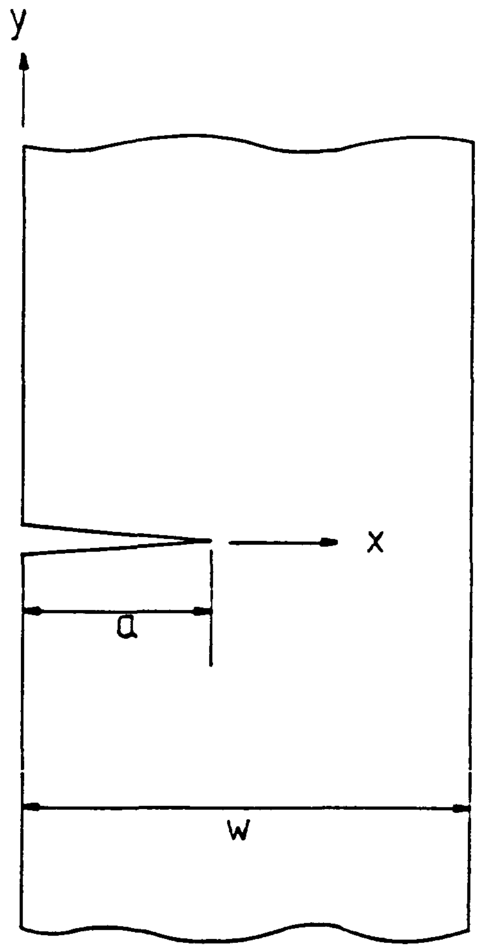

Fig.1. Continuous Surface Crack in An Infinite Strip

where m_1 and m_2 are functions of the ratio of crack depth to strip width (a/w). For the region of $0 \leq a/w \leq 0.5$ these values are given as:

$$m_1 = A_1 + B_1\left[\frac{a}{w}\right]^2 + C_1\left[\frac{a}{w}\right]^6$$

$$m_2 = A_2 + B_2\left[\frac{a}{w}\right]^2 + C_2\left[\frac{a}{w}\right]^2$$

where

$A_1=0.6147,\ B_1=17.1844,\ C_1=\ 8.7822$

$A_2=0.2502,\ B_2=\ 3.2899,\ C_2=70.0444.$

Then, $K_I(a)$, stress intensity factor with a function of crack depth, a, could be obtained from the following equation. That is,

$$K_I(a) = \int_0^a \sigma(x)M(x,a)dx \quad (2)$$

where $\sigma(x)$ is the arbitrary stress distribution along the normal plane to the crack depth in the uncracked structures.

2.2 Theoretical Approach

In order to obtain the exact solution for the semi-elliptical crack problems(Fig.2), 3–dimensional analysis is required. The main objective of this study was to simplify this complicated 3–dimensional analysis. This simplification could be perfomed through 2–dimensional approach based on the above equation (2) with some geometrical constraint factors such as a semi-elliptical crack shape and a finite crack length.

Since equation (1) was derived in the plate-shape geometry, it is required to modify this equation for the application in cylindrical structures(Fig.3).

The weight function is related to the crack shape and crack tip displacement. Crack tip displacement in the cylindrical structure would be more restricted than in the plate. The displacement at an arbitrary distance x from innner surface of the cylinder is a function of the ratio of distance from cylinder center to the crack tip[2]. According to Labbens and Heliot[2], the modified weight function for cylindrical structures could be given as:

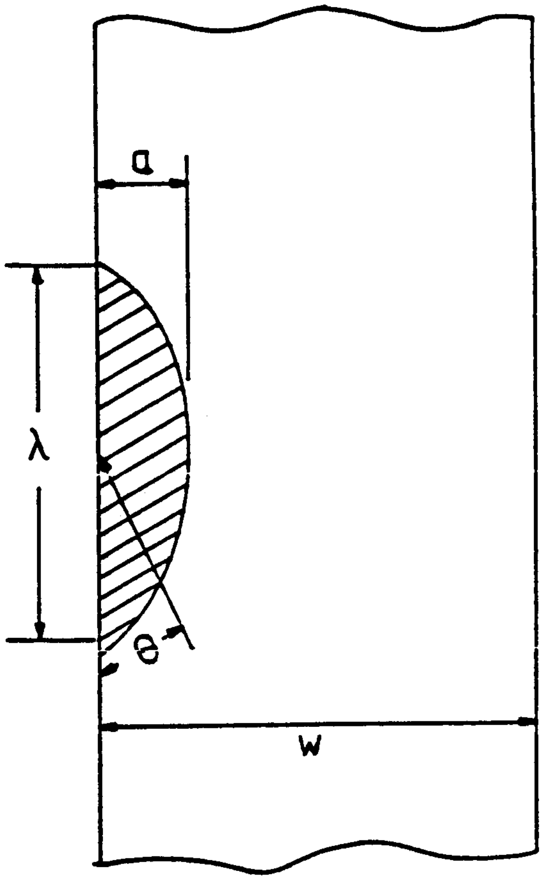

Fig.2. Semi-elliptical Surface Crack in An Infinte Strip

$$M(x,a) = \left[\frac{R_i+x}{R_i+a}\right]\left[\frac{1}{2\pi(a-x)}\right]^{1/2}\left[1+m_1\left[\frac{a-x}{a}\right] + m_2\left[\frac{a-x}{a}\right]^2\right] \quad (3)$$

where R_i is a inner radius.

1) Semi-elliptical Crack Shape

When reviewing the stress intensity factor solution for semi-elliptical cracked structure[7], the

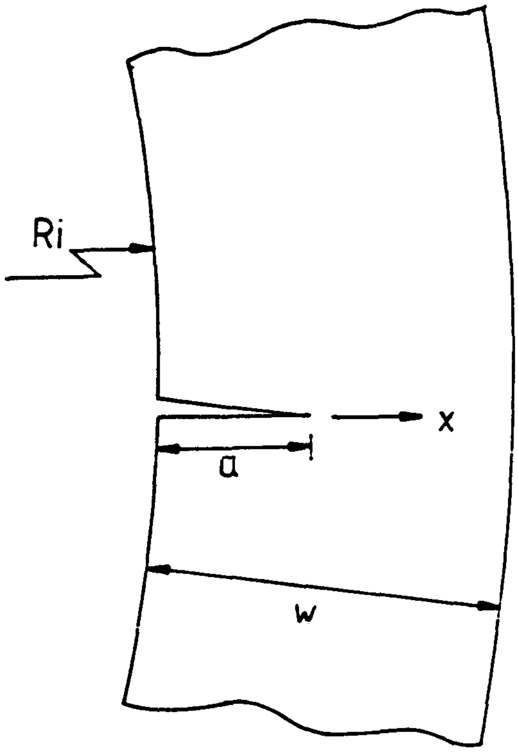

Fig.3. Continuous Surface Crack in A Cylinder

line integral Φ has been designated a shape factor because its value depends on the aspect ratio (crack depth/crack length : a/λ). Therefore, in this study the line integral Φ was classified as one of the constraint factor. Then the modified stress intensity factor is given as :

$$K_I(a)=\frac{1}{\Phi}\int_0^a \sigma(x)\, M(x,a)\left[\frac{R_i+x}{R_i+a}\right]dx \qquad (4)$$

$$\Phi=\int_0^{\pi/2}\left[1-\left[\frac{\lambda^2-a^2}{\lambda^2}\right]\sin^2\theta\right]^{1/2}d\theta$$

2) Finite Crack Length

Accroding to equation (1) the value of weight function at the free surface is $1+m_1+m_2$ and becomes infinite at the crack tip. Furthermore the value of weight function for semi-elliptical cracks with finite crack length is much less than that for continuous cracks at any given intermediate points between the free surface and the crack tip. Therfore m_1 and m_2 in the weight function for semi-elliptical cracks can be modified as :

$$m_1=A_1+B_1\left[\frac{1}{C\Phi^2}\right]\left[\frac{a}{w}\right]^2+C_1\left[\frac{1}{C\Phi^2}\right]\left[\frac{a}{w}\right]^6$$

$$m_2=A_2+B_2\left[\frac{1}{C\Phi^2}\right]\left[\frac{a}{w}\right]^2+C_2\left[\frac{1}{C\Phi^2}\right]\left[\frac{a}{w}\right]^6.$$

In The above equations Φ represents line integral and C is constant. If a/λ is O, the constant terms (B_1, C_1, B_2, and C_2) represent the values for continuous crack, and if a/λ is 1, these constants represent the values for the semi-circular crack.

The approximate stress intensity factor solution including these geometric contraint factor is given as :

$$K_I(a)=\frac{1}{\Phi}\int_0^a \sigma(x)\, M(x,a)dx \qquad (5)$$

$$M(x,a)=\left[\frac{R_i+x}{R_i+a}\right]\left[\frac{1}{2\pi(a-x)}\right]^{1/2}\left[1+m_1\left[\frac{a-x}{a}\right]+m_2\left[\frac{a-x}{a}\right]^2\right]$$

$$m_1=A_1+B_1\left[\frac{1}{C\Phi^2}\right]\left[\frac{a}{w}\right]^2+C_1\left[\frac{1}{C\Phi^2}\right]\left[\frac{a}{w}\right]^6$$

$$m_2=A_2+B_2\left[\frac{1}{C\Phi^2}\right]\left[\frac{a}{w}\right]^2+C_2\left[\frac{1}{C\Phi^2}\right]\left[\frac{a}{w}\right]^6.$$

The arbitrary stress distribution can be expressed by a series of polynomial, i.e., $\sigma(x)=A_0+A_1x+A_2x^2+A_3x^3+\cdots+A_nx^n$. However this stress distribution can be fitted by a third degree polynomial, i.e.,

$$\sigma(x)=A_0+A_1x+A_2x^2+A_3x^3. \qquad (6)$$

With inserting the above stress distribution, equation (5) becomes :

$$K_I(a)=\frac{1}{\Phi}\int_0^a\left[\left[\frac{R_i+x}{R_i+a}\right][A_0+A_1x+A_2x^2+A_3x^3]\cdot\left[\frac{1}{2\pi(a-x)}\right]^{1/2}\left[1+m_1\left[\frac{a-x}{a}\right]+m_2\left[\frac{a-x}{a}\right]^2\right]\right]dx \qquad (7)$$

$$K_I(a)=\frac{1}{\Phi}\ \frac{1}{R_i+a}\left[\frac{a}{2\pi}\right]^{1/2}\Big[A_0(R_i\cdot f_{01}+a\cdot f_{02})+$$

$$A_1(R_i\cdot f_{11}+2\cdot a\cdot f_{12})a\ +A_2(R_i\cdot 2\cdot f_{21}+6\cdot a\cdot f_{22})a^2$$

$$+A_3(R_i\cdot 6\cdot f_{31}+24\cdot a\cdot f_{32})a^3\Big] \tag{8}$$

where

$$f_{01}=2+\frac{2}{3}m_1+\frac{2}{5}m_2$$

$$f_{02}=\frac{4}{3}+\frac{4}{3\cdot 5}m_1+\frac{4}{5\cdot 7}m_2$$

$$f_{11}=f_{02}$$

$$F_{12}=\frac{8}{3\cdot 5}+\frac{8}{3\cdot 5\cdot 7}m_1+\frac{8}{5\cdot 7\cdot 9}m_2$$

$$f_{21}=f_{12}$$

$$f_{22}=\frac{16}{3\cdot 5\cdot 7}+\frac{16}{3\cdot 5\cdot 7\cdot 9}m_1+\frac{16}{5\cdot 7\cdot 9\cdot 11}m_2$$

$$f_{31}=f_{22}$$

$$f_{32}=\frac{32}{3\cdot 5\cdot 7\cdot 9}+\frac{32}{3\cdot 5\cdot 7\cdot 9\cdot 11}m_1+$$

$$\frac{32}{5\cdot 7\cdot 9\cdot 11\cdot 13}m_2$$

$$m_1=A_1+B_1\left[\frac{1}{C\Phi^2}\right]\left[\frac{a}{w}\right]^2+C_1\left[\frac{1}{C\Phi^2}\right]\left[\frac{a}{w}\right]^6$$

$$m_2=A_2+B_2\left[\frac{1}{C\Phi^2}\right]\left[\frac{a}{w}\right]^2+C_2\left[\frac{1}{C\Phi^2}\right]\left[\frac{a}{w}\right]^6$$

Equation (7) can be programmed very easily with FORTRAN or BASIC language. Therefore we can calcuate the stress intensity factor without relying on the Elastic Fracture Handbook[8].

3. Comparison With Other Methods

In order to judge the analytically developed expression for the K versus a/λ relationship, the present results were compared to those calculated by Labbens *et al.* [2] and pc-CRACK[9] (commercial software for fracture analysis). The parameter used for the comparison is magnification factor.

Labbens *et al.*[2] expressed the stress intensity factor as follows:

$$K_I=\sqrt{\pi\ a}\cdot\Big[A_0\cdot F_1+\frac{2\cdot a}{\pi}A_1\cdot F_2$$

$$+\frac{a^2}{2}A_2\cdot F_3+\frac{4\cdot a^3}{3\cdot\pi}A_3\cdot F_4 \tag{9}$$

where F_1, F_2, F_3, F_4 are magnification factors.

Through the use of equation (7) to (9), magnification factors for the present method are derived as

$$F_1=\sqrt{2}\,(R_i\cdot f_{01}+a\cdot f_{02})\cdot\frac{1}{(R_i+a)}\cdot\frac{1}{\pi}$$

$$F_2=\frac{1}{\sqrt{2}}\,(R_i\cdot f_{11}+a\cdot 2\cdot f_{12})\cdot\frac{1}{(R_i+a)}$$

$$F_3=\sqrt{2}\,(R_i\cdot 2\cdot f_{21}+a\cdot 3\cdot 2\cdot f_{22})\cdot\frac{1}{(R_i+a)}\cdot\frac{1}{\pi}$$

$$F_4=\frac{3}{\sqrt{8}}\,(R_i\cdot 3\cdot 2\cdot f_{31}+a\cdot 4\cdot 3\cdot 2\cdot f_{32})\cdot\frac{1}{(R_i+a)}$$

Fig.4 shows the magnification factors with the variation of crack aspect ration(a/λ) for the given crack depth/thickness ratio($a/w=0.25$). Magnification factors decrease as the crack shape changed from continuous to semi-elliptic.

When a/λ ratio is above 0.1, magnification factors exhibited almost constant values.

The relation between magnification factors and fractional crack depth(a/w) is shown in Fig.5, including the results of Labbens and pc-CRACK, for the continuous surface crack in cylinder.

As shown in Fig. 5, magnification factors calculated by using equation(9) are relatively consistent and in good agreement with those obtained from Labbens *et al.* [2] and pc-CRACK[9]

There are some discrepancies among these values, when a/w is above 0.5. However the use of present method seems reasonable in practical application, since the a/w ratio for real cracks is usually much smaller than 0.5. It should be noted that constant terms, C_1 and C_2, suggested by Lab-

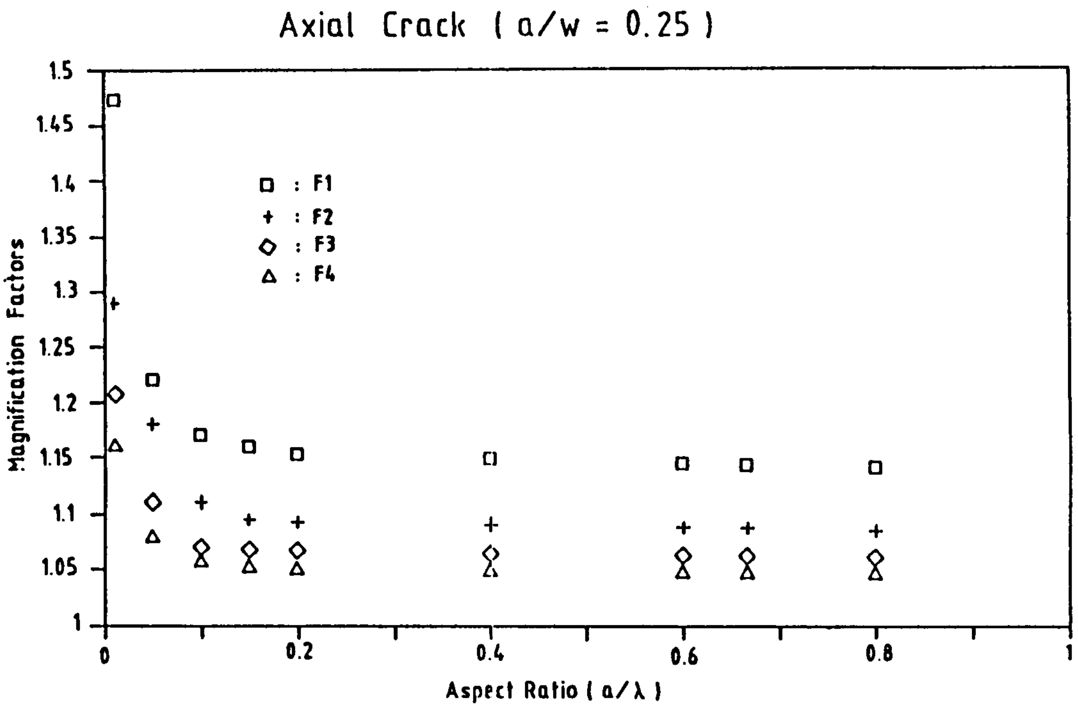

Fig.4. Magnification Factors vs. Aspect Ratio(a/ λ)

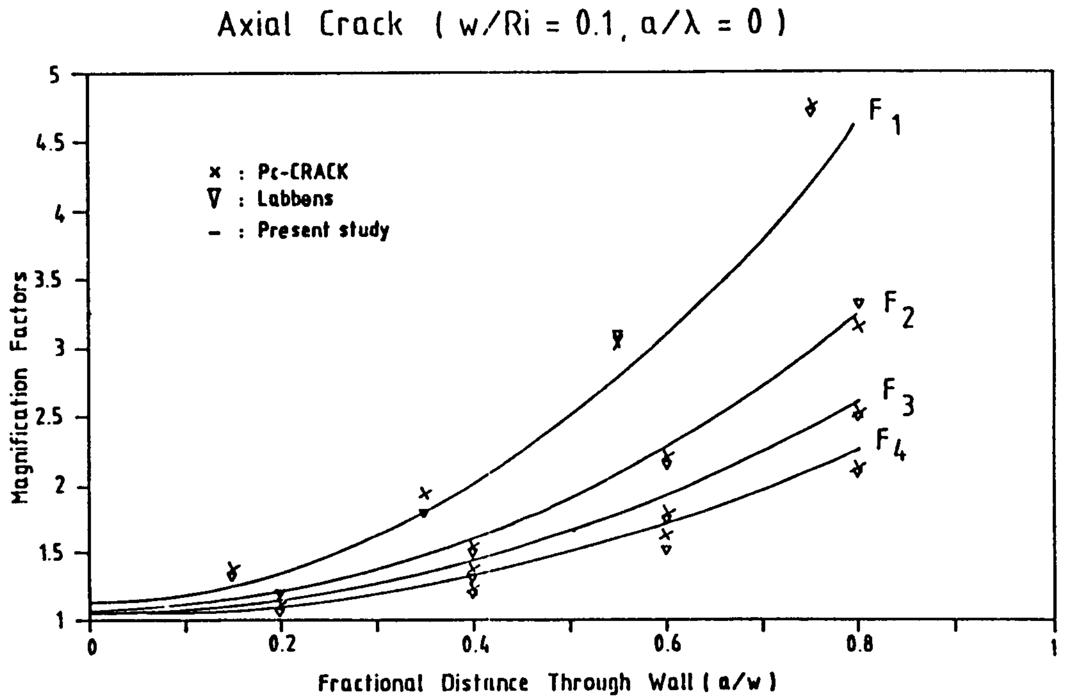

Fig.5. Magnification Factors

bens *et al.* [2] were neglected in order to minimize the data deviation, then m_1 and m_2 were represented as a second order polynomial terms.

4. Application Example

The example chosen is typical of nuclear pressure vessel (Outer Radius: R=100″, w/R=0.1). Two stress distributions are chosen, which were used in *Ref.* [10].

$\sigma_1(x)=40-8.9120x+0.974 \cdot x^2-0.0382 \cdot x^3$

$\sigma_2(x)=40-17.824 \cdot x+1.946 \cdot x^2-0.0765 \cdot x^3$

(1) Continuous Surface Crack in Cylinder (a/w =0.25, a/λ =0)

For the given stress distributions, stress intensity factors obtained from the present method were compared to the results of Ref. [10], as tabulated in Table 1.

Table–1 Stress Intensity Factors for Continuous Surface Crack

Calculation Method	Stress Intensity Factor	
	stress : σ_1	stress : σ_2
Maximum Stress	174.8	174.8
Linear Envelope	154.6	134.3
ASME	129.6	84.3
Present Study*	123.5	81.6
FEM by Buchalet	115.8	76.4

The values of stress intensity factor obtained from the present method is obviously closer to FEM values than that of other calculation methods.

(2) Semi-elliptical Surface Crack in Cylinder

Actual cracks in structural elements are often approximated by semi-elliptical cracks. Therefore comparison is extended to ASME-III APPENDIX–G type crack(a/λ =1/6, a/w=0.25). The results are shown in Table 2.

The values obtained from the present study is definitely less conservative than those of other calcuation methods [10].

Table–2 Stress Intensity Factors for ASME–III App.G–Type Crack

Calculation Method	Stress Intensity Factor	
	stress : σ_1	stress : σ_2
Maximum Stress	134.7	134.7
Linear Envelope	116.9	99.0
ASME	94.8	55.0
Present Study*	83.7	52.3
FEM by McGowan	75.8	46.9

(3) Variation of Stress Intensity Factors as a Function of a/w Ratio

By means of the present technique stress intensity factors could simply be calculated at any a/w ratio, as presented in FIg.6.

5. Conclusions

The semi-elliptical crack problems are considered to be important especially in the design and fracture analysis of pressure vessels and casings. The complicated 3–dimensional analysis is required in order to obtain the exact solution for the semi-elliptical crack problems. However, this complication could successfuflly be overcome through 2–dimensional approach based on the continuous crack solution, by taking into account geometric crack characters such as semi-elliptical crack shape and finite crack length. It would be a great advantage and was an objective herein to be able to estimate the K versus a/λ relationship for a semi-elliptical crack. Furthermore, the present solution appears less conservative than that of ASME–XI technique.

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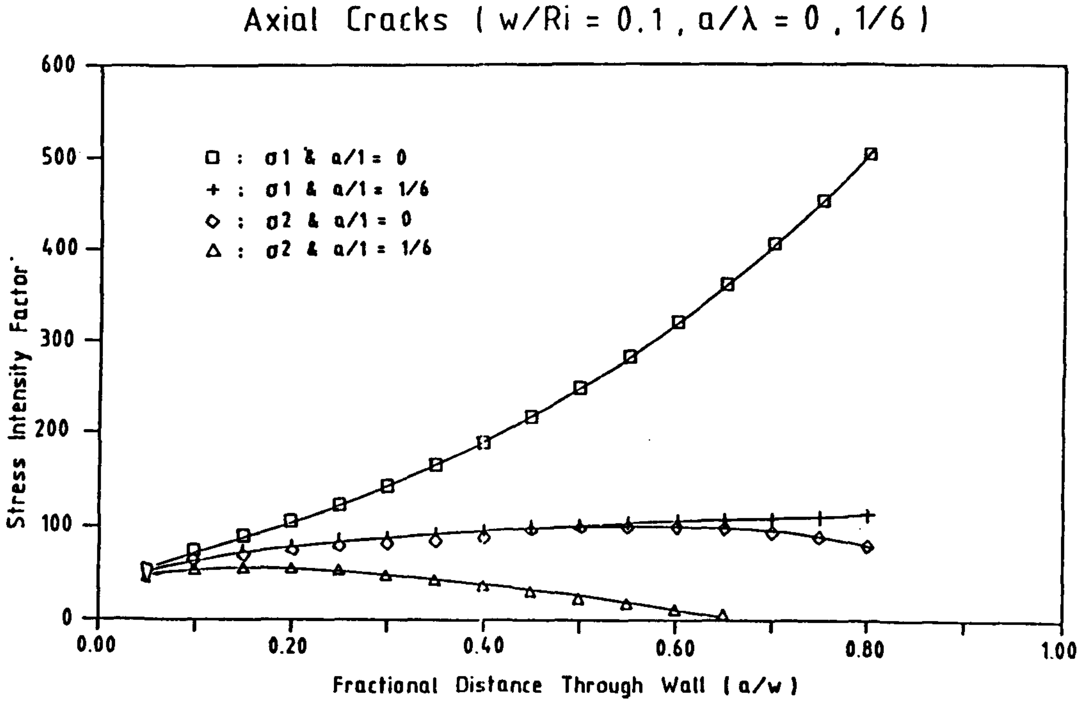

Fig.6. Stress Intensity Factors vs. Crack Depth with Various Stress Distribution

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