Journal of the Korean Nuclear Society
Volume 23, Number 3, September 1991

Application of Cubic Spline Synthesis in On-Line Core Axial Power Distribution Monitoring

Wang Kee In, Hyung Keun Yoo, Geun Sun Auh, Chong Chul Lee and Si Hwan Kim
Korea Atomic Energy Research Institute

(Received December 31, 1990)

실시간 노심출력분포 측정을 위한 3차 SPLINE합성법의 응용

인왕기 · 유형근 · 어근선 · 이종철 · 김시환
한국원자력연구소

(1990. 12. 31 접수)

Abstract

The Core Operating Limit Supervisory System (COLSS) is digital computer based on-line monitoring system that is designed to assist the operator in monitoring of the Limiting Conditions for Operation. A current COLSS calculates axial power distribution based on in-core detector signals using 5th order Fourier series method. It was found that the 5th order Fourier series method was not accurate for certain axial power shapes, especially saddle power shapes, resulting in thermal margin decrease.

A cubic spline synthesis was applied to the COLSS in order to improve the axial power distribution monitoring for the various axial power shapes. The results showed that the cubic spline synthesis simulated more accurately the axial power shapes, up to 5% in RMS errors, compared to those of the Fourier series.

요　약

COLSS는 정상 운전시 DNBR 및 LHR의 운전 제한 조건을 감시하는 디지탈 노심감시계통이다. 영광 3,4호기 COLSS는 현재 노내 계측기 신호를 입력으로 하여 5차의 Fourier 합성법에 의해 노심의 축방향 출력분포를 계산한다. 그러나 5차의 Fourier 합성법은 특정의 축방향 출력 형태, 특히 말 안장 모양의 출력분포에 대해서 그 정확성이 떨어져 노심의 운전 여유도를 감소시키는 요인이 되고 있다.

본 연구에서는 축방향 출력분포 계산의 정확성을 증대 시키기 위해 COLSS에 Cubic Spline 합성법을 적용 하였다. 그 결과, Fourier 합성법을 적용한 기존의 COLSS보다 RMS 오차의 관점에서 최고 5%까지 그 정확도가 향상 되었다.

Ⅰ. Introduction

Maintaining a nuclear power plant within its Limiting Conditions for Operation(LCOs) is a necessary condition for safe operation and acceptable transient consequences. These LCOs are delineated in the Technical Specifications. There are many systems in a nuclear power plant that are used to help the operators maintain the plant within the LCOs. One such system is the Core Operating Limit Supervisory System (COLSS)[1]. COLSS (Fig. 1) is a digital computer based on-line monitoring system that is designed to assist the operator in monitoring of the various types of LCOs.

The current COLSS axial power distribution is calculated by synthesizing in-core detector power signals using a 5th order Fourier series method[2]. For each of the five detector levels, the assembly relative powers calculated previously are averaged over all in-core locations with valid signals. This average at each level is then normalized to have a sum of 100%. The normalized detector signals are transformed into 5th order Fourier series weighting coefficients by evaluating the matrix product of a prestored coefficient matrix and the vector of detector signals. This prestored matrix depends only on the integral of the 5th order Fourier series over the axial length of the in-core detectors. The axial power distribution is then constructed by forming the sum, at each axial node, of the Fourier functions times their respective coefficients.

The current method appears to be inaccurate for certain axial shapes, especially saddle power shapes. It was believed that an axial power shape synthesis that allows the selection of coefficients specific to a particular set of axial power shapes would improve the accuracy of synthesizing the axial power shapes. Therefore, the cubic spline synthesis is applied to a axial power distribution monitoring in this study.

Ⅱ. Mathematical Modeling

The axial power shape can be obtained by utilizing the cubic spline synthesis technique[3]. The cubic spline synthesis assumes the axial power shape to be the sum of splines(Fig. 2) in which each spline is a piecewise cubic polynomials(Fig. 3) as shown below. The break points between splines are chosen based on relative detector signals.

$$\phi(z)=\sum_{j=1}^{9} A_j \mu_j(z) \tag{1}$$

where

$\phi(z)$–Power at core altitude z

A_j–Amplitude coefficients

$\mu_j(z)$–Cubic spline basic functions

The cubic spline basic functions(Fig. 3) are defined as follows

$$\begin{aligned}
\mu_j(z)&=f_1(\eta_1) && z_{i-2}\leq z\leq z_{i-1}\\
\mu_j(z)&=f_2(\eta_2) && z_{i-1}\leq z\leq z_{i}\\
\mu_j(z)&=f_2(\eta_3) && z_{i}\leq z\leq z_{i+1}\\
\mu_j(z)&=f_1(\eta_4) && z_{i+1}\leq z\leq z_{i-2}\\
\mu_j(z)&=0 && z>z_{i+2} \text{ or } z<z_{i-2}
\end{aligned}$$

where

$$\eta_1=\frac{z-z_{i-2}}{z_{i-1}-z_{i-2}} \qquad \eta_2=\frac{z-z_{i-1}}{z_i-z_{i-1}}$$

$$\eta_3=\frac{z_{i+1}-z}{z_{i+1}-z_i} \qquad \eta_4=\frac{z_{i+2}-z}{z_{i+2}-z_{i+1}}$$

$$f_1(\eta)=\eta^3/4 \qquad f_2(\eta)=1/4+3(\eta+\eta^2-\eta^3)/4$$

The various axial power shapes within the LCOs were categorized depending on the characteristics, i.e., center peak, top/bottom peak, flat, or saddle types. Appropriate number of nodes for each interval in Fig. 2 is assigned based on the categorized power shapes. The amplitude coefficients are then found to satisfy the following nine conditions.

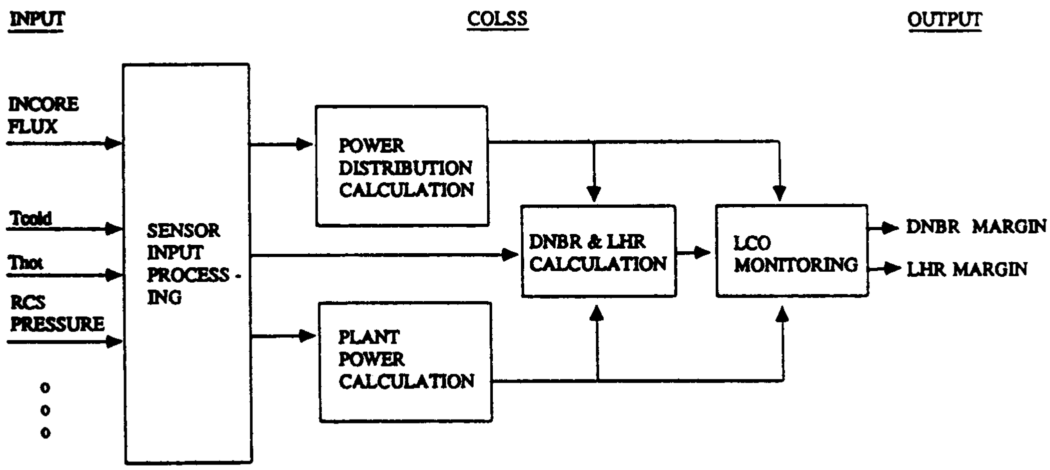

Fig. 1 Functional Block Diagram of COLSS

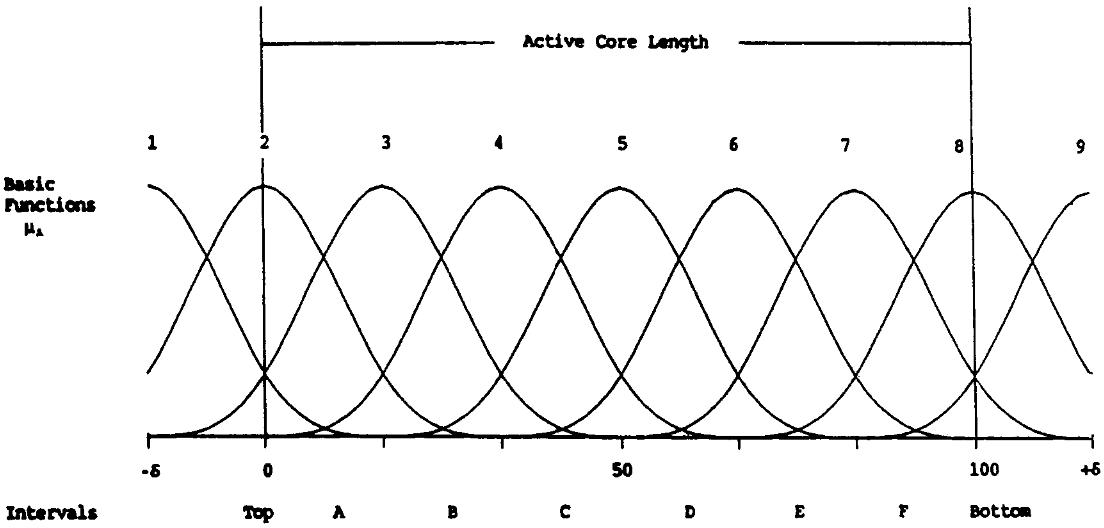

Fig. 2 Spline Nodal Assignment

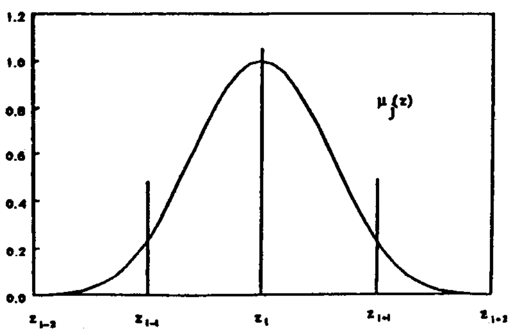

Fig. 3 Cubic Spline Basic Function

a. Five Detector Responses

$$D_i = \int_i \phi(z)dz \quad i=1,2,\cdots,5 \tag{2}$$

b. Two empirical boundary point powers

$$\phi(0) = \alpha 1 \times D_5 + \alpha 2 \text{ (Top)}$$
$$\phi(H) = \alpha 3 \times D_1 + \alpha 4 \text{ (Bottom)} \tag{3}$$

c. Two extrapolated boundary conditions

$$\phi(-\delta) = 0.0$$
$$\phi(H+\delta) = 0.0 \tag{4}$$

Where

D_i–Detector responses

$\alpha_1, \alpha_2, \alpha_3, \alpha_4$–Empirically correlated coefficients

δ–Extrapolated length

H–Core height

Using Eq.(1), Eqs(2),(3) and (4) can be expressed as :

$$\begin{aligned} B_1 &= H_{11}A_1 + H_{12}A_2 + \cdots + H_{19}A_9 \\ &\vdots \\ B_9 &= H_{91}A_1 + H_{92}A_2 + \cdots + H_{99}A_9 \end{aligned} \tag{5}$$

Eq.(5) is rewritten in matrix form for amplitude coefficients, A_j :

$$\mathbf{A} = \mathbf{H}^{-1}\mathbf{B} \tag{6}$$

Where

$\mathbf{A}$–Vector of spline amplitudes

$\mathbf{H}^{-1}$–Pre–calculated spline matrix for a selected spline node set

$\mathbf{B}$–Vector of detector responses and boundary point powers

Ⅲ. Results and Discussions

The core axial power shapes and in–core detector signals for various operating conditions were generated by using the best estimate neutronics computer code, FLAIR [4]. Based on the in–core detector signals, the two sets of axial power shapes were generated by Fourier and cubic spline methods, respectively. The error $[E_i^k]$ between the FLAIR (best estimate) and COLSS axial shape is defined as

$$E_i^k = \left[\frac{F_z(i)_{COLSS} - F_z(i)_{FLAIR}}{F_z(i)_{FLAIR}}\right]_i \times 100.0$$

$F_z(i)$–Normalized axial power at node i ($i = 1,2,\cdots,20$)

K–Case counter

The root–mean–square error (RMS^k) is then calculated by

$$RMS^K = \sqrt{\frac{\sum_{i=1}^{20} (E_i^K)^2}{20}}$$

The average RMS error is

$$RMS = \frac{1}{N}\sum_{K=1}^{N} RMS^K$$

N–Number of axial power shapes

The average RMS errors were computed for various power shapes at three typical burnup points (Table 1). The average RMS error decreased approximately by 1% for flat shapes and by 5% for center peak and saddle–type shapes when the cubic spline synthesis was applied. The core axial power shapes generated by the best estimate (B.E.) code, the Fourier method, and the

Table 1 Comparison of RMS Errors

(Unit : %)

		Fourier Series	Cubic Spline	Δ F.S-C.S
Center Peak	Nominal RMS	10.42	5.44	4.98
	Average RMS	10.21	6.26	3.95
Saddle-Type	Nominal RMS	16.01	10.46	5.55
	Average RMS	15.40	9.99	5.41
Flat	Nominal RMS	5.82	3.91	1.91
	Average RMS	6.70	6.13	0.57

+Note :

Nominal RMS – RMS based on nominal power shape at all-rods-out, equilibrium Xenon conditions.

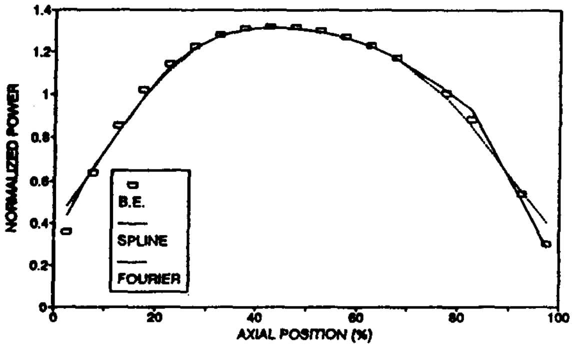

Fig. 4 Center Peak Axial Power Distributions

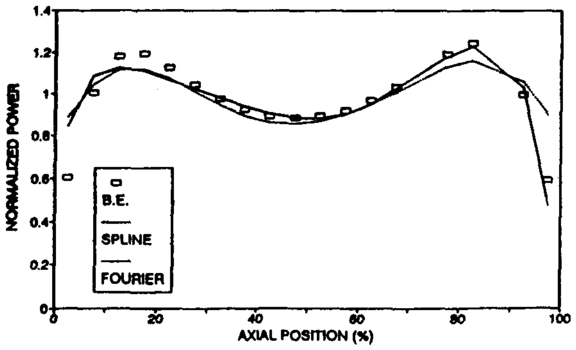

Fig. 5 Saddle-Type Axial Power Distributions

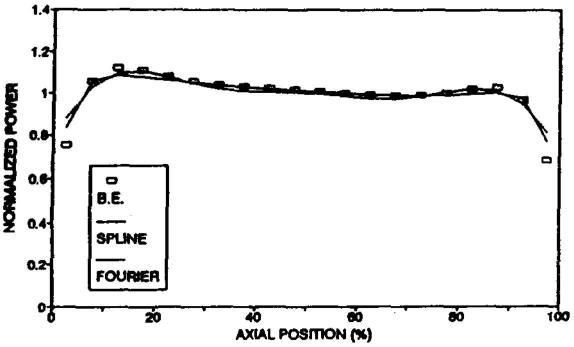

Fig. 6 Flat-Type Axial Power Distributions

cubic spline method for the nominal power shapes at the three burnup points are also shown in Figures 4, 5 and 6.

Due to the improved accuracy in axial power distribution synthesis, it is expected to decrease the uncertainties associated with the core thermal margin. However, the core thermal margin is influenced not only by the power shape accuracy but also by other factors. The complete evaluation should be conducted in future before the application of this method for plant operation.

Ⅳ. Conclusions

The cubic spline synthesis in COLSS axial power distribution monitoring is

ⅰ) more significantly accurate for saddle-type power shapes,

ⅱ) expected to increase core thermal margin.

Ⅴ. References

[1] Combustion Engineering Inc., "Overview Description of the Core Operating Limit Supervisory System (COLSS)", CEN-312-P, Revision 01-P, November 1986.

[2] Combustion Engineering Inc., "Assessment of the Accuracy of PWR Operating Limits as Determined by the Core Operating Limit Supervisory System (COLSS)", CENPS-D-169-P, July 1975.

[3] Kreysig, "Advanced Engineering Mathematics", Vol 2, 5th Edition, 1983, John Wiley & Sons, Inc..

[4] Combustion Engineering Inc., "User's Manual for FLAIR", CE-CES-99 Rev. 0-P, 1988.