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Out-of-Plane Vibrations of Angled Pipes Conveying Fluid

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내부유동을 포함한 굴곡된 파이프의 외평면 진동해석

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Abstract

This paper considered the out-of-plane motion of the piping system conveying fluid through the elbow connecting two straight pipes. The extended Hamilton's principle is used to derive equations of motion. It is found that dynamic instability does not exist for the clamped-clamped, clamped-pinned and pinned-pinned boundary conditions. The frequency equations for each boundary conditions are solved numerically to find the natural frequencies. The effects of fluid velocity and Coriolis force on the natural frequencies of piping system are investigated. It is shown that buckling-type instability may occur at certain critical velocities and fluid pressures. Equivalent critical velocity, which is defined as a function of flow velocity and fluid pressure, are calculated for various boundary conditions.

요 약

본 연구는 두개의 직선 pipe가 elbow로 연결된 piping system의 내부에 유체가 흐를때 발생하는 out-of-plane 운동을 다루었으며, Extended Hamilton's principle을 이용하여 운동방정식을 유도하였다. clamped-clamped, clamped-pinned, pinned-pinned인 경계조건을 갖는 piping system의 경우, dynamic instability는 일어나지 않음을 고찰하였으며, 각 경계조건에 대한 진동수 방정식으로부터 고유진동수의 수치해를 얻었다. 유체의 속도와 Coriolis 힘이 진동수에 미치는 영향을 고찰하였고, 유체의 속도와 압력이 어느값을 넘어설때 buckling-type instability가 일어남을 알았다. 그리고 유체의 속도와 압력의 함수로 등가임계속도를 정의하고 여러가지 경계조건에 대해 buckling이 일어나는 등가임계속도를 계산하였다.

1. Introduction

The dynamics and stability of pipes conveying fluid have been studied extensively over the past 40 years, because of its growing importance in aerospace and nuclear fields. The research on the flow-induced vibration of a straight pipe was originated from the vibration analysis of the Trans-Arabian pipeline by Ashley and Haviland[1]. Housner[2] studied the same problem by using Hamilton's principle. For a simply-supported straight pipe, he found that the pipe may buckle, like a column subject to axial loading, at critical flow velocity. Benjamin[3] considered the effect of fluid flow on the motion of pipes, divided into a series of articulated straight rigid pipes. He observed that the motion of the pipe is independent of fluid friction, theoretically and experimentally. Gregory and Paidoussis[4,5] studied the oscillations of a cantilevered straight pipe to support Benjamin's result mathematically. Also they showed that flutter may occur for a certain flow velocity.

For a curved pipe, Unny, Martin and Dubey[6] derived equations of motion and showed the relation between pipe angle and critical velocity for buckling. Chen[7], however, verified that their equations of motion have mistakes. And he derived the equations of motion for in-plane and out-of-plane motion of uniformly curved pipe using Hamilton's principle. Also he investigatied the relation between the frequency and fluid velocity and between the critical velocity and pipe angle. Hill and Davis[8] found that there was no initial tension force for Chen's equations and no buckling phenomena for some pipe shape by FEM using Galerkin's method.

In this paper, an angled pipe, composed of two straight pipes connected by an elbow, is considered. The force caused by momentum change at the elbow is considered as a follower force and the extended Hamilton's principle is used for the derivation of equation of motion. For the out-of-plane motion, the relations between natural frequency and flow velocty and also between the change of critical velocity and Coriolis force are investigated. Also it is shown that the initial force in static equilibrium have to be included in the equations of motion.

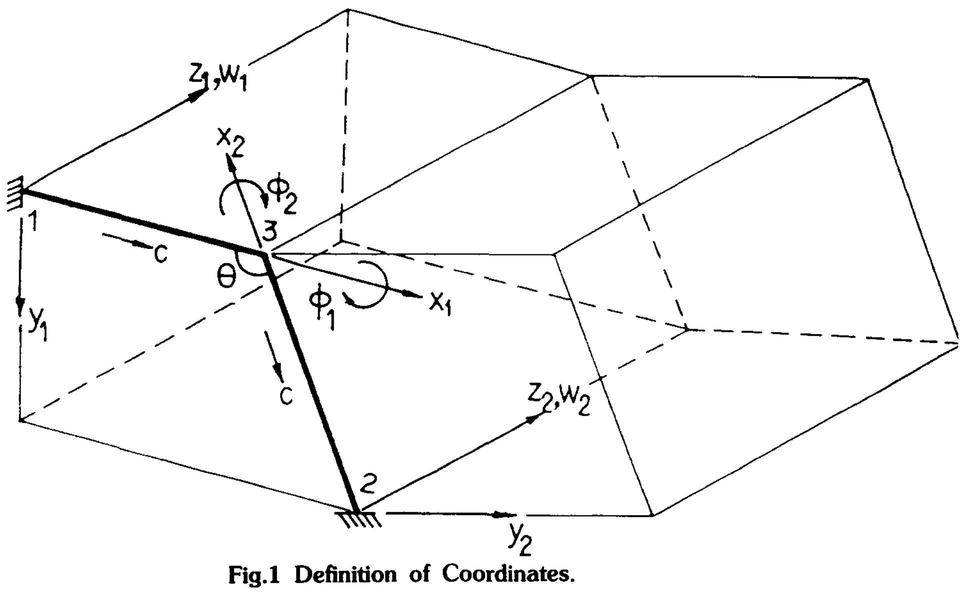

Fig.1 Definition of Coordinates.

2. Equations of Motion

The angled piping system considered herein consists of two pipes connected by an elbow to form angle θ as shown in Fig.1. The pipes have internal cross sectional area A_p, mass per unit length m_p, flexible rigidity EI, torsional rigidity GJ, lengths ℓ_1 and ℓ_2. m_f is the fluid mass per unit length and c is the constant fluid velocity between two points 1 and 2.

In the derivation of equations of motion, following assumptions are applied : (1) The effects of gravity and material damping are negligible, (2) the effects of rotatory inertia and shear force are negligible, (3) the pressure drop is negligible, and (4) all motions are small.

The extended Hamilton's principle provides the equations of motion as well as the boundary conditions, which is given in the form of

$$\int_{t1}^{t2}\left\{\delta T_p - \delta V_p + \delta W_f + \delta W_{pre} + \delta W_R + \delta\left(\sum_{j=1}^{3}\lambda_j g_j\right)\right\}dt = 0 \quad (1)$$

where T_p and V_p are the kinetic and potential energies associated with pipe, δW_f, δW_{pre} and δW_R are the virtual works by fluid acceleration, fluid pressure and the force caused by momentum change in elbow, respectively. g_j represent the constraint conditions for geometrical continuity of two pipes and λ_j correspondinq Largrange's multipliers.

Pipe deformations consist of transverse displacements w_1, w_2 and torsional displacements ϕ_1 and ϕ_2. Then kinetic and strain energies of pipe are given as follows :

$$\delta T_p = \delta\sum_{k=1}^{2}\int_o^{\ell_k}\frac{1}{2}\left\{m_p\left(\frac{\partial w_k}{\partial t}\right)^2 + J_o\left(\frac{\partial \phi_k}{\partial t}\right)^2\right\}dx_k \quad (2)$$

$$\delta V_p = \delta\sum_{k=1}^{2}\int_o^{\ell_k}\left\{\frac{EI}{2}\left(\frac{\partial w_k}{\partial x_k^2}\right)^2 + \frac{GJ}{2}\left(\frac{\partial \phi_k}{\partial x_k}\right)^2\right\}dx_k \quad (3)$$

where J_o is the torsional moment of inertia of pipe. The fluid may be accelerated by the pipe deformation. The accelerated fluid may cause force on the pipe in turn. The virtual work done on a pipe by the accelerated fluid[3, 8] is given by

$$\delta W_f = m_f\sum_{k=1}^{2}\int_o^{\ell_k}\left\{-\left(\frac{\partial^2 w_k}{\partial t^2} - (-1)^k 2c\frac{\partial^2 w_k}{\partial x_k \partial t} + c^2\frac{\partial^2 w_k}{\partial x_k^2}\right)\right\}\delta w_k\, dx_k \quad (4)$$

The fluid pressure also can deform pipe. Hence, the virtual work by fluid pressure is represented by

$$\delta W_{pre} = -\sum_{k=1}^{2}\int_o^{\ell_k} pA\frac{\partial x_k^2}{\partial t^2}\delta w_k\, dx_k \quad (5)$$

The fluid momentum change at the elbow may induce a force on the pipe[9]. The virtual work by the force is expressed as follows :

$$\delta W_R = R\sum_{k=1}^{2}\left[(1+\cos\theta)\left(\int_o^{\ell_k}\frac{\partial^2 w_k}{\partial x_k^2}\delta w_k\, dx_k - \frac{\partial w_k}{\partial x_k}\delta w_k\Big|_o^{\ell_k}\right) + \frac{\partial w_k}{\partial x_k}\delta w_k\Big|_{\ell_k}\right] \quad (6)$$

where, $R = m_f c^2 + pA$. The constraint conditions for the point 3 in Fig. 1 are as follows :

$$g_1 = w_1\Big|_{\ell_1} - w_2\Big|_{\ell_2}$$

$$g_2 = \phi_1\Big|_{\ell_1} - \frac{\partial w_2}{\partial x_2}\Big|_{\ell_2}\sin\theta - \phi_2\Big|_{\ell_2}\cos\theta \quad (7)$$

$$g_3 = \frac{\partial w_1}{\partial x_1}\Big|_{\ell_1} - \frac{\partial w_2}{\partial x_2}\Big|_{\ell_2}\cos\theta + \phi_2\Big|_{\ell_2}\sin\theta$$

By substituting Eqs. (2)~(7) into Eq.(1) and by eliminating Lagrange's multipliers, we obtain dynamic equations of motion and bounary conditions. They are concisely,

$$EI\frac{\partial^4 w_1}{\partial x_1^4} - (m_f c^2 + pA)\cos\theta\frac{\partial^2 w_1}{\partial x_1^2} + 2m_f c\frac{\partial^2 w_1}{\partial x_1 \partial t} + (m_f + m_p)\frac{\partial^2 w_1}{\partial t_2} = 0 \quad (8\text{–a})$$

$$GJ\frac{\partial^2 \phi_1}{\partial x_1^2} - J_o\frac{\partial^2 \phi_1}{\partial t^2} = 0 \quad (8\text{–b})$$

$$EI\frac{\partial^4 w_2}{\partial x_2^4} - (m_f c^2 + pA)\cos\theta\frac{\partial^2 w_2}{\partial x_2^2} - 2m_f c\frac{\partial^2 w_2}{\partial x_2 \partial t} + (m_f + m_p)\frac{\partial^2 w_2}{\partial t_2} = 0 \quad (8\text{–c})$$

Table 1. Boundary Conditions

Boundary	clamped-clamped	clamped-pinned	pinned-pinned
$x_1=0$	$w_1=0 \quad w_1'=0$ $\phi_1=0$	$w_1=0 \quad w_1'=0$ $\phi_1=0$	$w_1=0 \quad w_1''=0$ $\phi_1=0$
$x_2=0$	$w_2=0 \quad w_2'=0$ $\phi_2=0$	$w_2=0 \quad w_2''=0$ $\phi_2=0$	$w_2=0 \quad w_2''=0$ $\phi_2=0$
$x_1=\ell_1$ and $x_2=\ell_2$	$\phi_1=w_2'\sin\theta+\phi_2\cos\theta$ $w_1'=w_2'\sin\theta-\phi_2\cos\theta$ $w_1=w_2$ $EIw_2''+GJ\phi_1'\sin\theta+EIw_1''\cos\theta=0$ $EIw_2'''-(m_fc^2+pA)w_2'\cos\theta$ $+EIw_1'''-(m_fc^2+pA)w_1'\cos\theta=0$ $GJ\phi_2'+GJ\phi_1'\cos\theta-EIw_1''\sin\theta=0$		

$$GJ\frac{\partial^2\phi_2}{\partial x_2^2}-J_o\frac{\partial^2\phi_2}{\partial t^2}=0 \tag{8-d}$$

with boundary conditions summarized in table 1.

The second terms of Eqs. (8-a) and (8-c) show that pipes may experience compression force when elbow angle θ is obtuse and tension force when acute. In case of right angle, the axial force vanishes. Accordingly, instability phenomena may occur for the case of obtuse angle.

To nondimensionalize the dynamic equations of motion and boundary conditions, we introduce following nondimensional terms.

$$\begin{aligned}
&\frac{w_j}{\ell_j}=\bar{w}_j,\quad \frac{\phi_j}{\ell_j}=\bar{\phi}_j,\\
&\frac{x_j}{\ell_j}=\bar{x}_j,\quad (j=1,2),\quad \tau=\left(\frac{EI}{m_p+m_f}\right)^{\frac{1}{2}}t/L^2\\
&\gamma=\frac{m_f}{m_p+m_f},\quad C=\left(\frac{m_f}{EI}\right)^{\frac{1}{2}}Lc,\\
&\rho=\frac{pA}{EI}L^2,\quad k=\frac{EI}{GJ}\\
&\delta=\frac{J_o}{m_p+m_f}\frac{1}{L^2},\quad e_j=\frac{\ell_j}{L}\quad (e_1+e_2=1)
\end{aligned} \tag{9}$$

By use of Eqs.(9) into Eqs.(8), we obtain nondimensional dynamic equations of motion in forms of

$$\begin{aligned}
&\frac{\partial^4\bar{w}_1}{\partial\bar{x}_1^4}-(C^2+\rho)e_1^2\cos\theta\frac{\partial^2\bar{w}_1}{\partial\bar{x}_1^2}\\
&\qquad+2\gamma^{\frac{1}{2}}Ce_1^3\frac{\partial^2\bar{w}_1}{\partial\bar{x}_1\partial\tau}+e_1^4\frac{\partial^2\bar{w}_1}{\partial\tau^2}=0\\
&\frac{\partial^2\bar{\phi}_1}{\partial\bar{x}_1^2}-k\delta e_1^2\frac{\partial^2\bar{\phi}_1}{\partial\tau^2}=0\\
&\frac{\partial^4\bar{w}_2}{\partial\bar{x}_2^4}-(C^2+\rho)e_2^2\cos\theta\frac{\partial^2\bar{w}_2}{\partial\bar{x}_2^2}\\
&\qquad-2\gamma^{\frac{1}{2}}Ce_2^3\frac{\partial^2\bar{w}_2}{\partial\bar{x}_2\partial\tau}+e_2^4\frac{\partial^2\bar{w}_2}{\partial\tau^2}=0\\
&\frac{\partial^2\bar{\phi}_2}{\partial\bar{x}_2^2}-k\delta e_2^2\frac{\partial^2\bar{\phi}_2}{\partial\tau^2}=0
\end{aligned} \tag{10}$$

Nondimensional boundary conditions are shown in Table 2.

3. Free Vibration

We assume the solutions of Eqs.(10) as the forms of

$$\begin{aligned}
&\bar{w}_j(\bar{x}_j,\tau)=\Psi_j(\bar{x}_j)e^{i\Omega\tau}\\
&\bar{\phi}_j(\bar{x}_j,\tau)=\varphi_j(\bar{x}_j)e^{i\Omega\tau}
\end{aligned} \tag{11}$$

where $i=\sqrt{-1}$. Ω is the nondimensional frequency defined by $\Omega=\sqrt{(m_p+m_f)/EI}\,L^2\,\omega$. Substitution of Eqs.(11) into Eqs.(10) yields

Table 2. Nondimensionl Boundary Conditions.

Boundary	clamped–clamped	clamped–pinned	pinned–pinned
$\bar{x}_1=0$	$\bar{w}_1=0$ $\bar{w}_1'=0$ $\bar{\phi}_1=0$	$\bar{w}_1=0$ $\bar{w}_1'=0$ $\bar{\phi}_1=0$	$\bar{w}_1=0$ $\bar{w}_1''=0$ $\bar{\phi}_1=0$
$\bar{x}_2=0$	$\bar{w}_2=0$ $\bar{w}_2'=0$ $\bar{\phi}_2=0$	$\bar{w}_2=0$ $\bar{w}_2''=0$ $\bar{\phi}_2=0$	$\bar{w}_2=0$ $\bar{w}_2''=0$ $\bar{\phi}_2=0$
at $\bar{x}_1=\ell_1$ and $\bar{x}_2=\ell_2$	$\bar{\phi}_1=\bar{w}_2'\sin\theta+\bar{\phi}_2\cos\theta$ $\bar{w}_1'=\bar{w}_2'\cos\theta-\bar{\phi}_2\sin\theta$ $e_1\bar{w}_1=e_2\bar{w}_2$ $ke_1\,\bar{w}_2''+e_2\,\bar{\phi}_1'\sin\theta+ke_2\,\bar{w}_1''\cos\theta=0$ $e_1^2\,\bar{w}_2'''-e_1^2\,e_2^2(C^2+\rho)\cos\theta\;\bar{w}_2$ $+e_2^2\,\bar{w}_1'''-e_1^2\,e_2^2(C^2+\rho)\cos\theta\;\bar{w}_1'=0$ $e_1\bar{\phi}_2'+e_2\,\bar{\phi}_1'\cos\theta-e_2k\,\bar{w}_1''\sin\theta=0$		

$$\frac{d^4\Psi_1}{d\bar{x}_1^4}-(C^2+\rho)e_1^2\cos\theta\,\frac{d^2\Psi_1}{d\bar{x}_1^2}$$

$$+2i\Omega\gamma^{\frac{1}{2}}\,Ce_1^3\,\frac{d\Psi_1}{d\bar{x}_1}-e_1^4\Omega^2\Psi_1=0 \qquad (12\text{–a})$$

$$\frac{d^2\varphi_1}{d\bar{x}_1^2}+\Omega k^2\delta e_1^2\varphi=0 \qquad (12\text{–b})$$

$$\frac{d^4\Psi_2}{d\bar{x}_2^4}-(C^2+\rho)e_2^2\cos\theta\,\frac{d^2\Psi_2}{d\bar{x}_2^2}$$

$$-2i\Omega\gamma^{\frac{1}{2}}\,Ce_2^3\,\frac{d\Psi_2}{d\bar{x}_2}-e_2^4\Omega^2\Psi_2=0 \qquad (12\text{–c})$$

$$\frac{d^2\varphi_2}{d\bar{x}_2^2}+\Omega k^2\delta e_2^2\varphi_2=0 \qquad (12\text{–d})$$

Ω may be real, pure imaginary or complex number depending on system characteristics. The buckling–type instability may occur when Ω is zero. The system is dynamically stable when the imaginary part of complex Ω is positive, but unstable when negative.

Let $\bar{\Omega}$ and $\bar{\Psi}$ be the complex conjugates of Ω and Ψ. After multiplying $\bar{\Omega}$, $\bar{\Psi}$ to Eq. (12–a) and Ω, Ψ to the complex conjugate of Eq.(12–a). substract two equations and integrate the resulting equation by parts with respect to $\bar{x}_1$, $\bar{x}_2$ over the interval (0, 1) to obtain

$$\bar{\Omega}\bar{\Psi}\Psi_1'''\Big|_0^1-\bar{\Omega}\bar{\Psi}_1'\Psi_1''\Big|_0^1$$

$$-\Omega\Psi_1\bar{\Psi}_1'''\Big|_0^1+\bar{\Omega}\,\Psi_1'\,\bar{\Psi}_1''\Big|_0^1$$

$$-(C^2+\rho)e_1^2\cos\theta\;\;\bar{\Omega}\bar{\Psi}_1\Psi_1'\Big|_0^1$$

$$+(C^2+\rho)e_1^2\cos\theta\;\;\Omega\Psi_1\bar{\Psi}_1'\Big|_0^1$$

$$+2i\gamma^{\frac{1}{2}}Ce_1^3\Omega\bar{\Omega}\Psi_1\bar{\Psi}_1\Big|_0^1$$

$$-(C^2+\rho)e_1^2\cos\theta\,(\Omega-\bar{\Omega})\int_0^1|\,\Psi_1'\,|^2d\bar{x}_1$$

$$-(\Omega-\bar{\Omega})\int_0^1|\,\Psi_1''\,|^2d\bar{x}_1$$

$$-e_1^4\Omega\bar{\Omega}(\Omega-\bar{\Omega})\int_0^1|\,\Psi_1\,|^2d\bar{x}_1=0 \qquad (13)$$

Similary, for Eq.(12–b), (12–c) and Eq. (12–d), we obtain

$$\bar{\Omega}\bar{\varphi}_1\varphi_1'\Big|_0^1-\Omega\varphi_1\bar{\varphi}_1'\Big|_0^1$$

$$+(\Omega-\bar{\Omega})\int_0^1|\,\varphi_1'\,|^2d\bar{x}_1$$

$$+k\delta e_1^2\Omega\bar{\Omega}(\Omega-\bar{\Omega})\int_0^1|\,\varphi_1\,|^2d\bar{x}_1=0 \qquad (14)$$

$$\bar{\Omega}\bar{\Psi}_2\Psi_2'''\Big|_0^1-\bar{\Omega}\bar{\Psi}_2'\Psi_2''\Big|_0^1$$

$$-\Omega\Psi_2\bar{\Psi}_2'''\Big|_0^1+\Omega\Psi_2'\bar{\Psi}_2''\Big|_0^1$$

$$-(C^2+\rho)e_2^2\cos\theta\;\;\bar{\Omega}\bar{\Psi}_2\Psi_2'\Big|_0^1$$

$$+(C^2+\rho)e_2^2\cos\theta\ \ \Omega\Psi_2\bar{\Psi}_2'\Big|_0^1$$

$$+2i\gamma^{\frac{1}{2}}Ce_2^3\Omega\bar{\Omega}\Psi_2\bar{\Psi}_2\Big|_0^1$$

$$-(C^2+\rho)e_2^2\cos\theta\ (\Omega-\bar{\Omega})\int_0^1|\Psi_2'|^2d\bar{x}_2$$

$$-(\Omega-\bar{\Omega})\int_0^1|\Psi_2''|^2d\bar{x}_2$$

$$-e_2^4\Omega\bar{\Omega}(\Omega-\bar{\Omega})\int_0^1|\Psi_2|^2d\bar{x}_2=0 \qquad (15)$$

$$\bar{\Omega}\bar{\varphi}_2\varphi_2'\Big|_0^1-\Omega\varphi_2\bar{\varphi}_2'\Big|_0^1$$

$$+(\Omega-\bar{\Omega})\int_0^1|\varphi_2'|^2d\bar{x}_2$$

$$+k\delta e_2^2\Omega\bar{\Omega}(\Omega-\bar{\Omega})\int_0^1|\varphi_2|^2d\bar{x}_2=0 \qquad (16)$$

By adding Eqs.(14) and (16), substracting Eqs.(13) and (15), and by substituting boundary conditions of Table 2, we obtain

$$(\Omega-\bar{\Omega})\Big|\int_0^1|\Psi_1''|^2d\bar{x}_1+(C^2+\rho)$$

$$e_1^2\cos\theta\int_0^1|\Psi_1'|^2d\bar{x}_1+\int_0^1|\varphi_1'|^2d\bar{x}_1$$

$$+e_1^4|\Omega|^2\int_0^1|\Psi_1|^2d\bar{x}_1$$

$$+k\delta e_1^2|\Omega|^2\int_0^1|\varphi_1|^2d\bar{x}_1$$

$$+\int_0^1|\Psi_2''|^2d\bar{x}_2+(C^2+\rho)$$

$$e_2^2\cos\theta\int_0^1|\Psi_2'|^2d\bar{x}_2+\int_0^1|\varphi_2'|^2d\bar{x}_2$$

$$+e_2^4|\Omega|^2\int_0^1|\Psi_2|^2d\bar{x}_2$$

$$+k\delta e_2^2|\Omega|^2\int_0^1|\varphi_2|^2d\bar{x}_2\Big|=0 \qquad (17)$$

One note that Eq.(17) is always satisfied if $\Omega=\bar{\Omega}$, which implies Ω is real. Thus, the dynamic instability does not occur when the boundary conditions in table 2 are satisfied. This result is identical to the results by Paidoussis[12] and Chen[7] We concluded that dynamic instability does not exist for the conservative system with the clamped–clamped, clamped–pinned or pinned–pinned boundaries. Now, the solutions of Eqs.(12) are assumed as follows:

$$\begin{aligned}\Psi_1(\bar{x}_1)&=\sum_{n=1}^{4}A_n\ e^{is_{1n}\bar{x}_1}\\ \varphi_1(\bar{x}_1)&=\sum_{n=1}^{2}E_n\ n^{ir_{1n}\bar{x}_1}\\ \Psi_2(\bar{x}_2)&=\sum_{n=1}^{4}B_n\ e^{is_{2n}\bar{x}_2}\\ \varphi_2(\bar{x}_2)&=\sum_{n=1}^{2}E_n\ n^{ir_{2n}\bar{x}_2}\end{aligned} \qquad (18)$$

where A_n, E_n, B_n, F_n are the constants to be determinded from boundary conditions, and s_{1n}, s_{2n}, r_{1n}, r_{2n} are the roots of characteristic equations given by

$$\begin{aligned}&s_1^4+(C^2+\rho)e_1^2\cos\theta\ s_1^2\\ &\quad-2\Omega\gamma^{\frac{1}{2}}Ce_1^3s_1-e_1^4\Omega^2=0,\\ &r_1^2-\Omega^2k\ \delta e_1^2=0,\\ &s_2^4+(C^2+\rho)e_2^2\cos\theta\ s_2^2\\ &\quad-2\Omega\gamma^{\frac{1}{2}}Ce_2^3s_2-e_2^4\Omega^2=0,\\ &r_2^2-\Omega^2k\ \delta\ e_2^2=0\end{aligned} \qquad (19)$$

By substituting Eqs.(18) into the boundary conditions in table 2, the homogeneous equations are derived as the form of

$$[a_{jn}]\{\bar{A}\}=\{0\}.\ j,n=1,2,\cdots,10 \qquad (20)$$

where $\{\bar{A}\}=[A_1, A_2, A_3, A_4, E_1, B_1, B_2, B_3, B_4, F_1]^T$. The determinant of matrix $[a_{jn}]$ must be zero for the existence of nontrivial solution $\{\bar{A}\}$ That is,

$$\det[a_{jn}]=0 \qquad (21)$$

When the pipe is made of a homogenous material and has circular cross–section, k and δ become

$$\begin{aligned}k&=\frac{EI}{GJ}=\frac{E}{2G}=1+\nu\\ \delta&=2\mu(1-\gamma)\end{aligned} \qquad (22)$$

where ν is the Poisson's ratio and $\mu=I/A_p L^2$. From Eq.(21), the frequency equation is obtained in the form of

$$F(\Omega, \gamma, C, \rho, \theta, \nu, \mu, e_1)=0 \qquad (23)$$

Solving Eq.(23), the change of nondimensional natural frequency with respect to flow velocity is shown in Figs.2~4 when Poisson's ratio $\nu=0.3$, length ratio $e_1=0.5$, angle $\theta=120°$, pressure ratio $\rho=0$. The natural frequencies obtained from Eq.(23) are real for the boundary conditions specified in Table 2. The natural frequency decreases as the flow speed increases. And, when the flow velocity reaches a critical velocity, the natural frequency become zero to cause buckling-type instability. From Figs.2~4. we find that the first natural frequency decreases and the other frequency increases as the mass ratio increases. As the flow velocity increase, the difference of natural fre-

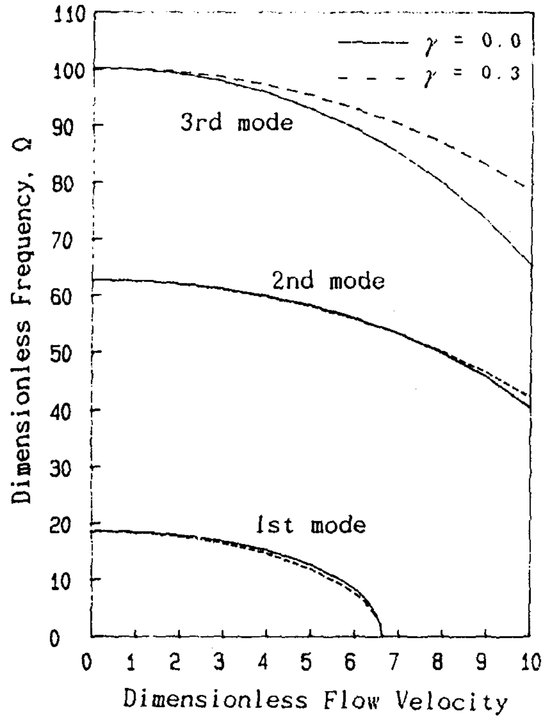

Fig.2. Natural Frequencies of Clamped-Clamped Pipe($\theta=120°$, $e_1=0.5$, $\rho=0$, $\nu=0.3$, $\mu=1.4\times10^{-5}$).

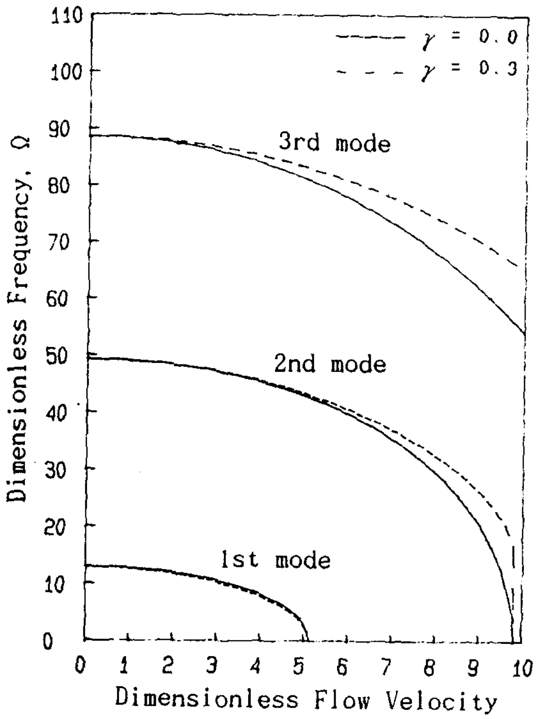

Fig.3 Natural Frequencies of Clamped-Pinned Pipe($\theta=120°$, $e_1=0.5$, $\rho=0$, $=0.3$, $\mu=1.4\times10^{-5}$).

quencies for different mass ratio becomes remarkable. However, with further increase in flow velocity, the natural frequencies for different value of vanish at the same value of flow velocity, which phenomena can be explained that the Coriolis force does not affect the stability of pipe. Also, the Coriolis effect is negligible when mass ratio is small.

4. Critical Velocity

The buckling-type instability boundary can be obtained by natural frequency $\Omega=0$ in Eqs.(11). To find the effects of a flow speed and pressure on buckling phenomena, we delete the time-dependent terms in Eqs.(10). Then we obtain

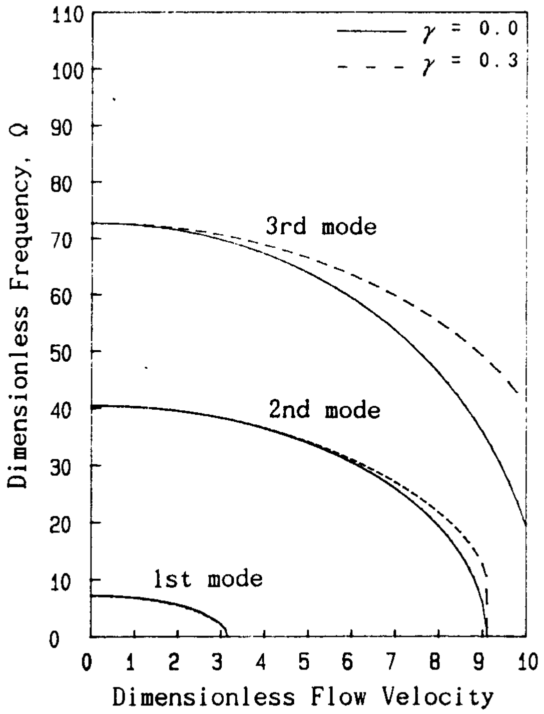

Fig.4 Natural Frequencies of Pinned–Pinned Pipe ($\theta=120°$, $e_1=0.5$, $\rho=0$, $=0.3$, $\mu=1.4\times10^{-5}$).

$$\frac{d^4\Psi_1}{d\bar{x}_1^4}-(C^2+\rho)e_1^2\cos\theta\,\frac{d^2\Psi_1}{d\bar{x}_1^2}=0,$$

$$\frac{d^2\varphi_1}{d\bar{x}_1^2}=0 \tag{24}$$

$$\frac{d^4\Psi_2}{d\bar{x}_2^4}-(C^2+\rho)e_2^2\cos\theta\,\frac{d^2\Psi_2}{d\bar{x}_2^2}=0,$$

$$\frac{d^2\varphi_2}{d\bar{x}_2^2}=0$$

The solutions of Eq.(24) are assumed as

$$\Psi_1(\bar{x}_1)=D_1+D_2\bar{x}_1+D_3\sin\alpha\,\bar{x}_1+D_4\cos\alpha\,\bar{x}_1$$

$$\varphi_1(\bar{x}_1)=M_1+M_2\bar{x}_1$$

$$\Psi_2(\bar{x}_2)=H_1+H_2\bar{x}_2+H_3\sin\beta\,\bar{x}_2+H_4\cos\beta\,\bar{x}_2$$

$$\varphi_2(\bar{x}_2)=N_1+N_2\bar{x}_2 \tag{25}$$

where

$$\alpha=V_{eq}\,e_1\sqrt{-\cos\theta},$$

$$\beta=V_{eq}\,e_2\sqrt{-\cos\theta}. \tag{26}$$

The constants of Eqs.(25) are determined from boundary conditions. To observe both effects of flow speed and pressure on static instability, the equivalent critical velocity V_{eq} is defined as

$$V_{eq}=\sqrt{C^2+\rho} \tag{26}$$

By substituting Eq.(25) into boundary conditions in table 2, we obtain six homgeneous equations in

Table 3. Equivalent Critical Velocities of a Clamped–Clamped Pipe.

e_1 / mode / angle	0.5		0.3	
	1st	2nd	1st	2nd
180°	6.2832	8.9869	6.2832	8.9868
170°	6.2961	9.0600	6.3261	8.9806
160°	6.2311	9.2877	6.4576	8.9646
150°	6.1875	9.6977	6.6868	8.9652
140°	6.1824	10.3471	7.0306	9.0678
130°	6.2918	11.3499	7.5249	9.4394
120°	6.6401	12.9507	8.2748	10.3395
110°	7.4963	15.7884	9.6246	12.2727
100°	9.8807	22.3959	12.9686	17.1282
95°	13.5511	31.8170	17.9539	24.1975
90°	∞	∞	∞	∞

Table 4. Equivalent Critical Velocities of a Clamped-Pinned Pipe.

angle \ mode \ e_l	0.5		0.3	
	1st	2nd	1st	2nd
180°	4.4934	7.7253	4.4934	7.7253
170°	4.5002	7,7529	4.5314	7.7412
160°	4.5219	7.8402	4.6496	7.7856
150°	4.5648	8.0070	4.8626	7.8557
140°	4.6496	8.3056	5.2005	7.9734
130°	4.8095	8.8407	5.7229	8.2202
120°	5.1392	9.8079	6.5577	8.7669
110°	5.8456	11.6678	8.0377	9.9911
100°	7.7315	16.2064	11.4771	13.2914
95°	10.6103	21.8048	16.3665	18.3154
90°	∞	∞	∞	∞

Table 5. Equivalent Critical Velocities of a Pinned-Pinned Pipe.

angle \ mode \ e_l	0.5		0.3	
	1st	2nd	1st	2nd
180°	3.1416	6.2832	3.1416	6.2832
170°	3.1345	6.3352	3.1452	6.2883
160°	3.1151	6.4973	3.1557	6.3111
150°	3.0892	6.7891	3.1734	6.3776
140°	3.0699	7.2514	3.2027	6.5400
130°	3.0806	7.9654	3.2597	6.8855
120°	3.1656	9.1052	3.3867	7.5661
110°	4.4276	11.1250	3.6980	8.9331
100°	4.2627	15.8231	4.6404	12.3425
95°	5.6426	22.5134	6.1816	17.3372
90°	∞	∞	∞	∞

a matrix form as

$$[b_{jn}]\{\bar{B}\} = \{0\} \quad (j,n=1,2,3,4,5,6) \tag{28}$$

For nontrivial solution of $\{\bar{B}\}$, determinant of $[b_{jn}]$ must be zero. Here the function of the equivalent critical velocity is

$$G(V_{eq}, e_1, \theta)=0 \tag{29}$$

The equivalent critical velocities V_{eq}, for length ratio $e_1=0.5$ and 0.3 were shown in tables 3~5 for various elbow angles θ. From tables 3~5, V_{eq} for $\theta=180^\circ$ is identical to the results by Paidoussis and Issid[12] and to the critical load of buckled beam. Generally, the V_{eq} increase as angle decrease and reach infinite value when angle is 90°

5. Conclusion

In the present study, the out-of-plane motion of a angled pipe is investigated to find the following results :

1. For clamped – clamped, clamped – pinned,

pinned – pinned boundary conditions, the natural frequency of the system is always real, which implies that dynamic instability does not occur.

2. Buckling-type instabilty does not occur for the acute angled piping system because it is subject to tension force. For the obtuse angled piping system, however, buckling-type instability can occur because it is now subject to compression force.
3. The obtuse angle piping system buckle at a certain critical flow velocity. Their natural frequency decrease as the flow velocity increase.
4. The Coriolis force does not affect the stability. When mass ratio is small, the Coriolis effect is negligible.
5. Initial tension force must be included in the theoretical analysis.

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