Journal of the Korean Nuclear Society
Volume 4, Number 3, September, 1972

<Original>

Digital Bandpass Filter Method-A Data Processing Technique

Dong Hoon Kim

Reactor Engineering Division, Atomic Energy Research Institute, Seoul, Korea

(Received April 24, 1972)

Abstract

A digital bandpass filter method is investigated for the data treatment in the measurement of the reactor transfer function. It was found that the technique is very effective in eliminating the effects of external noise. A computer code was programmed for this method. The frequency response of a digital filter is calculated, and the data treatment with this method using a digital computer was illustrated for triangular and square waveforms. Error and restriction involved in use of this method are discussed.

요 약

원자로의 전달함수의 측정 data의 처리를 위하여 digital bandpass filter 방법을 고안하였다. 이 방법에 의한 처리는 외부잡음을 제거하는데 대단히 유효하다는 것이 판명되었다. 이 방법에 의한 계산 코드를 작성하였으며 계산기를 사용하여 수치휠러의 주파수반응 및 수치처리를 삼각파형과 구형파형에 대하여 계산하였으며 그 결과를 표시하였다. 이 방법의 오차와 제한사항에 대하여서도 언급하였다.

1. Introduction

The dynamic behavior of nuclear reactors has been successfully analyzed by use of transfer functions. The reactor transfer function is normally measured either by exciting the reactor with sinusoidal reactivity change[1,2] or by using the inherent statistical fluctuations[3,4] in the neutron population of a reactor.

In evaluating transfer functions, there are several difficulties. One of the most common difficulties is that the effects of external disturbances on the measurement are uncertain, since the measured quantity is actually a combination of reactor fluctuations and instrument noise. To overcome this difficulty, several investigators[5,6,7] have studied the analog correlation method and electronic filtering techniques in the measurements of reactor transfer functions and the evaluation of the delayed-neutron fraction-neutron lifetime ratio.

In this paper, a digital bandpass filter method is pressented, which is an useful tool of data processing for the evaluation of the frequency response of a system in presence of the noise-like harmonics as in the measurement of the reactor transfer function.

2. Principle of the Method

Assume an ideal bandpass filter, having the bandwidth as shown in Fig. 1.

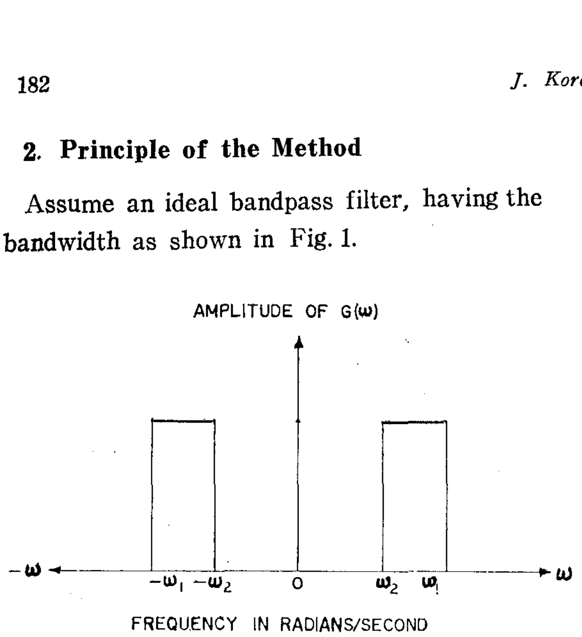

Fig. 1. Frequency response of ideal bandpass filter

$$G(\omega)=1 \quad \text{for } \omega_2\leq\omega\leq\omega_1,\ -\omega_1\leq\omega\leq-\omega_2$$
$$=0 \quad \text{for } \omega>\omega_1,\ -\omega_2<\omega<\omega_2,\ \omega<-\omega_1$$

It is well known that the impulse response of $G(\omega)$ is given by

$$g_1(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}G(\omega)e^{j\omega t}d\omega$$
$$=\frac{1}{\pi t}(\sin\omega_1 t-\sin\omega_2 t) \qquad (1)$$

on taking the inverse Fourier transform of $G(\omega)$.

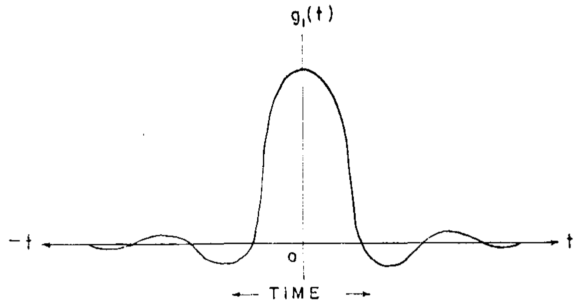

Fig. 2. Shape of impulse response of bandpass filter

Fig. 2 shows a form of $g_1(t)$ as a function of time, and it is seen that $g_1(t)$ is an even function.

If the system is linear, the convolution integral[8,9] between the input and impulse response functions is given as

$$y(t)=\int_{-\infty}^{\infty}x(t-\tau)g(\tau)d\tau \qquad (2)$$

where

$x(t)$=the input function to be treated

$g(t)$=the response of the bandpass filter to impulse function

In the actual practice, the length of the input data, $x(t)$ in our case, is finite. If we can choose a suitable time, T_d, such that

$$\left|\frac{g_1(\pm T_d)}{g_1(0)}\right|\ll 1, \qquad (3)$$

then

$$g(t)=0 \quad \text{for } t>|T_d|$$
$$=g_1(t) \quad \text{for } t\leq|T_d|$$

Eq. 2 becomes

$$y(t)\approx\int_{-T_d}^{T_d}x(t-\tau)g(\tau)d\tau \qquad (4)$$

Eq. 1 can be rewritten as

$$g(t)=g_1(t)=\frac{1}{\pi(t-T_d)}[\sin\omega_1(t-T_d)-\sin\omega_2(t-T_d)] \qquad (5)$$

for $-T_d\leq t\leq T_d$

On the other hand, the transfer function of the impulse response $g(t)$ is

$$G(\omega)=\frac{\int_{-\infty}^{\infty}g(t)e^{-j\omega t}dt}{\int_{-\infty}^{\infty}\delta(t)e^{-j\omega t}dt}=\int_{-T_d}^{T_d}g_1(t)e^{-j\omega t}dt$$
$$=\frac{1}{\pi}\int_0^{T_d}\frac{1}{t}[\sin(\omega_1+\omega)t+\sin(w_1-w)t-\sin(\omega_2+\omega)t-\sin(\omega_2-\omega)t]dt \qquad (6)$$

3. Computational Method

In order to determine T_d for the condition of Eq. 3, the impulse response of the bandpass filter is calculated using Eq. 1.

A simple trapezoidal approximation for computing the integral in Eq. 6 is employed to evaluate the transfer function, $G(\omega)$. Fig. 3 shows the calculated result of two cases with different values of T_d. ts and N in Fig. 3 indicate the sampling interval and the number of sampled data, respectively, and Nts is the time length, T_d, used in the calculation.

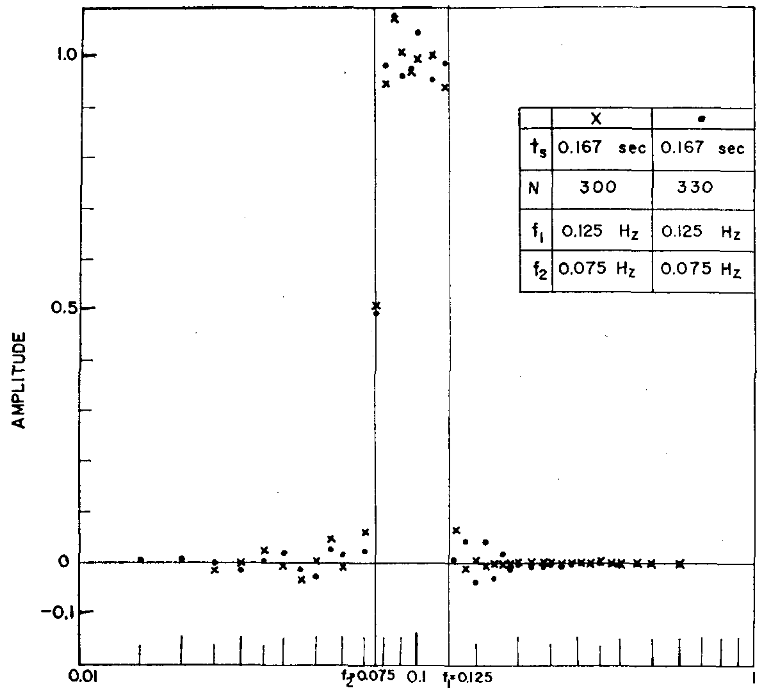

Fig. 3. Frequency response of digital filter, $G(2\pi f)$

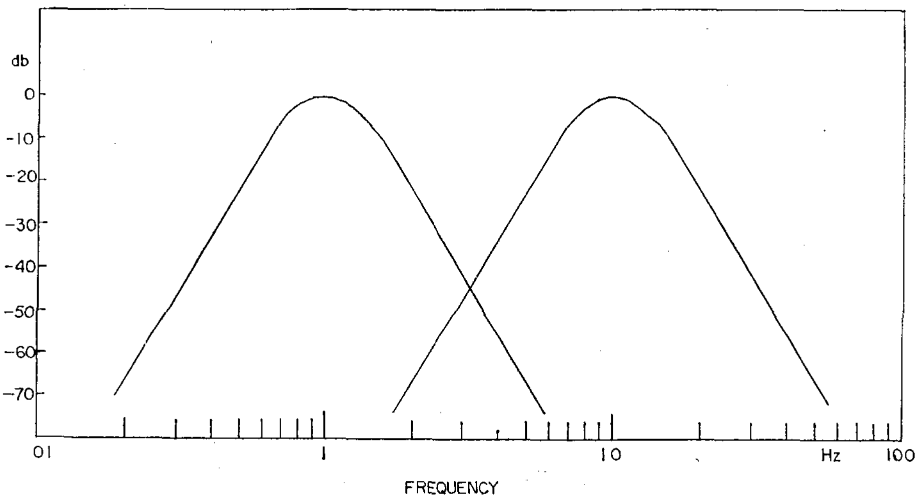

Fig. 4. Selective frequency characteristics of typical electronic bandpass filter

As seen in the figure, the gain at the center frequency is dependent on the length of the sampled data T_d, that is, the number of the sampled data N, with the fixed interval of ts. It should be noticed that the gain will be converged to 1(0 db) within the band when T_d approaches to the infinity. When the absolute values of the output data are needed, some correction for evaluating the real magnitude must be considered. However, if only the relative values such as the ratio of output-to-input is to be calculated, correction is not needed.

For the purpose of comparison, the frequency characteristics of typical electronic bandpass filter[10] is shown in Fig. 4.

Performing the calcuation of Eq. 4, let

$$Td=3T$$

and $$T=Nt_s,$$

where t_s is the sampling interval, and N is the number of the sampling in the duration of T. If $(8N+1)t_s$ is given for the length of $x(t)$, and $(6N+1)t_s$ is for that of $g(t)$, the length of the calculated data of $y(t)$ would be $(2N+1)t_s$ in Eq. 4. The graphycal interpretation for this calculation illustrated in Fig. 5.

If we express the functions in Eq. 4 in terms of number of the sampling intervals instead of t,

$$g(t)=G(k);\ k=1,\ 2,\cdots\cdots\cdots\cdots,\ (6N+1)$$
$$x(t)=X(i);\ i=1,\ 2,\cdots\cdots\cdots\cdots,\ (8N+1)$$
$$y(t)=Y(j);\ j=1,\ 2,\cdots\cdots\cdots\cdots,\ (2N+1)$$

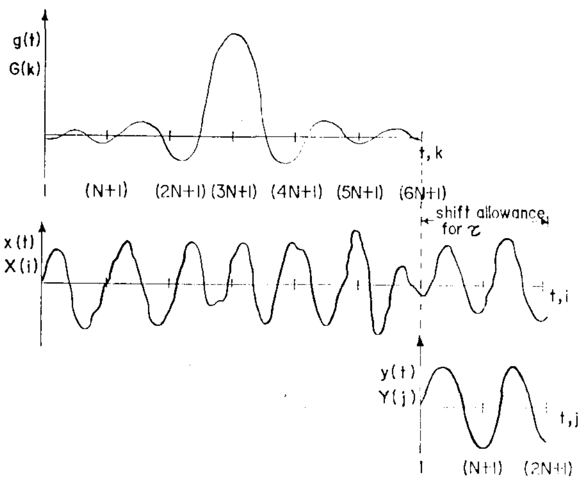

Fig. 5. Graphical interpretation of filter calculation

Considering that $G(k))$ is symmetrical about $(3N+1)t_s$ $(=Td)$, Eq. 4 by use of trapezoidal integration formula yields

$$Y(j)=t_s\{\frac{1}{2}[X(6N+j)+X(j)]G(1)$$
$$+X(3N+j)G(3N+1)$$
$$+\sum_{k=2}^{3N}[X(6N+1+j-k)X(j+k$$
$$-1)]G(k)\} \qquad (7)$$

A computer code is programmed by using Eq. 7. Several examples of this calculation are excercised. The fundamental frequencies obtained through each calculation for square and triangular waves of 0. 1 Hz are shown in Fig. 6. The filter used in the calculation is the same as shown in Fig. 3. The sampling interval is 0. 13 sec. for all cases, and the length of sampling time is taken as shown in Table 1.

As seen in Fig. 6. the result calculated on the amplitude for the square wave is 1070 (millivolts) with no corretion, whereas the theoretical amplitude is 1107(millivolts), and for the triangular wave, the amplitude is 740, which should be 760 theoretically. Error in-

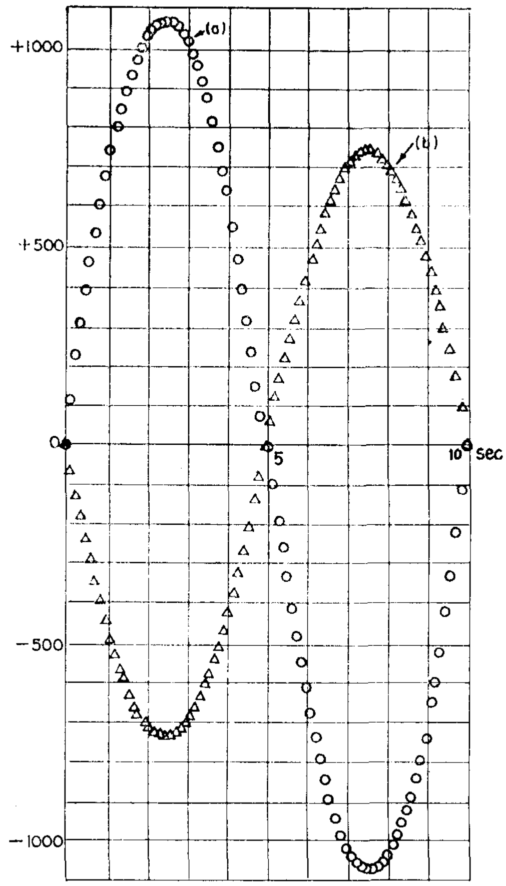

Fig. 6. Plots of fundamental frequencies of 0. 1Hz passed through digital filter for:

(a) 1744 P-P square wave
(b) 1880 P-P triangular wave

volved in this treatment could be limited within 3 per cent in the calculation.

Table 1. Length of sampling time in Fig. 4

Input treated	N value in $(8N+1)$
Square wave	382
Triangular wave	379

4. Discussion and Conclusion

The frequency characteristics of the digital filter is much better than the commercially available electronic bandpass filter, as seen in Fig. 3 and Fig, 4. The data obtained by this method are in good agreement with the theoretical values, if we choose the sufficient length of the experimental data. In the use of this method in the processing of experimental data, the following conditions should be considered.

$$f_{min}=\frac{1}{2T_d}$$

and

$$\Delta f=f_2-f_1>\frac{1}{2T_d}$$

where

f_{min}= the minimum frequency to be handled

Δf= the bandwidth of the digital filter.

As a result of semi-emperical investigation of this method, the following conditions were obtained for limiting error less than 2 per cent for the lowest frequency:

$$\tau_{max}\geqq\frac{1}{f_{min}}$$

and

$$T_d\geqq 10\tau_{max}$$

where τ_{max} is the maximum allowable shift for τ.

On the other hand, the maximum frequency is given by

$$f_{max}=\frac{1}{2t_s}$$

When we take a value of the maximum frequency close to $\frac{1}{2t_s}$, error would be large. For determining the sampling interval t_s, trial calculations were made for various values of t_s using a pure sinusoidal function, $x(t)=\sin t$.

We could get a very close function to the original input signal when we calculate with

$$t_s\leqq\frac{1}{20f_{max}}$$

However, in general, it can certainly be stated that the samples should be taken sufficiently close together to ensure that the function does not change by a singnificant amount within the sampling interval.

References

1) J.M. Harrer, *et al.*, Transfer Functions of Argonne CP-2 Reactor, Nucleonics, Vol. **10**, No. 8(1952)

2) G.R. Keepin, "Physics of Nuclear Kinetics" Addison-Wesley Publishing Co., Inc. (1965)

3) M.N. Moore, The Determination of Reactor Transfer Functions from measurements at Steady Operation, Nucl. Sci. Eng. **3**, 387 (1958)

4) J.A. Thie, "Reactor Noise" Rowman and Little Field Inc., New York (1963)

5) A.A. Wasserman, *et al.*, NERVA Reactor Transfer Function Measurements with Cross-correlation Techniques, AEC Symposium Series 9, "Neutron Noise, Waves, and Pulse Propagation" USAEC (1967)

6) S. Yamada, Reactor Noise Analysis of Swimming Pool Type Reactor, J. Nucl. Sci. Technol. **1**, No. 4:130 (1964)

7) J.D. Balcomb, *et al.*; A Cross-correlation Method for Measuring the Impulse Response of Reactor Systems, Nucl. Sci. Eng., **11**:159 (1961)

8) G. A. Campbell and R. M. Foster, "Fourier Integrals for Practical Applications" Van Nostrand Co. Inc., Princeton, N.J. (1948)

9) M.F. Gardner and J.L. Barnes, "Transients in Linear Systems" Vol. 1, John Wisley & Sons, Inc, N.Y. (1942)

10) T. Nomula, Reactivity Measurements by Reactor Noise Analysis using Two-Detector Correlation Method, J. Nucl. Sci. Technol. **3**(1):14 (1966)