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Energy Levels of ^{53}Mn by the Nilsson Model

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Abstract

The Nilsson model was used to predict energy levels of ^{53}Mn without taking band mixing into consideration. The results were compared with the experimental levels and the predictions with band mixing of Malik and Scholz. In both cases, with band mixing and without band mixing, the theoretical fit to experimental results is fair only for a few low-lying levels. However, it is found that the present calculation fits rather better to experiment than the band mixing calculation as far as higher levels are concerned.

요 약

에너지 band mixing을 고려함 없이 Nilsson의 핵구조 모형을 사용하여, ^{53}Mn의 에너지 준위를 계산하였다. 계산결과는 실험치 및 Malik과 Scholz의 에너지 band mixing에 의한 Nilsson 모형 계산값과 비교하였다. 두 경우, 즉 현재의 계산이나 에너지 band mixing을 고려한 Malik과 Scholz의 계산결과가, 둘 다 수개의 낮은 에너지 준위를 제외하고는, 실험치와 잘 맞지 않았다. 그러나 높은 에너지 준위에 관한한, 현재의 결과가 에너지 band mixing법에 의한 결과 보다 오히려 약간 더 나은 것이 발견되었다.

1. Introduction

Recently, the nucleus ^{53}Mn has been extensively investigated, experimentally using the ^{53}Cr$(p, n\gamma)^{53}$Mn reaction[1], and theoretically in terms of the intermediate coupling unified model approach[2].

The intermediate coupling calculations have successfully reproduced energy levels of ^{53}Mn. Other theoretical calculations for ^{53}Mn have been also reported based on the mixed configuration shell model[3] and the strong coupling Nilsson model[4].

The purpose of this work is to predict energy levels by the Nilsson model without taking band mixing into consideration and to compare the results with experimental values[1], and also with the results of the Nilsson model with band mixing by Malik and Scholz[4].

Although the mixed configuration shell model[3] and the intermediate coupling unified model[2, 5-7] give satisfactory spectroscopic interpretation for nuclei in this mass region, it will be worthwhile to clarify the previ-

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ously reported strong coupling Nilsson model results[4] by approaching the same model in a different aspect for ^{53}Mn.

2. Formalism

The single-particle states in the deformed nuclear field were first introduced by Nilsson[8]. Nilsson's Hamiltonian is given by[8]

$$H_p=\overset{0}{H}_0+H_\delta+C\bar{l}\cdot\bar{S}+D\bar{l}^2,$$

where $\overset{0}{H}_0+H_\delta\equiv H_0$ is the harmonic-oscillator Hamiltonian, $\overset{0}{H}_0$ a spherically symmetric term, H_δ the coupling of the particle to the axis of the deformation, $C\,\bar{l}\cdot\bar{S}$ the usual spin-orbit term and $D\,\bar{l}^2$ a correction to the oscillation potential.

The Hamiltonian can be written in terms of the new parameters[8], μ, κ, and η, introduced by Nilsson:

$$H_p=H_0+\kappa\hbar\bar{\omega}_0[-\frac{4}{3}\eta\sqrt{\frac{\pi}{5}}r^2Y_{20}-2\bar{l}\cdot\bar{s}-\mu\bar{l}^2]\equiv\bar{H}_0+\kappa\hbar\bar{\omega}_0R$$

The quantuam numbers l, $\wedge$, and $\sum$ corresponding to $\bar{l}^2$, $\bar{l}_z$, and S_z, respectively, for the single-particles, which all commute with $\overset{0}{H}_0$, are used as basic vectors:

$$\overset{0}{H}_0|Nl\wedge\Sigma>=\left(N+\frac{3}{2}\right)\hbar\omega_0|Nl\wedge\Sigma>,$$
$$\wedge+\Sigma=\Omega,$$

where the quantum number N represents the total number of oscillator quanta, and Ω is the quantum number corresponding to the commuting operator $j_z=l_z+s_z$

The eigenvalues and the expansion coefficients of the eigenvectors are obtained by diagonalizing the dimensionless matrix R in the representation chosen. Thn energy eigenvalues are then given as

$$E_\alpha=\left(N_\alpha+\frac{3}{2}\right)\hbar\omega_0(\delta)+\kappa\hbar\bar{\omega}_0r_\alpha(\eta)$$

κ and μ are used as free parameters varying from shell to shell. The total Hamiltonian of the nuclear system can be then written as

$$H=H_p+\frac{\hbar^2}{2J}[I^2+j^2-2(\bar{I}\cdot\bar{j})],$$

where H_p represents the single-particle Hamiltonian and is usually the Nilsson Hamiltonian.

The $(\bar{I}\cdot\bar{j})$ term is called Coriolis coupling or rotation-particle coupling term. In the usual procedure, the coupling term $(\bar{I}\cdot\bar{j})$ and the term j^2 are either omitted or treated in the first order perturbation theory.

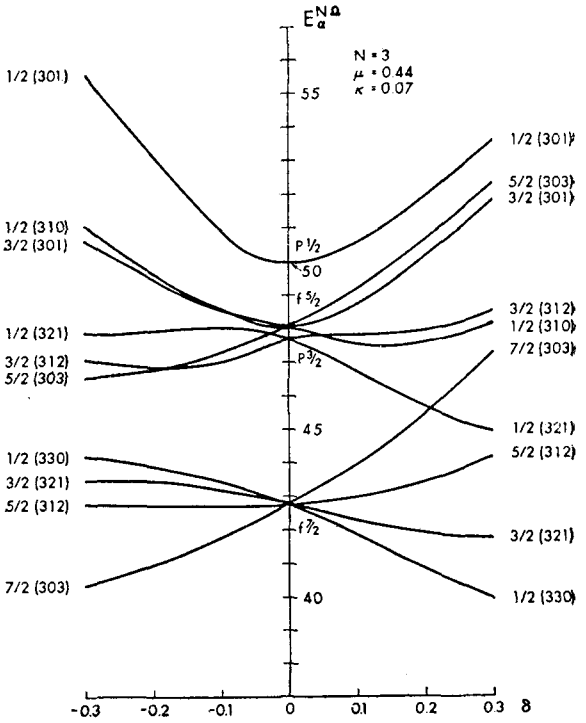

Fig. 1 The Nilsson model energy diagram. Parameters used are μ=0.44 and κ=0.07

The total wave function[9], |EIMK, describing the state of the nucleus is a product of the rotation wave function and the single particle wave function. In the odd-A nucleus, the total angular momentum I is given by the sum of the angular momenta of the

last odd particle and the rotational core, j and R, respectively:

$$\bar{I}=\bar{j}+\bar{R}$$

The energy difference in a rotational band between the excited level with spin I and the intrinsic state with spin K is given by[10]

$$\Delta E_{IK}=\frac{\hbar^2}{2J}[I(I+1)+a(-)^{I+\frac{1}{2}}(I+\frac{1}{2})\delta_{K,\frac{1}{2}}]$$
$$-\frac{\hbar^2}{2J}[K(K+1)+a(-)^{K+\frac{1}{2}}(K+\frac{1}{2})\delta_{K,\frac{1}{2}}]$$

When there are two rotational spectra corresponding to the close configuration K and $K+1$, the states with the same I of the bands will be mixed by the Coriolis term. Malik and Scholz[4] have calculated level spectra of ^{53}Mn using the strong-coupling rotational model with band mixing.

3. Method of Analysis

A computer program NILSSON[11] was used in the present Nilsson model calculations. The eigenvalues for the Nilsson-model Hamiltonian for the oscillator number $N=3$ with the specified values of the parameters, μ and κ, are plotted in Fig. 1 as a function of the deformation parameter δ. The values of the Nilsson-model parameters used in this calculation are those obtained in this mass region by Sood[12], i.e., $\mu=0.44$, $\kappa=0.07$.

Table 1 Parameters used in the Nilsson model calculations in ^{53}Mn.

μ	κ	δ	$A=\frac{\hbar^2}{2J}$	a
0.44	0.07	−0.1	179.9KeV	−2.0

This figure shows the expected relative ordering and spacing of various single-particle Nilsson states on which rotational levels are to be built.

The ground-state spin of the nucleus is usually determined by the lowest of a band based on the Nilsson state occupied by the last odd nucleon. The excited states are obtained from the rotational states based on this ground-state band and from other rotational states built on particle bands and hole bands. Particle bands are constructed by placing the last unpaired nucleon in any of the unoccupied and hole bands are constructed by lifting a core particle and pairing it with the odd nucleon. However, as far as the particle bands are concerned in Fig. 1, the lowest state of a band based on the Nilsson level occupied by the last odd proton does not pridict, at a first glance, the observed ground-state spin of ^{53}Mn, $J^\pi=\frac{7}{2}^-$, since the ground state configuration for $^{53}_{25}Mn_{28}$ may be taken to be

$$\nu\left\{^{48}_{20}Ca_{28};\ (\tfrac{1}{2})^2\,(\tfrac{3}{2})^2\,(\tfrac{5}{2})^1\right\}\text{ for a positive }\delta,$$

$$\nu\left\{^{48}_{20}Ca_{28};\ (\tfrac{7}{2})^2\,(\tfrac{5}{2})^2\,(\tfrac{3}{2})^1\right\}\text{for a negative }\delta,$$

where ν represents the proton configuration. That is, in the ground state configuration, the last odd proton occupies the Nilsson particle states $\frac{5}{2}[312]$ and $\frac{3}{2}[321]$ for a positive deformation and a negative deformation, respectively, indicating $J^\pi=\frac{5}{2}^-$ or $\frac{3}{2}^-$ for the ground-state spin depending on the sign of δ.

In the hole bands, the ground-state configuration for a negative δ,

$$\nu\left\{^{48}_{20}Ca_{28};\ (\tfrac{7}{2})^{-1}\,(\tfrac{5}{2})^2\,(\tfrac{3}{2})^2\right\},$$

would reproduce the correct spin, $J^\pi=\frac{7}{2}^-$, for the ground state of ^{53}Mn. Therefore, in the present calculations, it was assumed that the rotational spectra in the levels of ^{53}Mn were built on the two hole bands $\frac{7}{2}[303]$ and $\frac{5}{2}[312]$ for a negative δ.

The rotational constant $A=\frac{\hbar^2}{2I}$ is usually a free parameter. The present values of $A=179.9$keV and $a=-2.0$ were determined by the best fit of rotational levels to a given number of experimental levels. The parameters used in the present calculations are listed in Table 1.

4. Results and Discussion

The calculated energy levels of ^{53}Mn are shown in Fig. 2 and compared with the values computed with the band mixing by Malik and Scholz[4].

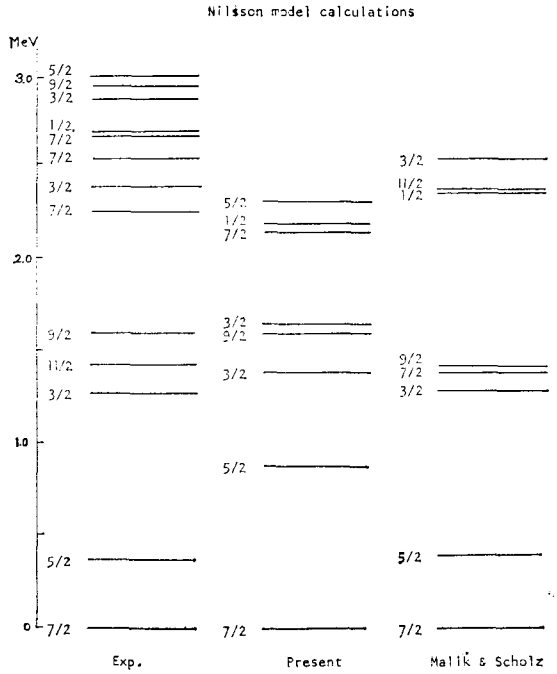

Fig. 2 The calculated energy lebels of ^{53}Mn based on the Nilsson model. The present calculated levels without band-mixing are compared with the observed levels and the computed levels with band-mixing by Malik and Scholz.

In this calculations, according to the usual procedure, μ, κ, and A were treated as free parameters. The values of μ and κ are those of Sood[12] which have been proved to be best fit for the $N=3$ oscillator number in this mass region. The value of $A=179.9$keV was obtained from the best fit to experimental levels of ^{53}Mn.

In the band mixing, Malik and Scholz included all bands based on the ten available single-particle and core-excited levels in the f-p shell and obtained the final spectra by diagonalizing the Coriolis interaction with rotational wave functions. They attributed the ground-state spin in $\frac{7}{2}^-$ in ^{53}Mn to the occurrence of a low-lying $\frac{7}{2}^-$ state in the decoupled $K=\frac{1}{2}$ band based on the Nilsson state $\frac{1}{2}[330]$ together with the strong Coriolis coupling.

In the present calculations, the four spins, $\frac{7}{2}^-$, $\frac{5}{2}^-$, $\frac{3}{2}^-$ and $\frac{9}{2}^-$ in the low-lying states are well reproduced, although the first excited state $\frac{5}{2}^-$ is predicted at a higher position. A spin of $\frac{7}{2}^-$ at about 2.2MeV would correspond to the experimental level of $\frac{7}{2}^-$ in this region. However, the model does not predict a low-lying spin of $\frac{11}{2}^-$ at 1.4 MeV.

In general, the present calculation gives a little better results than the band-mixing one, indicating that the straight calcula tion is good enough as far as the Nilsson model prediction for ^{53}Mn levels is concerned, without need for complicated band-mixing calculations.

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