Similarity Analysis of Reactor Physics Benchmark Experiments for Uncertainty Quantification of SFR TRU Burner

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1. Introduction

Korea Atomic Energy Research Institute (KAERI) has developed the Prototype Gen IV Sodium-cooled Fast Reactor (PGSFR) [1] to demonstrate the technology to reduce the long-lived radioactivity of the spent fuel by recycling the transuranic (TRU) elements. The PGSFR adopted a TRU burner type reactor which uses the U-TRU-Zr metallic fuel to leverage: 1) the higher thermal conductivity for enhanced safety [2] and 2) the hardened neutron spectrum for a higher TRU transmutation rate compared to the typical oxide fuel. However, this poses a challenge in the validation and uncertainty quantification of the reactor physics parameters calculated by the nuclear reactor design code, since the reactor is loaded with a nonnegligible amount of minor actinides that lacks measurement data for the cross section and there is no reactor physics benchmark experiment that faithfully mocks up the nuclear characteristics of the TRU burner with metallic fuel.

In the field of criticality safety analysis for the spent fuel storage and the transportation cask, the technical guidelines to determine the criticality bias and bias uncertainty based on the existing benchmark has been established [4], [5]. The guidelines propose several parameters to establish the area of applicability (AOA) and develop a trend line. These parameters include fissile enrichment, hydrogen-to-fissile nuclide ratio (H/X), energy of average lethargy causing fission (EALF), and etc. However, it is an ambiguous problem to determine the AOA based on the multiple parameters. To address this problem, the integral system parameter c_k , which represents the similarity coefficient between the two systems in terms of the criticality response when the nuclear data uncertainty is propagated, was proposed [6],[7] and incorporated in the TSUNAMI-IP module of the SCALE code package [8].

To evaluate the c_k rigorously, all the covariance nuclear data provided in ENDF-6 format [9] should be propagated to the criticality uncertainty. However, covariance of the energy and angular distributions has been usually ignored [10]. Recently, the nuclear data sampling tool SANDY [11] has been developed by SCK CEN which can stochastically sample the nuclear data considering all the covariances provided in ENDF-6 format.

This paper presents a computational framework for the similarity analysis comprising the nuclear data sampling code SANDY, the nuclear data processing code NJOY [12],

and the continuous-energy Monte Carlo (MC) code McCARD [13]. Subsequently, the established framework was applied to evaluate the c_k for existing fast reactor benchmark experiments against a typical TRU burner model.

2. SANDY/NJOY/McCARD Code System

Figure 1 shows a flow chart for the computational process of the SANDY/NJOY/McCARD code system which propagates the covariance nuclear data to the reactor physics parameter based on the stochastic sampling method.



Figure 1. Flow chart of the SANDY/NJOY/McCARD.

By coupling SANDY and NJOY codes, a total of N_s samples of nuclear data, $\vec{\mu}^{(n)}$, are obtained from a multivariate normal distribution of $N(\vec{\mu}, \mathbf{C})$, where $\vec{\mu}^{(n)}$ is the *n*-th sample of nuclear data for $n = 1 \text{ to } N_s$, $\vec{\mu}$ is the best-estimate of the nuclear data, and **C** is the covariance nuclear data for the neutron multiplicities (MF=31), the resonance parameters (MF=32), the cross sections (MF=33), the angular distribution of the secondary particle (MF=34), and the fission neutron spectrum (MF=35). These samples are then written in the ACE (A Compact ENDF) format for the continuous-energy MC codes.

Next, for each sampled nuclear data, the reactor physics parameters $Q_B^{(n)}$ and $Q_T^{(n)}$ of the benchmark experiments and the target core model, respectively, are calculated by the McCARD. The nuclear data-induced uncertainty $Q_i^{(n)}$ (for i = B or T) is calculated by:

$$\sigma(Q_i) = \sqrt{\frac{\sum_{n=1}^{N_s} (Q_i^{(n)} - \bar{Q}_i)}{N_s - 1}} \text{ for } i = B \text{ or } T, \qquad (1)$$

where $\bar{Q}_i = \frac{1}{N_s} \sum_{n=1}^{N_s} Q_i^{(n)}$. The standard deviation of the estimated nuclear data-induced uncertainty can be estimated by the bootstrap method.

The similarity coefficient c_k between the two critical system can be estimated by:

$$c_{k} = \frac{\sum_{n=1}^{N_{s}} (k_{B}^{(n)} - \bar{k}_{B}) (k_{T}^{(n)} - \bar{k}_{T})}{\sqrt{\sum_{n=1}^{N_{s}} (k_{B}^{(n)} - \bar{k}_{B})^{2} \sqrt{\sum_{n=1}^{N_{s}} (k_{T}^{(n)} - \bar{k}_{T})^{2}}}$$
(2)

where $k_B^{(n)}$ and $k_T^{(n)}$ denote the criticality calculated by the *n*-th sampled nuclear data for the target core model and the benchmark experiments, respectively, and $\bar{k}_i = \frac{1}{N_s} \sum_{n=1}^{N_s} k_i^{(n)}$.

The standard deviation of the c_k can be evaluated by the second moment of the probability density function of the Pearson correlation coefficient as below [14]:

$$\sigma(c_k) \cong \sqrt{\frac{(1-c_k^2)}{N_s} \left(1 + \frac{11c_k^2}{2N_s}\right)}.$$
(3)

3. Numerical Results

3.1. Description of Target Core Model

The target core model is a typical TRU burner using the U-TRU-Zr metallic fuel based on the PGSFR core design at the beginning of cycle (BOC) of the equilibrium (Mixed TRU comprising self-recycled and light water reactor spent fuels) MTRU core [1]. The average fissile enrichment is 14.0 wt%. The TRU and MA compositions are 25.2 wt% and 3.0 wt%, respectively. The radial cut view of the target core model is shown in Figure 2.



Figure 2. Radial cut view of target core model for the SFR TRU Burner [15].

3.2. Evaluation Similarity Coefficient of Benchmark Experiments

A total of 19 fast reactor benchmark experiments were selected for the similarity analysis, which are listed as: two benchmark experiments (BFS-55-1 and BFS-76-1A) obtained through the collaboration of KAERI-IPPE (Institute of Physics and Power Engineering) [16],[17], the CEFR (China Experimental Fast Reactor) benchmark experiment acquired from the IAEA-CRP (International Atomic Energy Agency-Coordinated Research Program) [18], 12 benchmark experiments (BFS-62-3A, FFTF, JOYO, ZPPR-10A, ZPPR-9, ZPPR-13A, ZPPR-12, ZPPR-2, ZPR-6-7A, ZPR-3-48, ZPR3-48B, and ZPR3-56B) provided in the IRPhE (International Reactor Physics Experiments database) [19], and 4 benchmarks (ZPPR-15A,B,C, and D) obtained through I-NERI (International Nuclear Energy Research Initiative) project [20] by KAERI-ANL (Argonne National Laboratory) collaboration.

Among the various core loadings in the benchmark experiments, a total of 63 core loadings were modelled by the McCARD that can retrieve the criticality measurement. The calculation conditions for the McCARD analysis were 50,000 histories per cycle and 60 active cycles, where the standard deviations of the k_{eff} are around 30 pcm. For the similarity analysis, 250 sets of nuclear data were randomly sampled based on the ENDF/B-VII.1 [21] which provides the covariance data for 183 major nuclides out of a total 423 nuclides.

The core loadings for the similarity analysis are summarized in Table I, along with their corresponding core indices imposed in this study. Figure 3 shows the similarity coefficient matrix calculated by the SANDY/NJOY/McCARD code system. The maximum standard deviation of the similarity coefficient is 0.06.

Table I. List of core loadings for similarity analysis

Database	Benchmark/Core	Core	Database	Benchmark/Core	Core
	Loading	Index		Loading	Index
KAERI- IPPE	BFS-55-1/Ref	1	ZPPR-15A (KAERI- ANL)	ZPPR-15A/L021	33
	BFS-76-1A/Ref	2		ZPPR-15A/L022	34
IAEA-CRP	CEFR/CRIT_72FA	3		ZPPR-15A/L025	35
	BFS-62-3A/Ref	4		ZPPR-15A/L026	36
	BFS-62-3A/CR13	5		ZPPR-15A/L027	37
	BFS-62-3A/CR16	6		ZPPR-15A/L028	38
	BFS-62-3A/CR35	7	ZPPR-15B (KAERI- ANL)	ZPPR-15B/L088	39
	BFS-62-3A/CR311	8		ZPPR-15B/L091	40
IDDhE	BFS-62-3A/SR22	9		ZPPR-15B/L092	41
IKPHE	BFS-62-3A/SR25	10		ZPPR-15B/L093	42
	BFS-62-3A/SVR-LEZ	11		ZPPR-15B/L094	43
	BFS-62-3A/SVR-	12		ZPPR-15B/L096	44
	MEZ				
	BFS-62-3A/SVR-	- 13		ZPPR-15B/L100	45
	PEZ				

Database	Benchmark/Core	Core	Database	Benchmark/Core	Core
	Loading	Index		Loading	Index
	BFS-62-3A/SVR-	14		ZPPR-15B/L102	46
	HEZ				
	FFTF/Ref	15		ZPPR-15B/L103	47
	JOYO/L64	16		ZPPR-15B/L104	48
	JOYO/L70	17		ZPPR-15B/L105	49
	ZPPR-10A/L007	18		ZPPR-15B/L106	50
	ZPPR-9/L013	19	ZPPR-15C (KAERI- ANL)	ZPPR-15C/L166	51
	ZPPR-13A/L024	20		ZPPR-15C/L167	52
	ZPPR-12/L009	21		ZPPR-15C/L168	53
	ZPPR-2/L090	22		ZPPR-15C/L169	54
	ZPR6-7A/L012	23	ZPPR-15D (KAERI- ANL)	ZPPR-15D/L184	55
	ZPR6-7A/L099	24		ZPPR-15D/L185	56
	ZPR3-48/L047	25		ZPPR-15D/L189	57
	ZPR3-48B/L006	26		ZPPR-15D/L190	58
	ZPR3-56B/L017	27		ZPPR-15D/L193	59
ZPPR-15A	ZPPR-15A/L015	28		ZPPR-15D/L194	60
(KAERI-	ZPPR-15A/L016	29		ZPPR-15D/L195	61
ANL)	ZPPR-15A/L018	30		ZPPR-15D/L198	62

Database	Benchmark/Core	Core	Database	Benchmark/Core	Core
	Loading	Index		Loading	Index
	ZPPR-15A/L019	31		ZPPR-15D/L199	63
	ZPPR-15A/L020	32	Target Core Model		Т

4. Summary and Conclusion

The SANDY/NJOY/McCARD framework has been established to evaluate nuclear-data induced uncertainty and the similarity coefficient c_k . Subsequently, the established framework was applied to evaluate the c_k for existing fast reactor benchmark experiments against a typical TRU burner model. Among the 63 core loadings of the benchmark experiments, there were 33 core loadings with the c_k greater than 0.8 and 32 core loadings with the c_k greater than 0.9. The highest similarity coefficient, 0.962, was observed in the ZPPR-15B with the L102 core loading. The results of this study can be utilized to quantify the uncertainty of the SFR TRU burner based on the well-established statistical methods for the criticality safety analysis.



Figure 3. Similarity coefficient matrix for 63 core loadings and target core model.

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