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APOLLO3® Homogenization Techniques for Transport Core Calculations - Application to the ASTRID CFV Core

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Abstract - *This paper presents a comparison of homogenization techniques implemented in the APOLLO3® platform for transport core calculations: standard scalar flux weighting and new flux-moments homogenization, in different combinations with (or without) leakage models. Besides the historical B1-homogeneous model, a new B-heterogeneous one has indeed been recently implemented in the TDT/MOC solver. First analyses have been performed on a very simple SFR core with a regular hexagonal lattice. They show that using the heterogeneous leakage model in association with the flux-moments homogenization strongly improves the prediction of Keff and void reactivity effects. These good results are confirmed when the application is done to the fissile assemblies of the more complex ASTRID CFV core.*

I. INTRODUCTION

Standard deterministic calculations lie generally on a two-step calculation scheme with a first step performed at the cell or assembly level using a 2D fine transport calculation that provides homogenized parameters for a global 3D core calculation in the second step, more often using diffusion theory.

The need for better accuracy in calculating neutronic parameters (such as reactivity, control rod worth, power distribution) leads to the development of transport core solvers taking advantage of the increasing performances of the computers. However, the magnitude of the computational problem posed by explicitly modeling the exact geometry with thousands of energy groups, hundreds of angular directions and millions of spatial nodes is still out of range of desktop computers with few tens of processors but requires expensive super-computers with often limited access. So, using transport theory for practical applications to three-dimensional reactor analysis still needs spatial homogenization and energy condensation.

Homogenization techniques have been widely studied when the core calculation is performed using diffusion theory, focusing on diffusion coefficients definition and introducing different techniques to attempt to preserve the neutron balance (discontinuity factors and SPH equivalence techniques are the most employed). Many thoughts and references can be found on these subjects in Ref. 1 and 2.

In the case of homogenization for transport calculations, diffusion coefficients are no more appearing and the flux-volume weighting of cross sections alone is not always sufficiently accurate (depending of the degree of spatial homogenization and energy collapsing). Discontinuity factors technique is not available in APOLLO3® and some past numerical experiments with APOLLO2 have shown that the transport-transport SPH equivalence was lacking of robustness when dealing with several tens of groups (Ref. 3). May be some recent developments proposed in Ref. 4 could help but we preferred to follow another way and try to improve the cross section weighting itself.

So, the angular flux moments weighting (Ref. 5), generalizing the P1-consistent method of the ECCO cell code (included in the ERANOS code system, Ref. 6), has been implemented in the APOLLO3® platform (Ref. 7) in order to improve the treatment of exchanges between assemblies of different types (fissile and control rod assemblies for example) and between core and reflector (as shown in Ref. 5). Besides, a B-heterogeneous leakage model (Ref. 8) has been implemented in the TDT/MOC (Ref. 9) for the treatment of fissile assemblies, following the work of Chiba and Van Rooijen (Ref. 10).

The paper presents first the flux-moments homogenization method in the absence of leakage model. In the second part, the B-heterogeneous leakage model is recalled and the above homogenization is adapted to deal with complex flux-moments (a possibility has also been introduced in APOLLO3® to use the leakage rate $\boldsymbol{DB}^2\boldsymbol{\Phi}$ with $\boldsymbol{DB}^2$ coming from a B1-homogeneous calculation to homogenize order 1 transfer cross sections). In the third part, the impact of different combinations (leakage model, homogenization method) on the effective multiplication factor and voiding effect are studied on a simplified "regular SFR core" before application to the more complex ASTRID CFV core (Ref. 11) in the final section.

II. ANGULAR FLUX MOMENTS WEIGHTING

1. General 3D case

The stationary transport equation discretized on microregion i and microgroup g can be written as:

$$\int_{\partial i} dS \left|\vec{\Omega}.\vec{n}\right|\left(\Psi_i^{g,+}(\vec{\Omega})-\Psi_i^{g,-}(\vec{\Omega})\right)+\Sigma_{t,i}^{g}\Psi_i^{g}(\vec{\Omega}) = \sum_{g'=1}^{N_g}\sum_{l=0}^{L}\frac{(21+1)}{4\pi}\Sigma_{sl,i}^{g'\to g}\sum_{m=-l}^{+l}\Phi_{l,i}^{m,g'}R_l^m(\vec{\Omega})+S_i^g(\vec{\Omega}) \quad (1)$$

In formula (1) the fluxes $\Psi_i^{g,\pm}(\vec{\Omega})$ are the boundary fluxes used in MOC calculations to compute the leakage contribution in the balance equation (Ref. 9), and the integration is done over the boundary of region i.

After homogenization/condensation $\left(\sum_{i\in I}\sum_{g\in G}(...)\right)$, a slightly different form is obtained:

$$\int_{\partial I} dS \left|\vec{\Omega}.\vec{n}\right|\left(\Psi_I^{G,+}(\vec{\Omega})-\Psi_I^{G,-}(\vec{\Omega})\right)+\Sigma_{t,I}^{G}(\vec{\Omega})\Psi_I^{G}(\vec{\Omega}) = \sum_{G'=1}^{N_G}\sum_{l=0}^{L}\frac{(21+1)}{4\pi}\sum_{m=-l}^{+l}\Sigma_{sl,I}^{m,G'\to G}\Phi_{l,I}^{m,G'}R_l^m(\vec{\Omega})+S_I^G(\vec{\Omega}) \quad (2)$$

Defining the homogenized and condensed angular fluxes and moments:

$$\Psi_I^G(\vec{\Omega})=\sum_{i\in I}\sum_{g\in G}\Psi_i^g(\vec{\Omega}) \quad , \quad \Phi_{l,I}^{m,G}=\sum_{i\in I}\sum_{g\in G}\Phi_{l,i}^{m,g}$$

the total cross section for macroregion I and macrogroup G is now dependent of the direction vector $\vec{\Omega}$:

$$\Sigma_{t,I}^{G}(\vec{\Omega})=\frac{\sum_{i\in I}\sum_{g\in G}\Sigma_{t,i}^{g}\Psi_i^g(\vec{\Omega})}{\sum_{i\in I}\sum_{g\in G}\Psi_i^g(\vec{\Omega})}$$

and transfer cross sections moments of the azimuthal rank m:

$$\Sigma_{sl,I}^{m,G'\to G}=\frac{\sum_{i\in I}\sum_{g'\in G'}\sum_{g\in G}\Sigma_{sl,i}^{g'\to g}\Phi_{l,i}^{m,g'}}{\sum_{i\in I}\sum_{g'\in G'}\Phi_{l,i}^{m,g'}}$$

In Ref. 5, we proposed to generalize the Pn-consistent method (Ref. 12) to 2D or 3D for collapsing these transfer cross sections, using a minimization strategy to eliminate the dependency on rank m and the anisotropy of the total cross section so that the transport equation keeps its standard form:

$$\hat{\Sigma}_{sl,I}^{G'\to G}=\frac{\sum_{m=-l}^{+l}\Phi_{l,I}^{m,G'}\sum_{i\in I}\sum_{g'\in G'}\sum_{g\in G}\Sigma_{sl,i}^{g'\to g}\Phi_{l,i}^{m,g'}}{\sum_{m=-l}^{+l}\left(\Phi_{l,I}^{m,G'}\right)^2}+\delta_{G,G'}\left(\Sigma_{t,I}^{G'}-\Sigma_{tl,I}^{G'}\right) \quad (3)$$

with the usual total cross section weighted by the scalar flux

$$\Sigma_{t,I}^{G}=\Sigma_{t0,I}^{0,G}=\frac{\sum_{i\in I}\sum_{g\in G}\Sigma_{t,i}^{g}\Phi_{0,i}^{0,g}}{\sum_{i\in I}\sum_{g\in G}\Phi_{0,i}^{0,g}}$$

and superior orders terms

$$\Sigma_{\substack{tl,I\\ l>0}}^{G}=\frac{\sum_{m=-l}^{+l}\left(\Phi_{l,I}^{m,G}\sum_{i\in I}\sum_{g\in G}\Sigma_{t,i}^{g}\Phi_{l,i}^{m,g}\right)}{\sum_{m=-l}^{+l}\left(\Phi_{l,I}^{m,G}\right)^2} \quad (4)$$

This method thus "captures" most of the flux anisotropy in the transfer cross sections (mainly in the order 1 terms). In the following, it will be called the **"moments"** homogenization.

2. Specific 1D plane case

In the particular case of 1D plane geometries, angular flux moments are zero for m≠0 when the axis perpendicular to the slabs is oriented along the polar direction (traditionally the z direction of vector $\vec{\Omega}$). So, considering the P1 transfer cross section, formula (3) is reduced to a current-weighting homogenization with a P1-correction for the G towards G terms:

$$\hat{\Sigma}_{sl,1}^{G'\to G}=\frac{\sum_{i\in I}\sum_{g'\in G'}\sum_{g\in G}\Sigma_{sl,i}^{g'\to g}J_i^{g'}}{\sum_{i\in I}\sum_{g'\in G'}J_i^{g'}} + \delta_{G,G'}\left(\frac{\sum_{i\in I}\sum_{g'\in G'}\Sigma_{t,i}^{g'}\Phi_i^{g'}}{\sum_{i\in I}\sum_{g'\in G'}\Phi_i^{g'}}-\frac{\sum_{i\in I}\sum_{g'\in G'}\Sigma_{t,i}^{g'}J_i^{g'}}{\sum_{i\in I}\sum_{g'\in G'}J_i^{g'}}\right) \quad (5)$$

where $\Phi_i^{g'}=\Phi_{0,i}^{0,g'}$ is the scalar flux and $J_i^{g'}=\Phi_{1,i}^{0,g'}$ is the integrated current.

This homogenization is relevant if a significant current is established in the calculated pattern (for example: clusters of fissile and non-fissile assemblies, core reflector interface as seen in Ref. 3).

When the pattern is a reflected fissile assembly, the current can be locally small and numerical cancellation can occur when the size of a macroregion I is growing (it necessarily occurs when the assembly is fully homogenized, generating numerical difficulties). To overcome this difficulty, an extension of the flux moments homogenization method has been proposed in APOLLO3® linked to the new heterogeneous leakage model implemented in the TDT/MOC solver.

III. HETEROGENEOUS LEAKAGE MODEL WITH FLUX MOMENTS HOMOGENIZATION

Assuming a fundamental mode factorization with:

$$\Psi^g(\vec{r},\vec{\Omega}) = \Psi_B^g(\vec{r},\vec{\Omega})e^{i\vec{B}.\vec{r}}$$

where $\Psi_B^g(\vec{r},\vec{\Omega})$ is a lattice-periodic complex distribution called "microscopic flux" and $\vec{B}$ an invariant vector that represents the macroscopic curvature of the flux within the core, the transport equation is transformed into a new eigenvalue problem including a leakage term. The MOC solver of APOLLO3® (TDT/MOC) directly solves the corresponding B heterogeneous equation in its complex form, which, once discretized in space, can be written:

$$\int_{\partial i} dS\,|\vec{\Omega}.\vec{n}|\left(\Psi_{B,i}^{g,+}(\vec{\Omega}) - \Psi_{B,i}^{g,-}(\vec{\Omega})\right) + \left(\Sigma_{t,i}^g + i\vec{B}.\vec{\Omega}\right)\Psi_{B,i}^g(\vec{\Omega})$$
$$= \sum_{g'=1}^{N_g}\sum_{l=0}^{L}\frac{(2l+1)}{4\pi}\Sigma_{sl,i}^{g'\to g}\sum_{m=-l}^{+l}\Phi_{B,l,i}^{m,g'}R_l^m(\vec{\Omega}) + S_{B,i}^g(\vec{\Omega}) \quad (6)$$

to get the complex periodic buckling-dependent flux:

$$\Psi_{B,i}^g(\vec{\Omega}) = \Psi_{\mathrm{Re},i}^g(\vec{\Omega}) - i\Psi_{\mathrm{Im},i}^g(\vec{\Omega})$$

The homogenization/condensation of Eq. 6 has a similar form than Eq. 2 with an additional leakage term $i\vec{B}.\vec{\Omega}\Psi_{B,I}^G(\vec{\Omega})$:

$$\int_{\partial I} dS\,|\vec{\Omega}.\vec{n}|\left(\Psi_{B,I}^{G,+}(\vec{\Omega}) - \Psi_{B,I}^{G,-}(\vec{\Omega})\right) + \left(\Sigma_{t,I}^G(\vec{\Omega}) + i\vec{B}.\vec{\Omega}\right)\Psi_{B,I}^G(\vec{\Omega})$$
$$= \sum_{G'=1}^{N_G}\sum_{l=0}^{L}\frac{(2l+1)}{4\pi}\sum_{m=-l}^{+l}\Sigma_{sl,I}^{m,G'\to G}\Phi_{B,l,I}^{m,G'}R_l^m(\vec{\Omega}) + S_I^G(\vec{\Omega}) \quad (7)$$

In Ref. 1 and with more details in Ref. 13, it has been shown that if the material distribution has a central symmetry (which is the case here) and if the homogenization is performed on macroregions invariant under the action of this central symmetry (this is also the case):

- the even moments of the homogenized flux are real,
- the odd moments are purely imaginary.

and so, because of cancellations between symmetric microregions inside the macroregions, real values are obtained for transfer cross sections and total cross sections of different orders: even ones are weighted by the real component and odd ones by the imaginary component. Formula (3) and (4) can be split into:

$$\underset{l\ even}{\Sigma_{tl,I}^G} = \frac{\sum_{m=-l}^{+l}\left(\Phi_{\mathrm{Re}l,I}^{m,G}\sum_{i\in I}\sum_{g\in G}\Sigma_{t,i}^g\Phi_{\mathrm{Re},l,i}^{m,g}\right)}{\sum_{m=-l}^{+l}\left(\Phi_{\mathrm{Re},l,I}^{m,G}\right)^2}$$

$$\underset{l\ odd}{\Sigma_{tl,I}^G} = \frac{\sum_{m=-l}^{+l}\left(\Phi_{\mathrm{Im},l,I}^{m,G}\sum_{i\in I}\sum_{g\in G}\Sigma_{t,i}^g\Phi_{\mathrm{Im},l,i}^{m,g}\right)}{\sum_{m=-l}^{+l}\left(\Phi_{\mathrm{Im},l,I}^{m,G}\right)^2}$$

$$\underset{l\ even}{\hat{\Sigma}_{sl,I}^{G'\to G}} = \frac{\sum_{m=-l}^{+l}\Phi_{\mathrm{Re},l,I}^{m,G'}\sum_{i\in I}\sum_{g'\in G'}\sum_{g\in G}\Sigma_{sl,i}^{g'\to g}\Phi_{\mathrm{Re},l,i}^{m,g'}}{\sum_{m=-l}^{+l}\left(\Phi_{\mathrm{Re},l,I}^{m,G'}\right)^2} + \delta_{G,G'}\left(\Sigma_{t,I}^{G'} - \underset{l\ even}{\Sigma_{tl,I}^G}\right)$$

$$\underset{l\ odd}{\hat{\Sigma}_{sl,I}^{G'\to G}} = \frac{\sum_{m=-l}^{+l}\Phi_{\mathrm{Im},l,I}^{m,G'}\sum_{i\in I}\sum_{g'\in G'}\sum_{g\in G}\Sigma_{sl,i}^{g'\to g}\Phi_{\mathrm{Im},l,i}^{m,g'}}{\sum_{m=-l}^{+l}\left(\Phi_{\mathrm{Im},l,I}^{m,G'}\right)^2} + \delta_{G,G'}\left(\Sigma_{t,I}^{G'} - \underset{l\ odd}{\Sigma_{tl,I}^G}\right)$$

These formulas have been recently implemented in APOLLO3® and will be mentioned in the following as **"moments"** homogenization as they are only a generalization of the formula without leakage model.

Besides, APOLLO3® includes also a more classical B1 homogeneous leakage model that provides uniform leakage coefficients at the assembly scale D^g so that the leakage term in the heterogeneous calculation is written $D^g B^2\Psi_{\mathrm{Re},i}^g(\vec{\Omega})$. The corresponding "fundamental" current is then $J_{\mathrm{hom},i}^g = D^g B\Phi_{\mathrm{Re},i}^g$ and can also be used to homogenize order 1 transfer cross sections in replacement of J_i^g in Eq. 5 (superior order transfer XS are then homogenized by the scalar flux). This last method is called **"homCurrent"** homogenization. Standard scalar flux-weighting of the whole cross sections is of course available in the code and will be simply called **"flux"** homogenization.

IV. RESULTS ON A SIMPLIFIED SFR CORE

1. Geometries

In order to compare and validate the different homogenization methods with minimum biases against Monte Carlo TRIPOLI-4® (Ref. 14) reference calculations, a very simple SFR core has been modeled. It is composed of a finite hexagonal lattice of CFV-like pins axially limited to the fissile height. This "regular" core has been made critical by adjusting the number of hexagonal rings in a configuration without reflector (3D Monte Carlo continuous energy TRIPOLI-4® simulations has been used for this, the corresponding geometry is given on Figures 1 and 2). The core is small, contained in a cylinder of 90 cm diameter and 80 cm in height. In a second configuration, a 14-cm thick steel reflector has been radially added to model a more realistic (and more challenging) situation (cf. Figure 3).

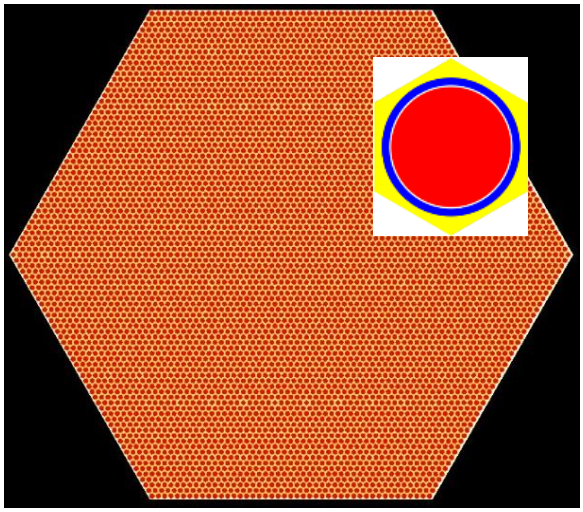

Fig. 1. Radial cross section of the bare "regular SFR core"

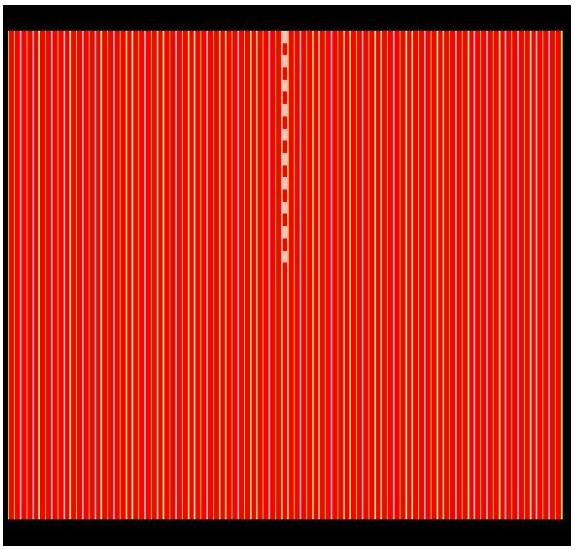

Fig. 2. Axial cross section of the bare "regular SFR core"

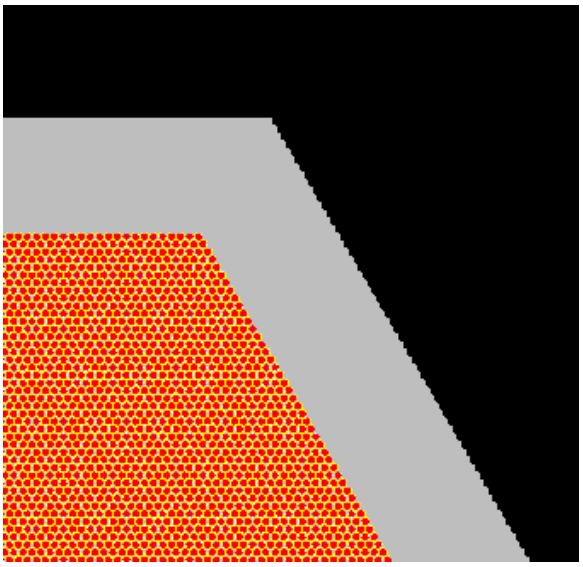

Fig. 3. Radial cross sections of the reflected "regular SFR core" (one quarter core)

2. Calculation routes

2D 1968-group TDT/MOC calculations using the sub-group method for self-shielding (Ref. 15) have been performed to generate 33-group cell-homogenized cross sections following the different methods presented above:

- Infinite fuel cell calculations for use with a leakage model (the cell picture is enclosed in Figure 1),
- Pseudo-1D traverse to explicitly describe the radial leakage and the interface with the void or the reflector. A reflective boundary condition is applied on the left and a vacuum one on the right (see Figure 4). Five sets of fuel homogenized cross sections are produced regarding the distance of the cells from the core boundary (different colors on Figure 5) and, when the reflector is present, five sets of cross sections are generated for the reflector itself.

The ECCO code (Ref. 16), which possesses a simplified heterogeneous model called "P1-consistent" (the fundamental current is supposed to be isotropic) and a current weighting homogenization technique has also been used for comparison in the bare configuration.

Fig. 4. MOC Traverse geometries (up: without reflector, down: with steel reflector)

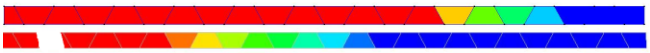

Fig. 5. Distinct radial zones for the homogenization (up: without reflector, down: with steel reflector)

The unstructured Sn solver MINARET of APOLLO3® (Ref. 17) allowed us to perform 33-group 3D core calculations for four configurations, with and without sodium, with and without reflector. A Chebyshev-Gauss product quadrature with 72 directions and P3 scattering anisotropy cross sections has been used. The thickness of the extruded triangular meshes is less than 3 cm in the radial plane (see Figures 6 and 7) and 5 cm in the axial direction. The finite elements are linear.

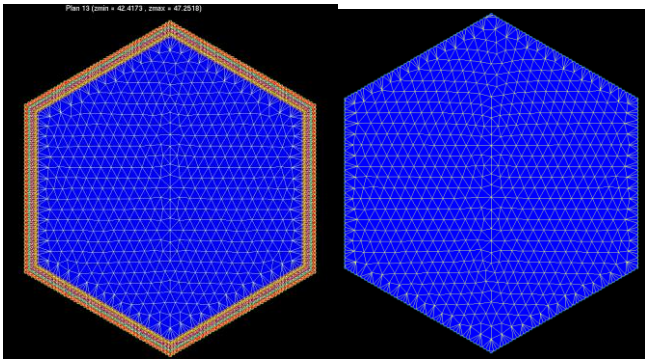

Fig. 6. MINARET Radial Mesh for the bare core (traverse homogenization on the left, cell one on the right)

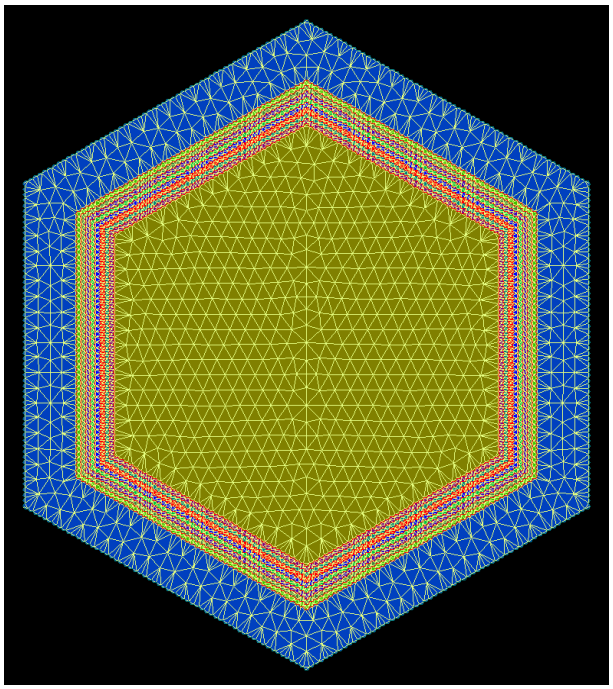

Fig. 7. MINARET Radial Mesh for the reflected core (traverse homogenization")

3. Bare core results

The results on effective multiplication factor K_{eff} and sodium void effect for different combinations of geometry, leakage model and homogenization techniques are shown in Table I. It can be observed that:

- all the calculations using a leakage model and the scalar flux homogenization (blue lines) provide poor results on K_{eff}: discrepancies with TRIPOLI-4® are larger than +500 pcm when the core is filled with sodium and up to +800 pcm when it is voided. The voiding effect is then overpredicted by 0.5 to 0.7 $. When a flux-weighting is performed, the best results are obtained without leakage model (black line).
- using the flux moments (red lines) coming from a traverse calculation or a cell calculation with heterogeneous leakage to collapse transfer cross sections significantly improves the APOLLO3® results. The biases on K_{eff} are less than 250 pcm ; for sodium void effect, they are reduced to 0.2 $. The agreement with Monte Carlo calculations is quite satisfactory.
- even if they are closer to TRIPOLI-4®, the B heterogeneous results are sensitive to the direction of the buckling vector: K_{eff} are underestimated when $\vec{B}$ is oriented in the z-direction (-132 pcm) because leakage channels are favored (this effect is enhanced when voiding: -198 pcm) and overestimated when belonging to the radial plane (respectively +4 and +87 pcm, purple line).
- the B1 model with the homogeneous leakage rate homogenization gives satisfactory results on the filled core configuration (+115 pcm) but not on the voiding one, which leads to an important overprediction of the void effect (+0,5$).
- the ECCO P1-consistent results are close to the B heterogeneous one with B oriented in the z-direction with a rather low discrepancy on the void effect (-0.2$). The error on K_{eff} is slightly increased against Monte Carlo results (-236 pcm and -311 pcm for the two configurations) probably because of symmetry assumptions and angular average of the coupling terms between the real and imaginary parts of Eq. 5 (see details in Ref. 6). For solvers based on the collision probability method like ECCO, this simplified heterogeneous model offers a real improvement compared to the B1 homogeneous one.

4. Steel-reflected core results

When a steel reflector is radially surrounding the core, the flux shape in the different groups are strongly different from the bare configuration when approaching the core reflector interface (see radial flux traverses Figure 8) with strong increases in the reflector for energies less than 5 keV (groups with g≥17). Using the fuel homogenized cross sections coming from the TDT/MOC traverse calculation described in IV.2 allows the influence of the reflector to be taken into account, which is not the case when XS are coming for the single cell calculation. Table II shows however that the trends are still the same than the one from the previous case without reflector:

- when using the scalar flux homogenization, the sodium void reactivity is strongly overpredicted (0.5 to 0.6 $) with cell homogenized cross sections and the same with Keff when XS are coming from the traverse calculation (more than 800 pcm discrepancy),
- Using the heterogeneous leakage and the flux moments homogenization together in the fuel cell calculation provides satisfactory results both in terms of reactivity and voiding effect, especially when the Buckling vector is z-oriented (leakage dominates in that direction because of the presence of the radial reflector). In this case, the error on K-effectives is less than 100 pcm, and is negligible on sodium void effect.

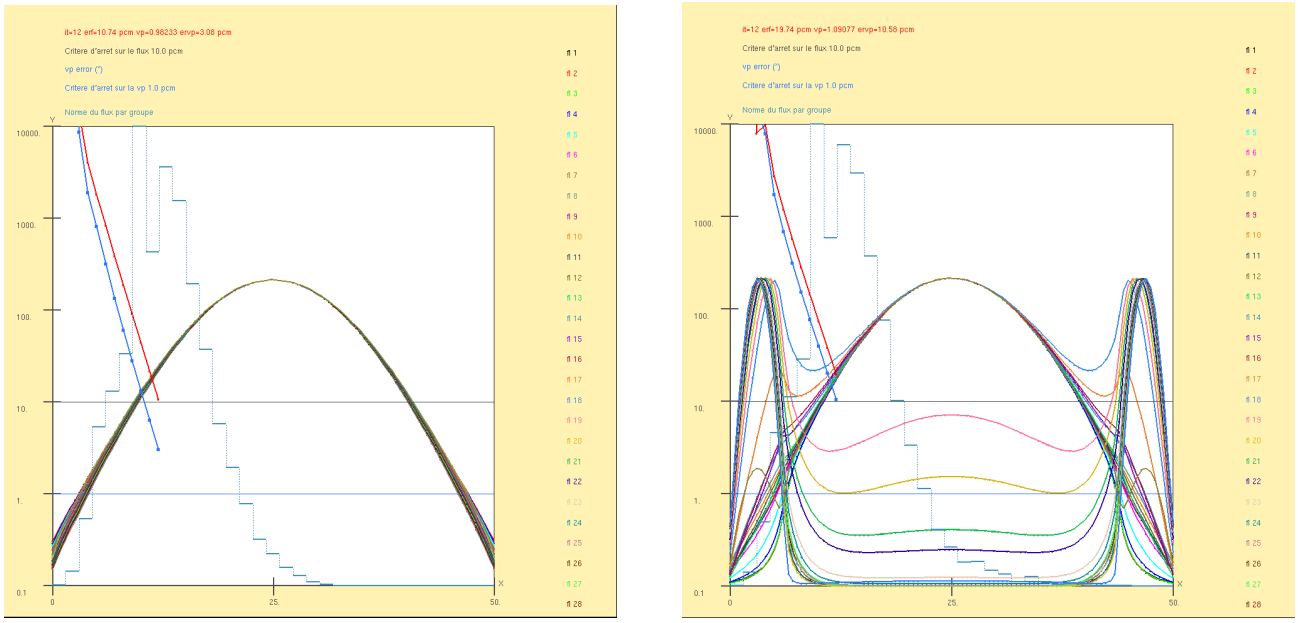

Fig. 8. MINARET Radial flux traverses without reflector (on the left) and with radial steel reflector (on the right)

Table I. Regular SFR core - Keff and Sodium void reactivity for different leakage and homogenization techniques for the bare configuration

TRIPOLI-4® * 1$ =~360 pcm	Keff		Sodium Void effect (pcm)
	Full	Voided	
	1.00478 ± 2 pcm	0.98137 ± 2 pcm	-2374 ± 3 pcm

MINARET cross sections origin						Δρ A3/T4 (pcm)		Voiding biases	
CODE Geometry	Leakage Model	Homogenization				Full	Voided	pcm	$*
ECCO Cell	**P1-cons**	**flux (Φ)**	**1.01055**	**0.98934**	**-2121**	**568**	**821**	**252**	**0.7**
	P1-cons	**hetCurrent (J)**	**1.00241**	**0.97839**	**-2449**	**-236**	**-311**	**-75**	**-0.2**
APOLLO3 Cell	**no**	**flux (Φ)**	**1.00830**	**0.98661**	**-2180**	**347**	**541**	**194**	**0.5**
	B1-hom	**flux (Φ)**	**1.00990**	**0.98840**	**-2154**	**504**	**724**	**220**	**0.6**
	B1-hom	**homCurrent (DB2Φ)**	**1.00594**	**0.98426**	**-2190**	**115**	**299**	**184**	**0.5**
	B-het (Bz)	**flux (Φ)**	**1.00991**	**0.98833**	**-2162**	**505**	**717**	**212**	**0.6**
	B-het (Bz)	**moments**	**1.00337**	**0.97927**	**-2453**	**-140**	**-219**	**-79**	**-0.2**
	B-het (Bx)	**moments**	**1.00499**	**0.98236**	**-2292**	**21**	**102**	**82**	**0.2**
APOLLO3 Traverse	**no**	**flux (Φ)**	**1.01137**	**0.98950**	**-2185**	**648**	**837**	**189**	**0.5**
	no	**moments**	**1.00730**	**0.98359**	**-2393**	**249**	**230**	**-19**	**-0.1**

Table II. Regular SFR core - Keff and Sodium void reactivity for different leakage and homogenization techniques for the steel reflected configuration

TRIPOLI-4® * 1$ =~360 pcm	Keff		Sodium Void effect (pcm)
	Full	Voided	
	1.09173 ± 2 pcm	1.07996 ± 2 pcm	-998 ± 3

MINARET cross sections origin						Δρ A3/T4 (pcm)		Voiding biases	
CODE Geometry	Leakage Model	Homogenization				Full	Voided	pcm	$*
APOLLO3 Cell	**no**	**flux (Φ)**	**1.00830**	**0.98661**	**-833**	**299**	**465**	**165**	**0.5**
	B1-hom	**flux (Φ)**	**1.00990**	**0.98840**	**-812**	**459**	**645**	**186**	**0.5**
	B1-hom	**homCurrent (DB2Φ)**	**1.00594**	**0.98426**	**-831**	**233**	**400**	**168**	**0.5**
	B-het (Bz)	**flux (Φ)**	**1.09876**	**1.08919**	**-800**	**586**	**785**	**199**	**0.6**
	B-het (Bz)	**moments**	**1.09272**	**1.08095**	**-996**	**83**	**85**	**2**	**0.0**
	B-het (Bx)	**moments**	**1.09375**	**1.08300**	**-908**	**169**	**260**	**91**	**0.25**
APOLLO3 Traverse	**no**	**flux (Φ)**	**1.10145**	**1.09089**	**-879**	**808**	**928**	**119**	**0.3**
	no	**moments**	**1.09864**	**1.08626**	**-1037**	**576**	**537**	**-39**	**-0.1**

V. RESULTS ON THE CFV-V1 CORE

The first version of the ASTRID core [Ref. 11] presented Figure 9 combines many geometric features (fertile, plenum sodium, absorbing plate, reduced core height) in order to obtain a negative void reactivity coefficient (-0.5$ at the end of Fuel Cycle). The accuracy of deterministic core calculations to predict the void effect is a key point of safety studies. The new homogenization techniques described and validated on simple configurations are now applied to the fissile assemblies of the larger and strongly heterogeneous CFV core.

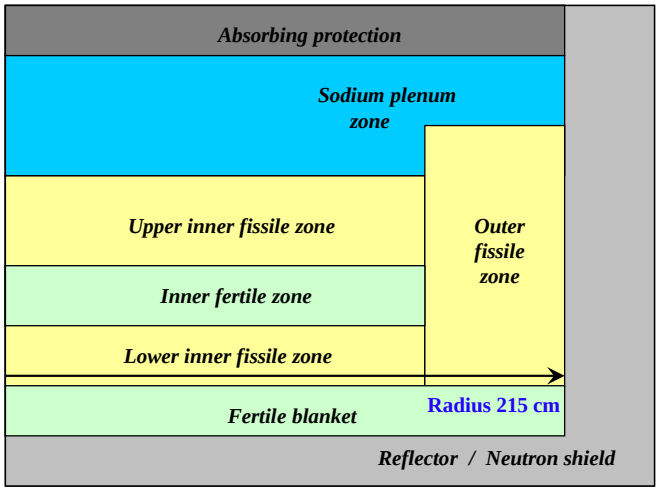

Fig. 9. Section of the ASTRID CFV-V1 core (1500 MWth)

Table III presents the results obtained for the more relevant choice of options. ;

- the APOLLO3® k-effectives using the B-heterogeneous leakage model associated with the flux-moments homogenization are in good agreement with TRIPOLI4® for the three configurations (nominal, voided and rodded) with small overestimations: +140, +240 and +165 pcm respectively (red lines).
- the sodium void effect is significantly improved when compared to the B1-homogeneous model and homogeneous fundamental current homogenization or when no leakage and scalar flux homogenization are used: the bias is reduced: 0.3 $ instead 0.6 $ in these two last cases.
- the control rod worth is also better estimated with an underestimation of only 1.4%, against 2.4 and 3.2 % respectively.
- the worst results regarding the sodium void effect are obtained with the B1-homogeneous model.

Table III. CFV-V1 core - Keff and Sodium void reactivity with different leakage and homogenization techniques for generating the fissile assemblies XS

Configuration	**Leak. Mod. Homog.**	**Nominal**	**Voided**		**Inserted control rods**	
Neutronic parameter		**k_{eff}**	**k_{eff}**	**$\Delta\rho_{Na}$ (pcm)**	**k_{eff}**	**$\Delta\rho_{CR}$ (pcm)**
TRIPOLI-4® (std. dev)		1.05408 ± 2 pcm	1.04533 ± 2 pcm	-794 ± 3	1.02836 ± 2 pcm	-2372 ± 3
APOLLO3® **Δρ AP3-T4 (pcm)**	**no leakage flux**	1.05536 **+150 ± 3**	1.04867 **+305 ± 2**	-604 **+190 ± 3**	1.03040 **+193 ± 2**	-2295 **+77 ± 3**
	B1-homog. homLeakage $DB^2\Phi$	1.05645 **+213 ± 3**	1.04999 **+425 ± 2**	-582 **+212 ± 3**	1.03115 **+263 ± 2**	-2322 **+51 ± 3**
	B-hetereg. (B2z) moments	1.05564 **+140 ± 3**	1.04792 **+236 ± 2**	-698 **+96 ± 3**	1.03020 **+165 ± 2**	-2339 **+33 ± 3**

VI. CONCLUSION

In the present work, homogenization techniques have been compared for the generation of cross sections for SFR transport core calculations with APOLLO3®. The flux moments weighting requires the description of a large pattern, representative of the core configuration, to get a relevant current for condensing transfer cross section for orders superior to zero. This method gives good results but is costly because of the size of the pattern and not very practical when dealing with fissile sub-assemblies with different environments in the core (absorbers, structure materials, reflector …). A more common modelization for these latter is to simulate an infinite lattice and use a leakage model.

A B1-homogeneous and a new B-heterogeneous leakage model are available within the TDT/MOC solver of APOLLO3®. Whatever the leakage model used, the scalar flux homogenization of the transfer cross sections of different orders gives poor results, both on Keff and sodium void effect, when dealing with a simplified SFR core with or without reflector. So, by analogy with the ECCO current weighting method and the flux moments weighting method, a new formula has been implemented to collapse transfer cross sections using reel (even orders) and imaginary (odd orders) parts of the complex periodic flux moments calculated within the B-heterogeneous leakage model.

Under the fundamental mode assumption, this homogenization technique is rigorous if the calculated assembly possesses a central symmetry. If not, the formula can still be applied but we are clearly out of the validity domain of the model and the results have to be checked carefully.

In the case of our simplified SFR core, using the B-heterogeneous leakage model in combination with the flux moments weighting method provides thus homogenized XS that give satisfactory results at the core calculation level when compared with reference Monte Carlo ones (error less than 0.2 $ on sodium void reactivity against 0.5 $ when using the B1 homogeneous leakage model).

This good result is confirmed when this methodology is applied to fissile assemblies of the heterogeneous CFV core selected for the ASTRID design studies: sodium void reactivity is overpredicted by only 0.3 $ and k-effectives by at most 240 pcm. Possible improvements will now come on the way to homogenize subcritical media (fertile assemblies, structure materials, reflectors, etc.). The use of 3D MOC solvers among other advanced techniques is now being investigated.

The B-heterogeneous leakage model and flux moments weighting homogenization is currently being applied to transport calculations of Light Water Reactors but this will be the topic of another paper.

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