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《**Original**》

Nuclear LS-Energy Matrix Elements with the Harmonic Oscillator Shell Model Wave Functions for the Configurations $(l_i l_{i+1} | l_i l_{i+1})$ and Sum Rules*

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調和 單振動子 波動函數를 쓴 原子核의 LS에너지 行列要素와 合法則

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Abstract

The nuclear LS-energy matrix elements have been calculculated with the harmonic oscillator shell model wave functions for the configurations $(l_i\ l_{i+1}|l_i\ l_{i+1})$ where $l_1=1s$, $l_2=1p$, $l_3=1d$, $2s$, $l_4=1f$, $2p$, $l_5=1g$, $2d$, $3s$. The resulting matrix elements are expressed in terms of both Talmi integrals I_l and Slater integrals F^k. In addition to this various sum rules are derived and applied to check the results of the calculations.

요 약

調和 單振動子 波動函數를 써서 原子核의 LS에너지 行列要素를 計算하였다. 範圍는 $l_1=1s$, $l_2=1p$, $l_3=1d$, $2s$, $l_4=1f$, $2p$, $l_5=1g$, $2d$, $3s$라 할 때 $(l_i\ l_{i+1}|l_i\ l_{i+1})$의 配置에 對한 것이었다. 計算結果는 Talmi積分 I_l과 Slater 積分 F^k를 써서 表示하였다. 또 여러가지 合法則을 誘導하고 이를써서 計算의 結果를 檢算하였다.

1. LS-Matrix Elements for Central Field

It is well known that an arbitrary central field $J(r_{12})$ can be expanded in a series of Legendre polynomials[1],

$$J(r_{12})=\sum_k J_k(r_1,r_2)\ P_k(\cos\ \theta_{12}), \tag{1.1}$$

where

$$J_k(r_1,r_2)=\frac{2k+1}{4\pi}\int J(r_{12})P_k(\cos\ \theta_{12})\sin\ \theta_{12}\ d\theta_{12}\ d\phi_{12}. \tag{1.2}$$

Using the addition theorem for spherical harm-

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onics[2)]

$$P_k(\cos\theta_{12})=\frac{4\pi}{2k+1}\sum_m Y_{km}^*(\theta_1,\phi_1)\,Y_{km}(\theta_2,\phi_2), \tag{1.3}$$

the angular function P_k can further be expanded in terms of spherical harmonic tensors[3)]

$$P_k(\cos\theta_{12})=\sum_m(-1)^m\, C_m^k(1)C_{-m}^k(2) =C_{(1)}^k\cdot C_{(2)}^k, \tag{1.4}$$

where we have employed the spherical harmonic tensor notation of Racah[4)] for Y_{km} given by

$$C_{(1)}^k=\left\{C_m^k(1)\right\}=\left\{\sqrt{\frac{4\pi}{2k+1}}\,Y_{km}(\theta_1,\phi_1)\right\}, \tag{1.5}$$

and

$$C_{(2)}^k=\left\{C_m^k(2)\right\}=\left\{\sqrt{\frac{4\pi}{2k+1}}\,Y_{km}(\theta_2,\phi_2)\right\}.$$

Hence the LS-matrix elements of $J(r_{12})$ between two states, $|n_1'l_1's_1', n_2'l_2's_2'; L'S'\rangle$ and $|n_1l_1s_1, n_2l_2s_2; LS\rangle$ can be written as

$$\begin{aligned}\langle J\rangle_{LS}=&\langle n_1'l_1's_1',\ n_2'l_2's_2';\ L'S'|J(r_{12})|n_1l_1s_1, n_2l_2s_2;LS\rangle\\ =&\sum_k F^k(n_1'l_1', n_2'l_2'; n_1l_1, n_2l_2)\langle(l_1', l_2')L', (s_1', s_2')S;J|C_{(1)}^k\cdot C_{(2)}^k|(l_1, l_2)L, (s_1, s_2)S;J\rangle\end{aligned} \tag{1.6}$$

The radial integral F^k (Slater integral) is given by

$$\begin{aligned}F^k&(n_1'l_1', n_2'l'_2; n_1l_1, n_2l_2)\\ &=\iint r_1^2r_2^2dr_1dr_2R_{n_1'l_1'}(r_1)R_{n_2'l_2'}(r_2)J_k(r_1,r_2) R_{n_1l_1}(r_1)R_{n_2l_2}(r_2)\end{aligned} \tag{1.7}$$

where $R_{nl}(r)$ is the radial wave function for the state $\psi_{nlm}(r)$.

2. Angular Integral A_L^k

Using the Wigner-Eckart theorem[4)]

$$\langle\alpha jm|T_q^k|\alpha'j'm'\rangle =(-1)^{2k}\frac{1}{\sqrt{2j+1}}\langle\alpha j\|T^k\|\alpha'j'\rangle C_{m'qm}^{j'kj} \tag{2.1}$$

and

$$\begin{aligned}\langle(j_1j_2)jm|T_{(1)}^kU_{(2)}^k|(j_1'j_2')j'm'\rangle =&\delta_{jj'}\delta_{mm'}(-1)^{j_1+j_2'-j}\langle j_1\|T^k\|j_1'\rangle\langle j_2\|U^k\|j_2'\rangle\\ &W(j_1j_2j_1'j_2':jk),\end{aligned} \tag{2.2}$$

where $C_{m'qm}^{j'kj}$ and $W(j_1j_2j_1':jk)$ are the Clebsch-Gordan coefficient and Racah coefficient respectively, the angular integral of the LS-matrix elements can be written as

$$\begin{aligned}\langle(l_1'l_2')L',\ (s_1's_2')S';J|C_{(1)}^k\cdot C_{(2)}^k|(l_1l_2)L, (s_1s_2)S;J\rangle=&(-1)^{l_1'+l_1-L}\delta_{SS'}\delta_{LL'}D_{l_1'l_1k}D_{l_2l_2k}\\ &W(l_1'l_2'l_1l_2:Lk),\end{aligned} \tag{2.3}$$

where

$$D_{l_1l_2k}=\sqrt{\frac{(2l_1+1)(2l_2+1)}{2k+1}}\,C_{0\ 0\ 0}^{l_1\ l_2\ k}. \tag{2.4}$$

Hence the LS-matrix elements can now be written

$$\begin{aligned}LS\text{-Matrix Element}=&\langle n_1'l_1's_1', n_2'l_2's_2'; L'S'|J(r_{12})|n_1l_1s_1, n_2l_2s_2;LS\rangle\\ =&\sum_k A_L^k(l_1'l_2'l_1l_2)F^k(n_1'l_1', n_2'l_2'; n_1l_1, n_2l_2),\end{aligned} \tag{2.5}$$

where

$$A_L^k(l_1'l_2'l_1l_2)=(-1)^{l_1'+l_1-L}D_{l_1'l_1k} D_{l_2'l_2k}W(l_1'l_2'l_1l_2:Lk). \tag{2.6}$$

3. Radial Integral F^k

In nuclear spectroscopy, it is in most cases impossible to evaluate the integral F^k from measured energy levels, as there are usually only very few levels measured and classified. Therefore if one is to use the Slater method, the F^k must be mathematically evaluated. Even for the central forces, however, we do not know exact form of the potential. We should therefore calculate the energy levels for different forms of the potential. However, even in simple cases such as the Yukawa potential, the calculation of the F^k is so complicated that Slater method is of little practical value.

To overcome these difficulties we make use of the fact that the interaction energy depends only on the relative coordinate $\boldsymbol{r}=\boldsymbol{r}_2-\boldsymbol{r}_1=\boldsymbol{r}_{12}$ and our procedure is to express also the wave functions as functions of $\boldsymbol{r}$ and the coordinate of center of mass $\boldsymbol{R}=(\boldsymbol{r}_1+\boldsymbol{r}_2)/2$. This transformation enables us, when calculating matrix element, to integrate immediately with respect to R, and we are left with a single integration of r which can usually be carried out without difficulty.

This coordinate transformation is always possible, but the functions of $\boldsymbol{r}$ and $\boldsymbol{R}$ generally turns out to be very complicated. The success of this procedure depends on the proper choice of the wave functions. The best choice would be that one which allows us to expand the wave function $\phi_{n_1l_1}(\boldsymbol{r}_1)\ \phi_{n_2l_2}(\boldsymbol{r}_2)$ in a finite sum of products of functions which depend on $\boldsymbol{r}$ and $\boldsymbol{R}$ respectively.

Talmi[5] had shown that the only wave functions satisfying these conditions are harmonic oscillator wave functions[6]:

$$R_{nl}(r)=N_{nl}e^{-(\nu_2)r^2}r^{l+1}v_{nl}(r) \tag{3.1}$$

where N_{nl} is the normalization factor

$$N_{nl}^2(\nu)=\frac{2^{l-n+3}(2l+2n-1)!!\nu^{l+\frac{3}{2}}}{\sqrt{\pi}\,(n-1)![(2l+1)!!]^2}, \tag{3.2}$$

$v_{nl}(r)$ is an associated Laguerre polynomials

$$v_{nl}(r)=L_{n+l-\frac{1}{2}}^{l+\frac{1}{2}}(\nu r^2)=\sum_{k=0}^{n-1}(-1)^k\,2^k\binom{n}{k}\frac{(2l+1)!!}{(2l+2k+1)!!}(\nu r^2)^k, \tag{3.3}$$

ν is given by $\nu=\omega m/\hbar$ and m and ω appear in the potential

$$V(r)=\frac{1}{2}m\omega^2r^2=\hbar\omega\nu r^2. \tag{3.4}$$

The first few functions which we have used in the present calculation are

$$R_{1l}(r)=N_{1l}e^{-\nu r^2/2}r^{l+1}$$

$$R_{2l}(r)=N_{2l}e^{-\nu r^2/2}r^{l+1}\ 1-\frac{2\nu}{2l+3}r^2$$

$$R_{3l}(r)=N_{3l}e^{-\nu r^2/2}r^{l+1}\ 1-\frac{4\nu r^2}{2l+3}+\frac{4\nu^2r^4}{(2l+3)(2l+5)}$$

$$R_{4l}(r)=N_{4l}e^{-\nu r^2/2}r^{l+1}\ 1-\frac{6\nu r^2}{2l+3}+\frac{12\nu^2r^4}{(2l+3)(2l+5)}-\frac{8\nu^3r^6}{(2l+3)(2l+5)(2l+7)} \tag{3.5}$$

and

$$N_{1l}^2=\frac{\nu^{l+3/2}2^{l+1}}{\sqrt{\pi}\,(2l+1)!!}$$

$$N_{2l}=\frac{\nu^{l+3/2}2^{l+1}(2l+3)}{\sqrt{\pi}\,(2l+1)!!}$$

$$N_{3l}=\frac{\nu^{l+3/2}2^{l}(2l+3)(2l+5)}{\sqrt{\pi}\,(2l+1)!!}$$

$$N_{4l}=\frac{\nu^{l+3/2}2^{l-2}(2l+3)(2l+5)(2l+7)}{\sqrt{\pi}\,3(2l+1)!!}.$$

In order to perform the transformation from the r_1, r_2, θ_{12} to r, R, α (angle between $\boldsymbol{r}$ and $\boldsymbol{R}$), one needs the following relations

$$r_1^2+r_2^2=\frac{4R^2+r^2}{2}$$

$$r_1^2r_2^2=\left(\frac{4R^2r^2}{4}\right)^2+R^2r^2\sin^2\alpha$$

$$r_1^kr_2^kP_k(\cos\theta_{12})=\sum_{n=0}^{m}C_{2n}\left(\frac{4R^2r^2}{4}\right)^{2n}\left[\left(\frac{4R^2-r^2}{4}\right)^2+R^2r^2\sin^2\alpha\right]^{m-n},\quad k=2m$$

$$r_1^2dr_1r_2^2dr_2d\cos\theta_{12}=R^2dRr^2drd\cos\alpha, \tag{3.6}$$

where the C_{2n} are defined by $P_k(\cos\theta_{12})=\sum_{n=0}^{\infty}C_{2n}\cos^{2n}\theta_{12}$. With the help of these relations the expression for F^k:

$$F^k(n_1'l_1',\ n_2'l_2';\ n_1l_1,\ n_2l_2)=\frac{2k+1}{2}\int_{-1}^{1}\int_0^\infty\int_0^\infty J(r)\frac{R_{n_1'l_1'}(r_1)R_{n_1l_1}(r_1)}{r_1^2}\frac{R_{n_2'l_2'}(r_2)R_{n_2l_2}(r_2)}{r_2^2}P_k(\cos\theta_{12})r_1^2r_2^2dr_1dr_2\,d\cos\theta_{12} \tag{3.7}$$

becomes

$$F^k(n_1'l_1',n_1'l_1';\ n_1,l_1,n_2l_2)=\frac{2k+1}{2}\int_{-1}^{1}\int_0^\infty\int_0^\infty J(r)\Phi_k(R,r,\alpha)R^2dRr^2dr\,d\cos\alpha, \tag{3.8}$$

where Φ_k is the transform of $R_{n_1'l_1'}(r_1)R_{n_1l_1}(r_1)R_{n_2'l_2'}(r_2)R_{n_2l_2}(r_2)P_k(\cos\theta_{12})/r_1^2r_2^2$, and is a polynomial in r,R and $\sin^2\alpha$ multiplied by exp $[-(4R^2+r^2)/2]$:

$$\Phi_k(R,r,\alpha)=N_{n_1'l_1'}N_{n_2'l_2'}N_{n_1l_1}N_{n_2l_2}e^{-(4R^2+r^2)/2}r_1^{l_1+l_1'}r_2^{l_2+l_2'}\cdot v_{n_1'l_1'}(r_1)v_{n_2'l_2'}(r_2)v_{n_1l_1}(r_1)v_{n_2l_2}(r_2)P_k(\cos\theta_{12}). \tag{3.9}$$

4. Talmi Integral I_l

At this stage it is convenient to introduce Talmi integral[5] defined by

$$I_l=N_{0l}^2\left(\frac{\nu}{2}\right)\int_0^\infty J(r)e^{\nu r^2/2}r^{2l+2}dr$$

$$=\sqrt{\frac{2}{\pi}}\frac{\nu^{l+3/2}}{(2l+1)!!}\int_0^\infty J(r)e^{-\nu r^2/2}r^{2l+2}dr, \qquad (4.1)$$

which alone contain the yet unspecified interaction potential $J(r)$. With a little tedious calculations it can now be shown that the Slater integral F^k takes the following form[7]:

$$F^k(n_1'l_1', n_2'l_2';\ n_1, l_1, n_2l_2)$$
$$=B\sum_{m,n} D_{mn}(2k+1)f_k(m,n)$$
$$=\frac{2k+1}{2}\pi B\sum_{n,n} D_{mn}\sum_{l=0}^{(m+n)/2} t_k{}^l(m,n)I_l, \qquad (4.2)$$

where

$$B=\nu^{-\left(3+\frac{l_1'+l_1+l_2'+l_2}{2}\right)}N_{n_1}{}'_{l_1}{}'(\nu)N_{n_2}{}'_{l_2}{}'(\nu)N_{n_1l_1}(\nu)$$
$$N_{n_2l_2}(\nu),$$
$$\sum_{m,n} D_{mn}x_1{}^m x_2{}^n = x_1{}^{l_1'l_1}x_2{}^{l_2'+l_2}v_{n_1}{}'_{l_2}{}'(x_1/\sqrt{\nu})$$
$$v_{n_2}{}'_{l_2}{}'(x_2/\sqrt{\nu})v_{n_1l_1}(x_1/\sqrt{\nu})v_{n_2l_2}(x_2/\sqrt{\nu}),$$

and

$$f_k(m,n)=f_k(n,m)=\frac{1}{2k+1}\int_0^\infty\int_0^\infty x_1{}^m x_2{}^n e^{-(x_1{}^2+x_2{}^2)}$$
$$J_k\left(\frac{x_1}{\sqrt{\nu}}, \frac{x_2}{\sqrt{\nu}}\right)x_1{}^2x_2{}^2dx_1dx_2$$
$$=\frac{\pi}{2}\sum_{l=0}^{(m+n)/2} t_k{}^l(m,n)I_l. \qquad (4.3)$$

In the last equation, the transformation from r_1 and r_2 to r and R has been carried out. The table for $t_k{}^l(m,n)$ in slightly different form has been computed by True and Ford[8].

5. Procedures for the Calculation

Following configurations have been considered in the present calculations:

$(l_il_{i+1}|l_il_{i+1})$

where

$l_1=1s,\ l_2=1p,\ l_3=1d, 2s,\ l_4=1f, 2p,$ and $l_5=1g, 2d, 3s.$

Exchange integrals are also calculated in addition to direct integrals.

The LS-matrix elements for the above configurations are calculated in three steps:

(1) **Computation of**

$$A_L{}^k(l_1'l_2'l_1l_2)=(-1)^{l_1'+l_1-L}D_{l_1}{}'_{l_1k}D_{l_2}{}'_{l_2k}$$
$$W(l_1'l_2'l_1l_2:Lk)$$

The numerical values for Clebsch-Gordan and Racah coefficients are taken from Rotenberg, Bivins, Metropolis and Wooten: The 3-j and 6-j Symbols, the Technical Press, MIT. (1959).

(2) **Calculation of F^k**

Using equations (4.1) through (4.3), F^k has been calculated and expressed in terms of Talmi integral I_l:

$$F^k=(\text{Common Factor})\sum_l f_l{}^kI_l. \qquad (5.1)$$

(3) **LS-Matrix Element**

The calculated coefficients of F_k and $A_L{}^k$ are substituted into

$$\langle J\rangle_{LS}=LS\text{-Matrix Element}=\sum_k A_L{}^kF^k \quad (5.2)$$

to yield the result in terms of I_l:

$$\langle J\rangle_{LS}=LS\text{-Energy Matrix Element}$$
$$=(\text{Common Factor})\sum_l a_l{}^LI_l. \qquad (5.3)$$

6. Sum Rules and Checking

In order to check the results various sum rules are derived and applied.

(1) **Checking of the calculation of $A_L{}^k$**

Using the definitions of Clebsch-Gordan coefficient and Racah coefficient we find the following relations between the two coefficients:[9]

$$\sqrt{(2e+1)(2f+1)}\,W(abcd:ef)$$
$$=\sum_{\alpha,\beta} C^{a\ b\ e}_{\alpha\ \beta}\ C^{e\ \ d\ \ c}_{\alpha+\beta\ \gamma-\alpha-\beta}\ C^{b\ \ d\ \ f}_{\beta\ \gamma-\alpha-\beta}\ C^{a\ \ f\ \ c}_{\alpha\ \gamma-\alpha}$$

$$\sqrt{(2e+1)(2f+1)}\ C^{a\ \ f\ \ c}_{\alpha\ \gamma-\alpha}W(abcd:ef)$$
$$=\sum_\beta C^{a\ b\ e}_{\alpha\ \beta}\ C^{e\ \ d\ \ c}_{\alpha+\beta\ \gamma-\alpha-\beta}\ C^{b\ \ d\ \ f}_{\beta\ \gamma-\alpha-\beta}\ C^{a\ f\ c}_{\alpha\ \beta}\ C^{e\ \ d\ c}_{\alpha+\beta\ \delta}$$
$$=\sum_f C^{b\ d\ f}_{\beta\ \delta}\ C^{a\ \ f\ \ c}_{\alpha\ \beta+\delta}\sqrt{(2e+1)(2f+1)}$$
$$W(abcd:ef). \qquad (6.1)$$

If we use these relations it is easy to derive the following sum rules for D_{abc} defined through equation (2.4):

$$D_{abe}D_{cde}=(-1)^{a+c+e}\sum(2k+1)D_{ack}D_{bdk}\, W(abcd:ef). \qquad (6.2)$$

For short range limit (SRL) for which the interaction potential takes the form

$$J(r)=J_0\delta(r),$$

Talmi integral becomes

$$I_l\left(\frac{\nu}{2}\right)=\sqrt{\frac{2}{\pi}}\frac{\nu^{l+3/2}}{(2l+1)!!}J_0\frac{1}{4\pi}\int\delta(r)e^{-\frac{\nu}{2}r^2}r^{2l}dr$$
$$=\delta_{0l}I_0, \qquad (6.3)$$

while F^k becomes

$$F^k=\frac{\pi}{2}B\sum^{mn}D_{mn}(2k+1)\sum_{l=0}^{m+n/2}t_k^l(mn)I_l^{SRL}$$
$$=\frac{\pi}{2}B\sum_{mn}D_{mn}(2k+1)\sum t_k^0(m,n)I_0\delta_{l0},$$

namely

$$F^k=(2k+1)F^0, \qquad (6.4)$$

where

$$F^0=\frac{\pi}{2}B\sum_{m,n}D_{mn}t_k^0(mn)I_0\delta_{l0}. \qquad (6.5)$$

Hence for the short range limit the LS-Energy matrix element takes the following form:

$$\langle J\rangle_{LS}=\langle n_1'l',n_2'l_2':LS|J(r)|n_1l_1,n_2l_2:LS\rangle$$
$$=\sum A_k^LF^k$$
$$=(-1)^{l_1'+l_1-L}D_{e_1'l_1l}D_{l_2'l_2'k}W(l_1'l_2'l_1l_2:Lk)$$
$$F^k(n_2'l_1'n_2'l_2'n_1l_1n_2l_2)$$
$$=F^0\sum A_k^L(2k+1)$$
$$=F^0\sum_k(-1)^{l_1'+l_1-L}D_{l2'l_1k}D_{l_2'l_2k}$$
$$W(l_1'l_2'l'l_2:Lk)(2k+1)$$
$$=D_{l_1'l_2'L}D_{l_1l_2L}F^0,$$

where use of the eq. (6.4) and the sum rule (6.2) has been made. Rewriting the above equation we finally obtain the required sum rule for checking the correctness of A_k^L:

$$\sum_k(2k+1)A_k^L=D_{l_1'l_2'L}D_{l_1l_2L}. \qquad (6.6)$$

For example as shown in ⟨Table 3⟩ the values of A_3^k for the configuration $(1f1d)(1f1d)$ are $A_3^0=1$, $A_3^2=-\frac{11}{105}$, $A_3^4=\frac{2}{21}$ while $D_{233}=-\frac{2}{\sqrt{3}}$, $D_{233}=-\frac{2}{\sqrt{3}}$, which together clearly satisfies the relation (6.6).

In addition to this eq. (6.5) suggests that for short range limit F^0 has single term containing I_0:

$$F^0=f_0^0I_0. \qquad (6.7)$$

Therefore it is expected that for the same configuration the coefficient of I_0, i.e., f_0^k for F^k with different k values must be proportional to $(2k+1)$:

$$\frac{f_0^k}{f_0^0}=(2k+1). \qquad (6.8)$$

For example the coefficients f_0^k for the configuration $(1f1d)$ $(1f1d)$ (see ⟨Table 4⟩ are

$$f_0^0=\frac{1}{160}\times33=\frac{33}{160}[2\times0+1],$$
$$f_0^2=\frac{3}{32}\times11=\frac{33}{160}[2\times2+1],$$
$$f_0^4=\frac{297}{160}\times1=\frac{33}{160}[2\times4+1],$$

and show the validity of the calculation.

(2) Sum Rules for F^k and $\langle J\rangle_{LS}$

For long range limit (=LRL) where the interaction potential takes the form

$$J(r)=J_0=\text{const}, \qquad (6.9)$$

Talmi integral takes the value:

$$I_l\left(\frac{\nu}{2}\right)=\sqrt{\frac{2}{\pi}}\frac{\nu^{l+3/2}}{(2l+1)!!}J_0\int_0^\infty e^{-\frac{\nu}{2}r^2}r^{2l+2}dr$$
$$=\frac{J_02^{l+2}}{\sqrt{\pi}(2l+1)!!}\int_0^\infty e^{-x^2}x^{2l+2}dx$$
$$=J_0, \qquad (6.10)$$

while the expansion coefficient $J_k(r_1,r_2)$ (eq. (1.2)) of $J(r)$ (eq. (1.1)) becomes

$$J_k(r_1,r_2)=\frac{2k+1}{4\pi}\int J(r)P_k(\cos\theta_{12})d\cos\theta_{12}\, d\phi_{12}$$
$$=\frac{2k+1}{4\pi}2\pi J_0\int_{-1}^1P_k(x)dx$$
$$=V_0\delta_{0k}. \qquad (6.11)$$

Therefore the Slater integral F^k and LS-Energy matrix element, $\langle J\rangle_{LS}$ become

$$F^k(n_1'l_1'n_2'l_2';n_1l_1n_2l_2)$$
$$=J_0\delta_{0k}\delta_{n_1'n_1}\delta_{n_2'n_2}\delta_{l_1'l_1}\delta_{l_2'l_2}$$
$$=J_0\sum f_l^k\delta_{0k}, \qquad (6.12)$$
$$\langle J\rangle_{LS}=\langle n_1'l_1',n_2'l_2':LS|J|n_1l_1,n_2l_2:LS\rangle$$
$$=J_0\delta_{n_1'n_1}\delta_{n_2'n_2}\delta_{l_1'l_1}\delta_{l_2'l_2}$$
$$=J_0\sum a_l^L, \qquad (6.13)$$

which is the required sum rules, mamely

$$\sum_l f_k{}^l = \begin{cases} 1 \text{ if configuration is diagonal and } k=0 \\ 0 \text{ otherwise} \end{cases} \tag{6.14}$$

$$\sum_l a_l{}^L = \begin{cases} 1 \text{ if configuration is diagonal} \\ 0 \text{ if configuration is off-diagonal.} \end{cases} \tag{6.15}$$

For example the coefficients $f_l{}^k$ of F^k and $a_l{}^L$ of $\langle J\rangle_{LS}$ for the configurations $(1p1d)$ $(1p1d)$ and $(1p1d)$ and $(1p1d)(2s1p)$ are:

Configuration	k	common Factor	f_0	f_1	f_2	f_3	common Factor
$(1p1d)$ $(1p1d)$	0	1/24	7	5	5	7	1
$(1p1d)$ $(1p1d)$	2	35/24	1	−1	−1	1	0
$(1d1p)$ $(2s\,1p)$	1	$\sqrt{10}/16$	−1	1	−7	7	0

Configuration	L	common Factor	a_0	a_1	a_2	a_3	common Factor
$(1p1d)$ $(1p1d)$	1	1/12	7	−1	−1	7	1
	2	1/2	0	1	1	0	1
	3	1/8	3	1	1	3	1
	1	5/24	1	−1	7	−7	0

respectively and show the validity of the eq. (6.14) and eq (6.15).

7. Results

The result are listed in five tables at the end of this paper, They are:

TABLE of $D_{l_1 l_2 k}$

TABLE of $t_k{}^l(m,n)$

TABLE of $A_L{}^k(l_1'l_2'l_1l_2)=(-1)^{l_1'+l_1-L}D_{l_1'l_1k}D_{l_2'l_2k}W(l_1'l_2'l_1l_2:Lk)$

TABLE of Slater Integral $F^k({}_{n_1}l_1', n_2'l_2'; n_1l_1, n_2l_2)=(\text{common factor})\sum f_l{}^k I_l$

TABLE of LS-Energy Matrix Elements:

$$\langle J\rangle_{LS}=\langle n_1'l_1', n_2'l_2';\ LS|J(r_{12})|n_1l_1, n_2l_2; LS\rangle_J = (\text{Common Factor})\sum a_l{}^L I_l.$$

8. Conclusions

The Slater Integral F^k and nuclear LS-matrix elements so far has been computed by various authors[5,10] only for several physically interesting configurations involving lower excited states. But excitation becomes higher more and more complicated configuration are needed for consideration. In this paper we have computed F and LS-matrix element for all the configurations of the form

$$\langle l_i\ l_{i+1}|l_i\ l_{i+1}\rangle,$$

where

⟨Table 1⟩ Table of $D_{l_1 l_2 k}$

l_1	l_2 \ k	0	1	2	3	4	5	6	7	8
0	0	1								
	1		1							
	2			1						
	3				1					
	4					1				
1	1	$-\sqrt{3}$		$\sqrt{6/5}$						
	2		$-\sqrt{2}$		$3/\sqrt{7}$					
	3			$-3/\sqrt{5}$		$\sqrt{4/3}$				
	4				$-2\sqrt{3/7}$		$\sqrt{15/11}$			
2	2	$\sqrt{5}$		$-\sqrt{10/7}$		$\sqrt{10/7}$				
	3		$\sqrt{3}$		$-2/\sqrt{3}$		$5\sqrt{2/33}$			
	4			$\sqrt{18/7}$		$-10/\sqrt{77}$		$15/\sqrt{143}$		
3	3	$-\sqrt{7}$		$2\sqrt{7/15}$		$-2\sqrt{7/22}$		$10\sqrt{7/429}$		
	4		-2		$3\sqrt{2/11}$		$-6\sqrt{5/143}$		$7\sqrt{5/143}$	
4	4	3		$-6\sqrt{5/77}$		$27\sqrt{2}/\sqrt{1001}$		$-6\sqrt{5/143}$		$21\sqrt{10/2431}$

$l_1=1s,\ l_2=1p,\ l_3=1d, 2s,\ l_4=1f, 2p,$
$l_5=1g, 2d, 3s.$
to complement previous authors.

In addition as shown in section 6, we found various sum rules and these in turn are used to check the varidity of conputation. These rules are satisfied as shown in section 6.

⟨Table 2⟩ Tables of $t^l_k(m,n)$

m	n	k	Lommon Factor \ l	0	1	2	3	4	5	6	7	8
0	0	0	$1/2^3$	1								
2	0	0	$3/2^5$	1	1							
2	2	0	$3/2^7$	5	2	5						
		2	$3/2^7$	5	−10	5						
4	0	0	$3/2^7$	5	10	5						
4	2	0	$15/2^9$	7	5	5	7					
		2	$15/2^9$	7	−7	−7	7					
4	4	0	$15/2^{11}$	63	28	58	28	63				
		2	$105/2^{11}$	9	−8	−2	−8	9				
		4	$945/2^{11}$	1	−4	6	−4	1				
6	0	0	$105/2^9$	1	3	3	1					
6	2	0	$105/2^{11}$	9	12	6	12	9				
		2	$945/2^{11}$	1	0	−2	0	1				
6	4	0	$105/2^{13}$	99	63	78	28	63	99			
		2	$945/2^{13}$	11	−5	−6	−6	−5	11			
		4	$10,395/2^{13}$	1	−3	2	2	−3	1			
6	6	0	$105/2^{15}$	1,287	594	1,161	636	1,161	594	1,287		
		2	$945/2^{15}$	143	−66	−15	−124	−15	−66	143		
		4	$10,395/2^{15}$	13	−34	19	4	19	−34	13		
		6	$135,135/2^{15}$	1	−6	15	−20	15	−6	1		
8	0	0	$945/2^{11}$	1	4	6	4	1				
8	2	0	$945/2^{13}$	11	23	14	14	23	11			
		2	$10,395/2^{13}$	1	1	−2	−2	1	1			
8	4	0	$945/2^{15}$	143	154	97	172	97	154	143		
		2	$10,395/2^{15}$	13	2	−13	−4	−13	2	13		
		4	$135,135/2^{15}$	1	−2	−1	4	−1	−2	1		
8	6	0	$945/2^{17}$	2,145	1,287	1,749	1,539	1,530	1,749	1,287	2,145	
		2	$10,395/2^{17}$	195	−39	−45	−111	−111	−45	−39	195	
		4	$135,135/2^{17}$	15	−31	3	13	13	3	−31	15	
		6	$2,027,025/2^{17}$	1	−5	9	−5	−5	9	−5	1	
8	8	0	$945/2^{19}$	36,465	17,160	32,604	18,744	31,974	18,744	32,604	17,160	36,465
		2	$10,395/2^{19}$	3,315	−780	156	−2,292	−798	−2,292	156	−780	3,315
		4	$135,135/2^{19}$	255	−480	68	−32	378	−32	68	−480	255
		6	$2,027,025/2^{19}$	17	−76	116	−52	−10	−52	116	−76	17
		8	$34,459,425/2^{19}$	1	−8	28	−56	70	−56	28	−8	1
1	1	1	$3/2^5$	1	−1							
3	1	1	$15/2^7$	1	0	−1						
3	3	1	$15/2^9$	7	−1	1	−7					
		3	$105/2^9$	1	−3	3	−1					
5	1	1	$105/2^9$	1	1	−1	−1					
5	3	1	$105/2^{11}$	9	2	0	−2	−9				

m	n	k	Lommon Factor \ l	0	1	2	3	4	5	6	7	8
5	3	3	$945/2^{11}$	1	−2	0	2	−1				
5	5	1	$105/2^{13}$	99	9	38	−38	−9	−99			
		3	$945/2^{13}$	11	−19	2	−2	19	−11			
		5	$10,395/2^{13}$	1	−5	10	−10	5	−1			
7	1	1	$945/2^{11}$	1	2	0	−2	−1				
7	3	1	$945/2^{13}$	11	9	−2	2	−9	−11			
		3	$10,395/2^{13}$	1	−1	−2	2	1	1			
7	5	1	$645/2^{15}$	143	44	43	0	−43	−44	−143		
		3	$10,395/2^{15}$	13	−16	−7	0	7	16	−13		
		5	$135,135/2^{15}$	1	4	5	0	−5	4	−1		
7	7	1	$945/2^{17}$	2,145	429	1,089	−267	267	−1,089	−429	−2,145	
		3	$10,395/2^{17}$	195	−221	−41	−137	137	41	221	−195	
		5	$135,135/2^{17}$	15	−53	55	−5	5	−55	53	−15	
		7	$2,027,025/2^{17}$	1	−7	21	−35	35	−21	7	−1	

⟨Table 3⟩ **Tables of A_L^k $(l_1'l_2'l_1l_2)$** (1/7)

Configuration	L \ k	0	1	2	3	4	5	6	7	8	$D_{l_1'l_2'L}$	$D_{l_1l_2L}$
$(1s1p)$ $(1s1p)$	1	1									1	1
$(1p1s)$	1		$\frac{1}{3}$								1	1
$(1p1d)$ $(1p1d)$	1	1		$\frac{1}{5}$							$-\sqrt{2}$	$-\sqrt{2}$
	2	1		$-\frac{1}{5}$							0	0
	3	1		$\frac{2}{35}$							$\frac{3}{\sqrt{7}}$	$\frac{3}{\sqrt{7}}$
$(1d1p)$	1		$\frac{1}{15}$		$\frac{9}{35}$						$-\sqrt{2}$	$-\sqrt{2}$
	2		$\frac{1}{5}$		$-\frac{3}{35}$						0	0
	3		$\frac{2}{5}$		$\frac{3}{245}$						$\frac{3}{\sqrt{7}}$	$\frac{3}{\sqrt{7}}$
$(1p2s)$	1			$-\frac{\sqrt{2}}{5}$							$-\sqrt{2}$	1
$(2s1p)$	1		$-\frac{\sqrt{2}}{3}$								$-\sqrt{2}$	1
$(1p2s)$ $(1p2s)$	1	1									1	1
$(2s1p)$	1		$\frac{1}{3}$								1	1
$(1d1f)$ $(1d1f)$	1	1		$\frac{8}{35}$		$\frac{2}{21}$					$\sqrt{3}$	$\sqrt{3}$
	2	1		$\frac{2}{35}$		$-\frac{1}{7}$					0	0
	3	1		$-\frac{11}{105}$		$\frac{2}{21}$					$-\frac{2}{\sqrt{3}}$	$-\frac{2}{\sqrt{3}}$
	4	1		$-\frac{1}{7}$		$-\frac{2}{63}$					0	0
	5	1		$\frac{2}{21}$		$\frac{1}{231}$					$\frac{5\sqrt{2}}{\sqrt{33}}$	$\frac{5\sqrt{2}}{\sqrt{33}}$
$(1f1d)$	1			$\frac{1}{35}$		$\frac{8}{105}$		$\frac{50}{231}$			$\sqrt{3}$	$\sqrt{3}$

Configuration	L \ k	0	1	2	3	4	5	6	7	8	$D_{l_1'l_2'L}$	$D_{l_1l_2L}$
	2		$\frac{3}{35}$		$\frac{2}{15}$		$-\frac{25}{231}$				0	0
	3		$\frac{6}{35}$		$\frac{19}{315}$		$\frac{25}{693}$				$-\frac{2}{\sqrt{3}}$	$-\frac{2}{\sqrt{3}}$
	4		$\frac{6}{25}$		$-\frac{1}{9}$		$-\frac{5}{693}$				0	0
	5		$\frac{3}{7}$		$\frac{2}{63}$		$\frac{5}{7623}$				$\frac{5\sqrt{2}}{\sqrt{33}}$	$\frac{5\sqrt{2}}{\sqrt{33}}$
$(1d1f)$ $(1d2p)$	1			$-\frac{\sqrt{6}}{35}$		$-\frac{2\sqrt{6}}{21}$					$\sqrt{3}$	$-\sqrt{2}$
	2			$-\frac{6}{35}$		$\frac{2}{21}$					0	0
	3			$-\frac{12\sqrt{21}}{245}$		$-\frac{2\sqrt{21}}{441}$					$-\frac{2}{\sqrt{3}}$	$\frac{3}{\sqrt{7}}$
$(2p1d)$	1		$-\frac{\sqrt{6}}{5}$		$-\frac{2\sqrt{6}}{35}$						$\sqrt{3}$	$-\sqrt{2}$
	2		$-\frac{2}{5}$		$\frac{6}{35}$						0	0
	3		$-\frac{2\sqrt{21}}{35}$		$-\frac{4\sqrt{21}}{245}$						$-\frac{2}{\sqrt{3}}$	$\frac{3}{\sqrt{7}}$
$(2s1f)$	3			$-\frac{2\sqrt{3}}{15}$							$-\frac{2}{\sqrt{3}}$	1
$(1f2s)$	3				$-\frac{2\sqrt{3}}{21}$						$-\frac{2}{\sqrt{3}}$	1
$(2s2p)$	1			$\frac{\sqrt{3}}{5}$							$\sqrt{3}$	1
$(2p2s)$	1				$\frac{\sqrt{3}}{7}$						$\sqrt{3}$	1
$(1d2p)$ $(1d2p)$	1	1		$\frac{1}{5}$							$-\sqrt{2}$	$-\sqrt{2}$
	2	1		$-\frac{1}{5}$							0	0
	3	1		$\frac{2}{35}$							$\frac{3}{\sqrt{7}}$	$\frac{3}{\sqrt{7}}$
$(2p1d)$	1		$\frac{1}{15}$		$\frac{9}{35}$						$-\sqrt{2}$	$-\sqrt{2}$
	2		$\frac{1}{5}$		$-\frac{3}{35}$						0	0
	3		$\frac{2}{5}$		$\frac{3}{245}$						$\frac{3}{\sqrt{7}}$	$\frac{3}{\sqrt{7}}$
$(2s1f)$	3			$\frac{3\sqrt{7}}{35}$							$\frac{3}{\sqrt{7}}$	1
$(1f2s)$	3		$\frac{\sqrt{7}}{7}$								$\frac{3}{\sqrt{7}}$	1
$(2s2p)$	1			$-\frac{\sqrt{2}}{5}$							$-\sqrt{2}$	1
$(2p2s)$	1		$-\frac{\sqrt{2}}{3}$								$-\sqrt{2}$	1
$(2s1f)$ $(2s1f)$	3	1									1	1
$(1f2s)$	3				$\frac{1}{7}$						1	1
$(2s2p)$	•										•	•
$(2s1f)$ $(2p2s)$	•										•	•
$(2s2p)$ $(2s2p)$	1	1									1	1
$(2p2s)$	1		$\frac{1}{3}$								1	1

Configuration	$L \backslash k$	0	1	2	3	4	5	6	7	8	$D_{l_1'l_2'L}$	$D_{l_1l_2L}$
$(1f1g)(1f1g)$	1	1		$\frac{5}{21}$		$\frac{9}{77}$		$\frac{25}{429}$			-2	-2
	2	1		$\frac{1}{7}$		$-\frac{3}{77}$		$-\frac{15}{143}$			0	0
	3	1		$\frac{2}{77}$		$-\frac{69}{847}$		$\frac{150}{1,573}$			$3\sqrt{\frac{2}{11}}$	$3\sqrt{\frac{2}{11}}$
	4	1		$-\frac{94}{1,155}$		$\frac{9}{847}$		$-\frac{250}{4,719}$			0	0
	5	1		$-\frac{53}{385}$		$\frac{867}{11,011}$		$\frac{375}{20,449}$			$-6\sqrt{\frac{5}{143}}$	$-6\sqrt{\frac{5}{143}}$
	6	1		$-\frac{1}{11}$		$-\frac{87}{1,573}$		$-\frac{75}{20,449}$			0	0
	7	1		$\frac{4}{33}$		$\frac{18}{1,573}$		$\frac{20}{61,347}$			$7\sqrt{\frac{5}{143}}$	$7\sqrt{\frac{5}{143}}$
$(1g1f)$	1		$\frac{1}{63}$		$\frac{3}{77}$		$\frac{75}{10,001}$		$\frac{245}{1,287}$		-2	-2
	2		$\frac{1}{21}$		$\frac{1}{11}$		$\frac{85}{1,001}$		$-\frac{49}{429}$		0	0
	3		$\frac{2}{21}$		$\frac{97}{847}$		$-\frac{230}{11,011}$		$\frac{245}{4,719}$		$3\sqrt{\frac{2}{11}}$	$3\sqrt{\frac{2}{11}}$
	4		$\frac{10}{63}$		$\frac{9}{121}$		$-\frac{738}{11,011}$		$-\frac{245}{14,157}$		0	0
	5		$\frac{5}{21}$		$-\frac{23}{847}$		$\frac{8,779}{143,143}$		$\frac{245}{61,347}$		$-6\sqrt{\frac{5}{143}}$	$-6\sqrt{\frac{5}{143}}$
	6		$\frac{1}{3}$		$-\frac{13}{121}$		$-\frac{445}{20,449}$		$-\frac{35}{61,347}$		0	0
	7		$\frac{4}{9}$		$\frac{6}{121}$		$\frac{60}{20,449}$		$\frac{7}{184,041}$		$7\sqrt{\frac{5}{143}}$	$7\sqrt{\frac{5}{143}}$
$(1f2d)$	1			$-\frac{2\sqrt{3}}{105}$		$-\frac{10\sqrt{3}}{231}$		$-\frac{50\sqrt{3}}{429}$			-2	$\sqrt{3}$
	2			$-\frac{\sqrt{10}}{35}$		$-\frac{8\sqrt{10}}{231}$		$\frac{5\sqrt{10}}{143}$			0	0
	3			$-\frac{2\sqrt{66}}{105}$		$-\frac{5\sqrt{66}}{2,541}$		$-\frac{25\sqrt{66}}{4,719}$			$3\sqrt{\frac{2}{11}}$	$-\frac{2}{\sqrt{3}}$
	4			$-\frac{2\sqrt{110}}{105}$		$\frac{23\sqrt{110}}{2,541}$		$\frac{5\sqrt{110}}{4,719}$			0	0
	5			$-\frac{\sqrt{390}}{105}$		$-\frac{2\sqrt{390}}{847}$		$-\frac{5\sqrt{390}}{61,347}$			$-6\sqrt{\frac{5}{143}}$	$-6\sqrt{\frac{5}{33}}$
$(1f1g)(2d1f)$	1		$-\frac{2\sqrt{3}}{7}$		$-\frac{2\sqrt{3}}{21}$		$-\frac{10\sqrt{3}}{231}$				-2	$\sqrt{3}$
	2		$-\frac{\sqrt{10}}{7}$		0		$\frac{3\sqrt{10}}{77}$				0	0
	3		$-\frac{\sqrt{66}}{21}$		$\frac{\sqrt{66}}{77}$		$-\frac{10\sqrt{66}}{847}$				$3\sqrt{\frac{2}{11}}$	$-\frac{2}{\sqrt{3}}$
	4		$-\frac{\sqrt{110}}{35}$		$\frac{\sqrt{110}}{165}$		$\frac{10\sqrt{110}}{2,541}$				0	0
	5		$-\frac{\sqrt{390}}{105}$		$-\frac{2\sqrt{390}}{385}$		$-\frac{5\sqrt{390}}{11,011}$				$-6\sqrt{\frac{5}{143}}$	$5\sqrt{\frac{2}{33}}$
$(1f3s)$	3					$\frac{\sqrt{22}}{33}$					$3\sqrt{\frac{2}{11}}$	1
$(3s1f)$	3				$\frac{3\sqrt{22}}{77}$						$3\sqrt{\frac{2}{11}}$	1
$(2p1g)$	3			$-\frac{3\sqrt{462}}{539}$		$-\frac{3\sqrt{462}}{539}$					$3\sqrt{\frac{2}{11}}$	$-2\sqrt{\frac{3}{7}}$
	4			$-\frac{9\sqrt{66}}{385}$		$\frac{\sqrt{66}}{77}$					0	0

Configuration	$L \backslash k$	0	1	2	3	4	5	6	7	8	$D_{l_1'l_2'L}$	$D_{l_1l_2L}$
	5			$-\frac{12\sqrt{39}}{385}$		$-\frac{6\sqrt{39}}{1,573}$					$-6\sqrt{\frac{5}{143}}$	$\sqrt{\frac{15}{11}}$
(1*g*2*p*)	3				$-\frac{\sqrt{462}}{539}$		$-\frac{5\sqrt{462}}{847}$				$3\sqrt{\frac{2}{11}}$	$-2\sqrt{\frac{3}{7}}$
	4				$-\frac{\sqrt{66}}{77}$		$\frac{\sqrt{66}}{121}$				0	0
	5				$-\frac{2\sqrt{39}}{77}$		$-\frac{4\sqrt{39}}{1,573}$				$-6\sqrt{\frac{5}{143}}$	$\sqrt{\frac{15}{11}}$
(2*p*2*d*)	1			$\frac{9\sqrt{2}}{35}$		$\frac{5\sqrt{2}}{63}$					-2	$-\sqrt{2}$
	2			$\frac{3\sqrt{10}}{35}$		$-\frac{\sqrt{10}}{21}$					0	0
	3			$\frac{9\sqrt{154}}{735}$		$\frac{10\sqrt{154}}{1,617}$					$3\sqrt{\frac{2}{11}}$	$\frac{3}{\sqrt{7}}$
(2*d*2*p*)	1				$\frac{\sqrt{2}}{21}$		$\frac{5\sqrt{2}}{33}$				-2	$-\sqrt{2}$
	2				$\frac{\sqrt{10}}{21}$		$-\frac{\sqrt{10}}{33}$				0	0
	3				$\frac{2\sqrt{154}}{147}$		$\frac{5\sqrt{154}}{2,541}$				$3\sqrt{\frac{2}{11}}$	$\frac{3}{\sqrt{7}}$
(2*p*3*s*)	1					$-\frac{2}{9}$					-2	1
(3*s*2*p*)	1				$-\frac{2}{7}$						-2	1
(1*f*2*d*) (1*f*2*d*)	1	1		$\frac{8}{35}$		$\frac{2}{21}$					$\sqrt{3}$	$\sqrt{3}$
	2	1		$\frac{2}{35}$		$-\frac{1}{7}$					0	0
	3	1		$-\frac{11}{105}$		$\frac{2}{21}$					$-\frac{2}{\sqrt{3}}$	$-\frac{2}{\sqrt{3}}$
	4	1		$-\frac{1}{7}$		$-\frac{2}{63}$					0	0
	5	1		$\frac{2}{21}$		$\frac{1}{231}$					$5\sqrt{\frac{2}{33}}$	$5\sqrt{\frac{2}{33}}$
(2*d*1*f*)	1		$\frac{1}{35}$		$\frac{8}{105}$		$\frac{50}{231}$				$\sqrt{3}$	$\sqrt{3}$
	2		$\frac{3}{35}$		$\frac{2}{15}$		$-\frac{25}{231}$				0	0
	3		$\frac{6}{35}$		$\frac{38}{630}$		$\frac{25}{693}$				$-\frac{2}{\sqrt{3}}$	$-\frac{2}{\sqrt{3}}$
	4		$\frac{2}{7}$		$-\frac{1}{9}$		$-\frac{5}{693}$				0	0
	5		$\frac{3}{7}$		$\frac{2}{63}$		$\frac{5}{7,623}$				$5\sqrt{\frac{2}{33}}$	$5\sqrt{\frac{2}{33}}$
(1*f*3*s*)	3			$-\frac{2}{5\sqrt{3}}$							$-\frac{2}{\sqrt{3}}$	1
(3*s*1*f*)	3				$-\frac{2}{7\sqrt{3}}$						$-\frac{2}{\sqrt{3}}$	1
(2*p*1*g*)	3			$\frac{3}{35\sqrt{7}}$		$\frac{25\sqrt{7}}{441}$					$-\frac{2}{\sqrt{3}}$	$-2\sqrt{\frac{3}{7}}$
	4			$\frac{\sqrt{15}}{35}$		$-\frac{\sqrt{15}}{63}$					0	0
	5			$\frac{3\sqrt{10}}{35}$		$\frac{2\sqrt{10}}{693}$					$5\sqrt{\frac{2}{33}}$	$\sqrt{\frac{15}{11}}$
(1*g*2*p*)	3		$\frac{\sqrt{7}}{21}$		$\frac{9\sqrt{7}}{147}$						$-\frac{2}{\sqrt{3}}$	$-2\sqrt{\frac{3}{7}}$

Configuration	L \ k	0	1	2	3	4	5	6	7	8	$D_{l_1'l_2'L}$	$D_{l_1l_2L}$
	4		$\frac{1}{\sqrt{15}}$		$-\frac{\sqrt{15}}{35}$						0	0
	5		$\frac{2\sqrt{10}}{15}$		$\frac{3\sqrt{10}}{385}$						$5\sqrt{\frac{2}{33}}$	$\sqrt{\frac{15}{11}}$
$(2p2d)$	1			$-\frac{\sqrt{6}}{35}$		$-\frac{2\sqrt{6}}{21}$					$\sqrt{3}$	$-\sqrt{2}$
	2			$-\frac{6}{35}$		$\frac{2}{21}$					0	0
	3			$-\frac{12\sqrt{21}}{245}$		$-\frac{2\sqrt{21}}{441}$					$-\frac{2}{\sqrt{3}}$	$\frac{3}{\sqrt{7}}$
$(2d2p)$	1		$-\frac{\sqrt{5}}{5}$		$-\frac{2\sqrt{6}}{35}$						$\sqrt{3}$	$-\sqrt{2}$
	2		$-\frac{2}{5}$		$\frac{6}{35}$						0	0
	3		$-\frac{2\sqrt{21}}{35}$		$-\frac{4\sqrt{21}}{245}$						$-\frac{2}{\sqrt{3}}$	$\frac{3}{\sqrt{7}}$
$(2p3s)$	1			$\frac{\sqrt{3}}{5}$							$\sqrt{3}$	1
$(1f2d)$ $(3s2p)$	1				$\frac{\sqrt{3}}{7}$						$\sqrt{3}$	1
$(1f3s)$ $(1f3s)$	3	1									1	1
$(3s1f)$	3				$\frac{1}{7}$						1	1
$(2p1g)$	3					$-\frac{2\sqrt{21}}{63}$					1	$-2\sqrt{\frac{3}{7}}$
$(1g2p)$	3		$-\frac{2\sqrt{21}}{21}$								1	$-2\frac{3}{7}$
$(2p2d)$	3			$\frac{3\sqrt{7}}{35}$							1	$\frac{3}{\sqrt{7}}$
$(2d2p)$	3		$\frac{\sqrt{7}}{7}$								1	$\frac{3}{\sqrt{7}}$
$(2p3s)$	·										·	·
$(3s2p)$	·										·	·
$(2p1g)$ $(2p1g)$	3	1		$\frac{1}{7}$							$-2\sqrt{\frac{3}{7}}$	$-2\sqrt{\frac{3}{7}}$
	4	1		$-\frac{1}{5}$							0	0
	5	1		$\frac{4}{55}$							$\sqrt{\frac{15}{11}}$	$\sqrt{\frac{15}{11}}$
$(1g2p)$	3				$\frac{1}{147}$		$\frac{15}{99}$				$-2\sqrt{\frac{3}{7}}$	$-2\sqrt{\frac{3}{7}}$
	4				$\frac{1}{21}$		$-\frac{1}{33}$				0	0
	5				$\frac{4}{21}$		$\frac{1}{363}$				$\sqrt{\frac{15}{11}}$	$\sqrt{\frac{15}{11}}$
$(2p2d)$	3			$-\frac{6\sqrt{3}}{35}$							$-2\sqrt{\frac{3}{7}}$	$\frac{3}{\sqrt{7}}$
$(2d2p)$	3				$-\frac{6\sqrt{3}}{49}$						$-2\sqrt{\frac{3}{7}}$	$\frac{3}{\sqrt{7}}$
$(2p3s)$	·										·	·
$(3s2p)$	·										·	·
$(2p2d)$ $(2p2d)$	1	1		$\frac{1}{5}$							$-\sqrt{2}$	$-\sqrt{2}$
	2	1		$-\frac{1}{5}$							0	0

Configuration	$L \backslash k$	0	1	2	3	4	5	6	7	8	$D_{l_1'l_2'L}$	$D_{l_1l_2L}$
	3	1		$\frac{2}{35}$							$\frac{3}{\sqrt{7}}$	$\frac{3}{\sqrt{7}}$
(2d2p)	1		$\frac{1}{15}$		$\frac{9}{35}$						$-\sqrt{2}$	$-\sqrt{2}$
(2p2d) (2d2p)	2		$\frac{1}{5}$		$-\frac{3}{35}$						0	0
	3		$\frac{2}{5}$		$\frac{3}{245}$						$\frac{3}{\sqrt{7}}$	$\frac{3}{\sqrt{7}}$
(2p3s)	1			$-\frac{\sqrt{2}}{5}$							$-\sqrt{2}$	1
(3s2p)	1		$-\frac{\sqrt{2}}{3}$								$-\sqrt{2}$	1
(2p3s) (2p3s)	1	1									1	1
(3s2p)	1		$\frac{1}{3}$								1	1

〈Table 4〉 Tables of Slater Integral F^k

Configuration	k	Common Factor	f_0^k	f_1^k	f_2^k	f_3^k	f_4^k	f_5^k	f_6^k	f_7^k
(1s1p) (1s1p)	0	1/2	1	1						
(1p1s)	1	3/2	1	−1						
(1p1d) (1p1d)	0	1/24	7	5	5	7				
	2	35/24	1	−1	−1	1				
(1d1p)	1	1/8	7	−1	1	−7				
	3	49/24	1	−3	3	−1				
(1p2s)	2	$\sqrt{10}/48$	−5	−25	65	−35				
(2s1p)	1	$\sqrt{10}/16$	−1	1	−7	7				
(1p2s) (1p2s)	0	1/48	11	37	−35	35				
(2s1p)	1	1/16	11	−41	65	−35				
(1f1d) (1f1d)	0	1/160	33	21	26	26	21	33		
	2	3/32	11	−5	−6	−6	−5	11		
	4	297/160	1	−3	2	2	−3	1		
(1d1f)	1	1/160	99	9	38	−38	−9	−99		
	3	21/160	11	−19	2	−2	19	−11		
	5	363/160	1	−5	10	−10	5	−1		
(1f1d) (2p1d)	2	$\sqrt{14}/192$	−9	−35	34	−26	135	−99		
	4	$3\sqrt{12}/320$	−9	−63	342	−558	387	−99		
(1d2p)	1	$\sqrt{14}/320$	−9	11	−38	18	−81	99		
	3	$21\sqrt{14}/320$	−1	−1	−2	22	−29	11		
(1f2s)	2	$3\sqrt{12}/64$	−5	5	−6	6	11	−11		
(2s1f)	3	$21\sqrt{10}/320$	−5	7	−2	14	−25	11		
(2p2s)	2	$\sqrt{35}/192$	+15	−25	14	86	−189	99		
(2s2p)	3	$7\sqrt{35}/320$	15	−63	198	−366	315	−99		
(1d2p) (1d2p)	0	1/960	133	381	−114	686	−819	693		
	2	7/192	19	15	−114	206	−225	99		
(2p1d)	1	1/320	133	−317	366	−686	1,197	−693		
	3	49/960	19	−111	318	−478	351	−99		
(2s1f)	2	$\sqrt{35}/192$	15	−25	14	86	−189	99		
(1f2s)	1	$\sqrt{35}/320$	15	37	−22	−66	135	−99		

Configuration	k	Common Factor	f_0^k	f_1^k	f_2^k	f_3^k	f_4^k	f_5^k	f_6^k	f_7^k
$(2s2p)$	2	$\sqrt{10}/384$	5	−465	1,482	−2,282	1,953	−693		
$(2p2s)$	1	$\sqrt{10}/640$	5	41	−606	1,442	−1,575	693		
$(2s1f)$ $(2s1f)$	0	1/64	9	9	38	−10	−15	33		
$(1f2s)$	3	21/64	3	−7	14	−30	31	−11		
$(2s2p)$	·	no k								
$(2p2s)$	·	no k								
$(2s2p)$ $(2s2p)$	0	5/1920	109	129	−582	1,610	−1,577	693		
$(2p2s)$	1	1/128	109	−521	1,602	−2,450	1,953	−693		
$(1f1g)(1f1g)$	0	3/13440	715	429	583	513	513	583	429	715
	2	11/896	65	−13	−15	−37	−37	−15	−13	65
	4	429/4480	15	−31	3	13	13	3	−31	15
	6	1859/896	1	−5	9	−5	−5	9	−5	1
$(1g1f)$	1	3/4480	715	143	363	−89	89	−363	−143	−715
	3	11/1920	195	−221	−41	−137	137	41	221	−195
	5	1573/13440	15	−53	55	−5	5	−55	53	−15
	7	2145/896	1	−7	21	−35	35	−21	7	−1
$(1f2d)$	2	$\sqrt{2}/1792$	−143	−495	285	−515	1,011	−429	2,431	−2,145
	4	$99\sqrt{2}/8960$	−13	−73	227	−113	97	−515	585	−195
$(1f1g)$ $(1f2d)$	6	$1859\sqrt{2}/8960$	−1	−9	75	−205	285	−219	89	−15
$(2d1f)$	1	$3\sqrt{2}/8960$	−143	187	−487	267	−869	473	−1,573	2,145
	3	$11\sqrt{2}/1280$	−13	−3	−57	137	−39	183	−403	195
	5	$1573\sqrt{2}/8960$	−1	−3	15	5	−75	111	−67	15
$(1f3s)$	4	$99\sqrt{14}/17920$	−5	97	−221	209	−391	779	−663	195
$(3s1f)$	3	$77\sqrt{14}/17920$	−5	39	−21	−17	57	−339	481	−195
$(2p1g)$	2	$11\sqrt{14}/5376$	−65	59	−85	71	−19	65	169	−195
	4	$99\sqrt{14}\sqrt{26880}$	−5	11	−13	27	−23	−23	41	−15
$(1g2p)$	3	$77\sqrt{14}/26880$	−65	61	−29	137	−67	119	−351	195
	5	$121\sqrt{14}/26880$	−5	13	−5	5	−55	95	−63	15
$(2p2d)$	2	$\sqrt{7}/1792$	253	−425	305	115	−281	1,749	−3,861	2,145
	4	$99\sqrt{7}/8960$	23	−87	183	−127	−387	915	−715	195
$(2d2p)$	3	$\sqrt{7}/1280$	253	−867	1,677	−1,787	2,319	−5,313	5,863	−2,145
	5	$121\sqrt{7}/8960$	23	−141	555	−1,465	2,325	−2,103	1,001	−195
$(2p3s)$	4	9/2560	115	273	201	−3,379	10,071	−13,629	8,723	−2,145
$(3s2p)$	3	7/2560	−115	411	−939	1,787	−4,257	7,689	−6,721	2,145
$(1f2d)$ $(1f2d)$	0	1/26880	2,673	7,299	−1,479	11,331	−6,309	18,513	−24,453	19,305
	2	3/1792	297	439	−1,251	1,075	−1,781	3,509	−4,433	2,145
	4	297/8960	27	−39	−101	449	−951	1,187	−767	195
$(2d1f)$	1	1/8960	2,673	−5,463	6,413	−9,851	11,475	−18,117	32,175	−19,305
	3	3/1280	297	−1,227	2,097	−1,899	3,075	−6,633	6,435	−2,145
	5	363/8960	27	−213	855	−2,025	2,865	−2,367	1,053	−195
$(1f3s)$	2	$3\sqrt{7}/3584$	183	−115	345	−955	3,251	−5,489	5,291	−2,145
$(3s1f)$	3	387/2560	183	747	−1,407	2,427	−5,709	9,273	−7,293	2,145
$(2p1g)$	2	$\sqrt{7}/1792$	253	−425	305	115	−281	1,749	−3,861	2,145
	4	$99\sqrt{7}/8960$	23	−87	183	−127	−387	915	−715	195
$(1g2p)$	1	$3\sqrt{7}/8960$	253	633	−223	13	39	−1,573	3,003	−2,145
	3	$77\sqrt{7}/8960$	23	23	−153	143	109	−483	533	−195
$(2p2d)$	2	$\sqrt{14}/10752$	117	−5,051	9,589	−14,195	33,399	−57,321	52,767	−19,305

Configuration	k	Common Factor	f_0^k	f_1^k	f_2^k	f_3^k	f_4^k	f_5^k	f_6^k	f_7^k
	4	$27\sqrt{14}/17920$	13	−1,031	5,281	−13,219	20,231	−18,997	9,867	−2,145
(2d2p)	1	$\sqrt{14}/17920$	117	743	−5,343	8,731	−20,385	41,877	−45,045	19,305
	3	$27\sqrt{14}/23040$	13	−253	−67	3,019	−8,465	11,473	−7,865	2,145
(1f2d) (2p3s)	2	$\sqrt{2}/3072$	237	2,555	10,195	29,795	−62,289	81,081	−60,489	19,305
(3s2p)	3	$7\sqrt{2}/5120$	79	−981	6,891	−22,441	41,037	−44,319	36,169	−6,435
(1f3s) (1f3s)	0	1/15360	305	2,583	6,261	3,771	−21,429	43,461	−39,897	19,305
(3s1f)	3	21/5120	145	−711	1,989	−4,867	9,747	−12,309	8,151	−2,145
(2p1g)	4	9/2560	−115	273	201	−3,379	10,071	−13,629	8,723	−2,145
(1g2p)	1	3/2560	−115	−249	721	−1,453	183	2,629	−3,861	2,145
(2p2d)	2	$\sqrt{2}/3072$	237	2,555	−10,195	29,795	−62,289	81,081	−60,489	19,305
(2d2p)	1	$\sqrt{2}/5120$	237	1,597	−4,347	−4,003	32,751	−59,697	52,767	−19,305
(2p2s)	·	no k								
(3s2p)	·	no k								
(2p1g) (2p1g)	0	1/3840	385	507	1,209	−501	1,899	−231	−1,573	2,145
	2	11/758	35	21	15	−231	249	15	−299	195
(1g2p)	3	7/384	385	−811	1,289	−1,707	1,867	−4,169	5,291	−2,145
	5	121/3840	35	−149	415	−1,065	1,865	−1,855	949	−195
(2p2d)	2	$\sqrt{2}/512$	−63	245	−1,495	2,485	−149	−4,167	5,291	−2,145
(2d2p)	3	$7\sqrt{2}/2560$	−63	−253	433	1,707	−6,389	9,713	−7,293	2,145
(2p1g) (2p3s)	·	no k								
(3s2p)	·	no k								
(2p2d) (2p2d)	0	7/53760	1,393	279	2,061	6,071	−28,269	56,133	−50,193	19,305
	2	1/1536	1,393	−3,549	3,081	18,515	−66,069	92,961	−65,637	19,305
(2d2p)	1	7/17920	1,393	−3,523	10,873	−25,411	53,595	−75,537	57,915	−19,305
	3	7/7680	1,393	−9,903	42,933	−109,051	166,995	−156,717	83,655	−19,305
(2p3s)	2	$5\sqrt{7}/107520$	−889	−27,825	153,195	−456,365	812,133	−858,627	513,513	−135,135
(3s2p)	1	$\sqrt{7}/5120$	−127	1,123	−11,863	41,683	−83,781	99,297	−65,637	19,305
(2p3s) (2p3s)	0	1/153600	27,275	132,705	−682,965	2,037,385	−3,408,615	3,669,435	−2,297,295	675,675
(3s2p)	1	1/10240	5,455	−44,953	207,547	−546,301	886,221	−886,347	513,513	−135,135

〈Table 5〉 **Tables of LS-Energy Matrix Elements (1/9)**

Configuration	L	Common Factor	a_0^L	a_1^L	a_2^L	a_3^L	a_4^L	a_5^L	a_6^L	a_7^L
(1s1p) (1s1p)	1	1/2	1	1						
(1p1s)	1	1/2	1	−1						
(1p1d) (1p1d)	1	1/12	7	−1	−1	7				
	2	1/2	0	1	1	0				
	3	1/8	3	1	1	3				
(1d1p)	1	1/12	7	−19	19	−7				
	2	1/2	0	1	−1	0				
	3	1/8	3	−1	1	−3				
(1p2s)	1	$\sqrt{5}/24$	1	5	−13	7				
(2s1p)	1	$\sqrt{5}/24$	1	−1	7	−7				
(1p2s) (1p2s)	1	1/48	11	37	−35	35				
(2s1p)	1	1/48	11	−41	65	−35				
(1d1f) (1p1f)	1	1/160	99	−81	62	62	−81	99		
	2	1/10	0	9	−4	−4	9	0		

Configuration	L	Common Factor	a_0^L	a_1^L	a_2^L	a_3^L	a_4^L	a_5^L	a_6^L	a_7^L
	3	1/40	11	−14	23	23	−14	11		
	4	1/8	0	3	1	1	3	0		
	5	1/16	5	1	2	2	1	5		
$(1d1f)$ $(1f1d)$	1	1/160	99	−423	790	−790	423	−99		
	2	3/10	0	3	−8	8	−3	0		
	3	1/40	11	−22	35	−35	22	−11		
	4	1/8	0	3	−1	1	−3	0		
	5	1/16	5	−1	2	−2	1	−5		
$(1d2p)$	1	$\sqrt{21}/480$	9	59	−298	482	−351	99		
	2	$\sqrt{14}/40$	0	−1	11	−19	9	0		
	3	$\sqrt{6}/160$	3	13	−26	34	−57	33		
$(2p1d)$	1	$\sqrt{21}/160$	3	−1	10	−30	51	−33		
	2	$\sqrt{14}/40$	0	−1	1	9	−9	0		
	3	$\sqrt{6}/160$	3	−1	10	−30	51	−33		
$(2s1f)$	3	$\sqrt{30}/160$	5	−5	6	−6	−11	11		
$(1f2s)$	3	$\sqrt{30}/160$	5	−7	2	−14	25	−11		
$(2s2p)$	1	$\sqrt{105}/960$	15	−25	14	86	−189	99		
$(2p2s)$	1	$\sqrt{105}/320$	5	−21	66	−122	105	−33		
$(1d2p)$ $(1d2p)$	1	1/480	133	243	−456	1,064	−1,197	693		
	2	1/80	0	23	57	−63	63	0		
	3	1/320	57	137	−114	366	−423	297		
$(2p1d)$	1	1/480	133	−731	2,040	−3,080	2,331	−693		
	2	1/80	0	23	−93	133	−63	0		
	3	1/320	57	−149	210	−370	549	−297		
$(2s1f)$	3	$\sqrt{5}/320$	15	−25	14	86	−189	99		
$(1f2s)$	3	$\sqrt{5}/320$	15	37	−22	−66	135	−99		
$(2s2p)$	1	$\sqrt{5}/960$	−5	465	−1,482	2,282	−1,953	+693		
$(2p2s)$	1	1/64	−5	−41	606	−1,442	1,575	−693		
$(2s1f)$ $(2s1f)$	3	1/64	9	9	38	−10	−15	33		
$(1f2s)$	3	1/64	9	−21	42	−90	93	−33		
$(2s2p)$	•	no L								
$(2p2s)$	•	no L								
$(2s2p)$ $(2s2p)$	1	1/384	109	129	−582	1,610	−1,575	693		
$(2s2p)$	1	1/384	109	−521	1,602	−2,450	1,953	−693		
$(1f1g)$ $(1f1g)$	1	1/1120	715	−1,001	1,353	−507	−507	1,353	−1,001	715
	2	1/112	0	143	−209	122	122	−209	143	0
	3	1/2240	585	−1,469	4,217	−2,213	−2,213	4,217	−1,469	585
	4	1/1120	0	702	−942	800	800	−942	702	0
	5	1/1120	225	−343	583	95	95	583	−343	225
	6	1/16	0	5	1	2	2	1	5	0
	7	1/128	35	5	15	9	9	15	5	35
$(1g1f)$	1	1/1120	715	−4,147	11,253	−17,949	17,949	−11,253	4,147	−715
	2	1/112	0	143	−583	1,058	−1,058	583	−143	0
	3	1/2240	585	−1,963	5,537	−9,931	9,931	−5,537	1,963	−585
	4	1/1120	0	702	−1,434	1,592	−1,592	1,434	−702	0
	5	1/1120	225	−437	739	−407	407	−739	437	−225
	6	1/16	0	5	−1	2	−2	1	−5	0

Configuration	L	Common Factor	a_0^L	a_1^L	a_2^L	a_3^L	a_4^L	a_5^L	a_6^L	a_7^L
	7	1/128	35	−5	15	−9	9	−15	5	−35
$(1f2d)$	1	$\sqrt{6}/4480$	143	1,155	−8,625	22,475	−31,131	24,849	−11,011	2,145
	2	$3\sqrt{5}/560$	0	−11	169	−536	752	−517	143	0
	3	$\sqrt{33}/2240$	13	75	−405	1,045	−1,461	1,149	−611	195
$(1f1g)(1f2d)$	4	$3\sqrt{55}/1120$	0	−3	27	−38	46	−71	39	0
	5	$\sqrt{195}/2240$	5	21	−39	41	−57	87	−133	75
$(2d1f)$	1	$\sqrt{6}/4480$	143	33	−93	−787	3,069	−4,653	4,433	−2,145
	2	$3\sqrt{5}/560$	0	−11	47	8	−176	275	−143	0
	3	$\sqrt{33}/2240$	13	13	−133	3	739	−973	533	−195
	4	$3\sqrt{551120}$	0	−3	9	6	−34	61	−39	0
	5	$\sqrt{195}/2240$	5	−1	13	−33	47	−83	127	−75
$(1f3s)$	3	$3\sqrt{77}/8960$	−5	97	−221	209	−391	779	−663	195
$(3s1f)$	3	$3\sqrt{77}/8960$	−5	39	−21	−17	57	−339	481	−195
$(2p1g)$	3	$\sqrt{33}/2240$	65	−113	139	−251	199	169	−403	195
	4	$3\sqrt{231}/1120$	0	3	−3	10	−10	−13	13	0
	5	$\sqrt{546}/4480$	25	−31	41	−55	35	11	−101	75
$(1g2p)$	3	$\sqrt{33}/2240$	65	−151	59	−77	607	−1,049	741	−195
	4	$3\sqrt{231}/1120$	0	3	−1	−2	−18	31	−13	0
	5	$\sqrt{546}/4480$	25	−29	13	−49	59	−103	159	−75
$(2p2d)$	1	$\sqrt{14}/4480$	253	−615	915	−425	−1,701	4,719	−5,291	2,145
	2	$3\sqrt{70}/2240$	0	19	−61	54	142	−297	143	0
	3	$3\sqrt{22}/8960$	69	−185	305	−165	−593	1,557	−1,573	585
$(2d2p)$	1	$\sqrt{14}/4480$	253	−1,437	5,367	−13,727	21,699	−20,163	10,153	−2,145
	2	$3\sqrt{70}/2240$	0	19	−123	398	−646	495	−143	0
	3	$3\sqrt{22}/8960$	69	−271	681	−1,211	1,807	−2,349	1,859	−585
$(2p3s)$	1	1/1280	115	−273	−201	3,379	−10,071	13,629	−8,723	2,145
$(3s2p)$	1	1/2180	115	−411	939	−1,787	4,257	−7,689	6,721	−2,145
$(1f2d)(1f2d)$	1	1/8960	2,673	2,835	−7,639	20,163	−35,109	51,777	−45,045	19,305
	2	1/560	0	279	170	−897	2,295	−2,574	1,287	0
	3	1/2240	297	160	−346	3,697	−6,551	8,558	−5,720	2,145
	4	1/448	0	93	157	−138	534	−627	429	0
	5	1/896	135	301	−241	589	−587	1,271	−1,547	975
$(2d1f)$	1	1/8960	2,673	−18,855	70,717	−162,427	230,355	−197,109	93,951	−19,305
	2	3/1120	0	186	−1,132	3,058	−4,290	3,036	−858	0
	3	3/2240	99	−440	1,246	−2,551	3,615	−3,542	2,288	−715
	4	1/448	5	93	−265	346	−575	825	−429	0
	5	3/896	45	−107	145	−199	255	−425	611	−325
$(1f2d)\ (1f3s)$	3	$\sqrt{21}/8960$	183	115	−345	955	−3,251	5,489	−5,291	2,145
$(3s1f)$	3	$3\sqrt{21}/8960$	61	−249	469	−809	1,903	−3,091	2,431	−715
$(2p1g)$	3	1/2240	253	−900	1,830	−1,235	−3,831	9,174	−7,436	2,145
	4	$\sqrt{105}/2240$	0	19	−61	54	142	−297	143	0
	5	$\sqrt{70}/8960$	115	−207	182	13	−231	1,011	−1,859	975
$(1g2p)$	3	1/2240	253	348	−1,318	1,183	909	−4.378	5,148	−2,145
	4	$\sqrt{105}/2240$	0	19	73	−78	−58	187	−143	0
	5	$\sqrt{70}/8960$	115	267	−181	91	81	−919	1,521	−975
$(2p2d)$	1	$\sqrt{21}/26880$	−117	8,675	−42,109	104,003	−160,839	154,737	−83,655	19,305
	2	$\sqrt{14}/2240$	0	−151	1,355	−3,742	5,310	−4,059	1,287	0

Configuration	L	Common Factor	a_0^L	a_1^L	a_2^L	a_3^L	a_4^L	a_5^L	a_6^L	a_7^L
	3	$\sqrt{6}/8960$	−39	1,885	−5,003	9,721	−18,213	24,519	−19,305	6,435
(2*d*2*p*)	1	$\sqrt{21}/8960$	−39	155	1,149	−5,369	14,235	−22,143	18,447	−6,435
	2	$\sqrt{14}/2240$	0	−151	237	922	−2,790	3,069	−1,287	0
	3	$\sqrt{6}/8960$	−39	155	1,149	−5,369	14,235	−22,143	18,447	−6,435
(2*p*3*s*)	1	$\sqrt{6}/15360$	237	2,555	−10,195	29,795	−62,289	81,081	−60,489	19,305
(3*s*2*p*)	1	$\sqrt{6}/15360$	237	−2,943	20,673	−67,323	123,111	−132,957	78,507	−19,305
(1*f*3*s*) (1*f*3*s*)	3	1/5120	435	861	2,087	1,257	−7,143	14,487	−13,299	6,435
(3*s*1*f*)	3	1/5120	435	−2,133	5,967	−14,601	29,241	−36,927	24,435	−6,435
(2*p*1*g*)	3	$\sqrt{21}/8960$	115	−273	−201	3,379	−10,071	13,629	−8,723	2,145
(1*g*2*p*)	3	$\sqrt{21}/8960$	115	249	−721	1,453	−183	−2,629	3,861	−2,145
(2*p*2*d*)	3	$\sqrt{14}/3584$	237	2,555	−10,195	29,795	−62,289	81,081	−60,489	19,305
(2*d*2*p*)	3	$\sqrt{14}/3584$	237	1,597	−4,347	−4,003	32,751	−59,697	52,767	−19,305
(2*p*3*s*)	·	no L								
(3*s*2*p*)	·	no L								
(2*p*1*g*) (2*p*1*g*)	3	1/2240	385	392	774	−1,351	2,249	−66	−2,288	2,145
	4	1/320	0	23	87	175	−70	−33	143	0
	5	1/1280	175	197	423	−475	965	−57	−923	975
(1*g*2*p*)	3	1/2240	385	−1,616	4,474	−11,437	19,997	−19,954	10,296	−2,145
	4	1/320	0	23	−91	278	−518	451	−143	0
	5	1/1280	175	−377	619	−877	1,037	−2,059	2,457	−975
(2*p*2*d*)	3	$3\sqrt{6}/8960$	63	−245	1,495	−2,485	149	4,169	−5,291	2,145
(2*d*2*p*)	3	$3\sqrt{6}/8960$	63	253	−433	−1,757	6,389	−9,713	7,293	−2,145
(2*p*1*g*) (2*p*3*s*)	·	nn L								
(3*s*2*p*)	·	*no* L								
(2*p*2*d*) (2*p*2*d*)	1	1/3840	1,393	−1,635	2,571	12,743	−47,169	74,547	−57,915	19,305
	2	1/640	0	319	−85	−962	3,150	−3,069	1,287	0
	3	1/17920	4,179	−1,715	6,863	28,609	−110,007	192,951	−160,875	57,915
(2*d*2*p*)	1	1/3840	1,393	−9,265	39,727	−100,687	155,655	−148,588	81,081	−19,305
	2	1/640	0	319	−1,603	4,182	−5,670	4,059	−1,287	0
	3	1/17920	4,179	−11,845	39,031	−92,961	183,465	−242,847	178,893	−57,915
(2*p*3*s*)	1	$\sqrt{14}/15360$	127	3,875	−21,885	65,195	−116,019	122,661	−73,359	19,305
(3*s*2*p*)	1	$\sqrt{14}/15360$	127	−1,123	11,863	−41,683	83,781	−99,297	65,637	−19,305
(2*p*3*s*) (2*p*3*s*)	1	1/30720	5,455	26,541	−136,593	407,477	−681,723	733,887	−459,459	135,135
(3*s*2*p*)	1	1/30720	5,455	−44,953	207.547	−546,301	886,221	−886,347	513,513	−135,135

References

1. See, for example, Condon and Shortley, "Theory of Atomic Spectra," Cambridge University Press, (1935).
2. The phase convention used for our Y_{km} is the one adopted by Condon and Shortley, ibid.
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   Our $t^l_k(m,n)$ has been renormalized according to
   $t^l_k(m,n)=(2l+1)!!\ C^{2l}_k(m'n)$
   where $C^{2l}_k(m,n)$ is that of True and Ford so that

the numerator of our $t^{l}_{k}(m,n)$ takes much more simpler form.

9. Biedenharn, O.R.N.L. Report No. 1098 (1952).
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