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A Comparative Study on the Fault Diagnosis Using Fuzzy Set Concept

Won-Guk Hwang
Kyung Hee University
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Fuzzy집합개념을 이용한 고장진단에 관한 비교연구

황 원 국
경희대학교
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Abstract

This paper provides a comparative study on methodologies for solutions of the inverse problems of certain basic fuzzy relational equations, with which fuzzy set is defined as mapping from sets into complete Brouwerian lattice. Three different algorithms developed so far are discussed and applied to fault diagnosis problem for the main coolant pump of nuclear power plants.

초 록

본논문은 Fuzzy관련공식의 역문제(逆問題)에 대한 해석방법론의 비교연구를 제시하고 있다. 여기서 Fuzzy 집합은 완전한 Brouwerian격자에의 투영으로 정의된다.

현재까지 관련공식의 역해석은 세가지 다른 결과로 연구되고 있어, 이를 이용하여 원자력발전소 주급수펌프의 고장진단에 적용하여 검토하였다.

I. Introduction

In 1965 L.A. Zadeh formulated the initial statement of fuzzy set theory.[1] Since then this mathematical subdiscipline has gone through a substantial theoretical development. Correspondingly there has been a florescence of applications of this basic mathematical framework to a variety of fields such as management science, process control, artificial intelligence, decision making, languages, biology, systems engineering, man-machine studies, and more. However, there are only a few papers in the field of nuclear engineering, but research results begins to appear for the topics such as fuzzy diagnosis and fault tree analysis.

Ordinary set theory principle underlies modern mathematics. Fundamental to this basic set theory is the notion that an item is either a member or not of a set like "true or false" of the two-valued logic. However, the fact is that in the real world membership in a set is not always so crisp. Fuzzy set theory is based on a recognition that certain sets have imprecise boundaries. Fuzzy sets or subsets are those ill-specified and not distinct collections of objects with unsharp boundaries in which the transition from membership to nonmembership in a subset of a reference set is gradual rather than abrupt. This concept is more appropriate for real environment.

Typically we speak of tall men and efficient women. Membership in such sets cannot be characterized definitely, but be more adequately considered in terms of degrees of believing. A fuzzy set is characterized by a membership function, defined as a real number in the interval $[0, 1]$. For example, a membership measure $\mu_A(x)=0.8$ defines that x is a member of set A to a degree of 0.8 on a scale where zero is no membership at all and one is complete membership. It is clear that fuzzy set theory can be reduced to ordinary set theory by constraining membership to the extremes of the range zero (false) or one (true).

It has been pointed out that the estimation of causal relationships between the initial damage and the end effects in radiation biology based on fuzzy target theory may be considered as a problem of fuzzy inference, or fuzzy reasoning in fuzzy logic. The relations between the causes and the symptoms used in medical diagnosis are also treated as more or less fuzzy relations, because of the complex nature of human judgement and the medical, physiological and psychological factors involved. Such an approach can be extended to the failure diagnosis of complex industrial systems. Basically, even in a fully automated plant, the critical diagnostic changes must be done by the human operators who usually express their control and diagnostic strategies linguistically as a set of heuristic decision rules. It turns out that for most of the time it is difficult to convert such qualitative diagnostic stragegies into quantitative rules due to the imprecise nature of such rules. The imprecision is associated with the complexity of the system under consideration. In fact as far as fuzzy control of comprex industrial processes is concerned, this matter is now well established that the process operator may control a complex process more effectively than an automatic system; when the operators experience difficulty this can often be attributed to the rate or manner of information display or the depth of decision evaluation. In fact only recently Tsukamoto and Terano(2), Terano, et al.(3), Shahinpoor and Wells(4), and Pappis and Sugeno(8) have applied the notion of fuzzy logic to fault diagnosis problems. The notion is to use all available information including the fuzzy ones obtained by the human operators to initiate a failure diagnosis. In complex industrial systems, the early detection of any abnormal state, the diagnosis of the cause and the suitable treatment are necessary for efficient operation and preventing from initiating accidents. Shahinpoor and Wells(4) have initiated the fuzzy diagnosis methodology development for nuclear power plant. They give numerical example for realistic reactor plant situations and specific results of the failure diagnosis for a main coolant pump.

Before continuing to next section, a fundamental clarification should be made for the reader that concerns how the imprecision of fuzzy set theory or possibility theory differs from the imprecision dealt with by probability theory(5). Basically the difference is that probability theory deals with randomness of future events, whereas possibility theory deals with the imprecision of current or past events. Randomness deals with the uncertainty regarding the occurence or nonoccurence of some events, while the imprecision of fuzzy set deals with the membership or nonmembership of an object in a set with imprecise boundaries.

A typical probabilistic statement is "there is a 10 percent chance that the next person in the room will be less than five feet tall." A typical possibilistic statement is "that man is short." The probabilistic statement refers to a precise set of people under five feet tall. The imprecision in this case has to do with the event relating to the next person in the room. The fuzzy statement is not imprecision here and has to do with the vagueness of the concept of "short" itself linguistically.

There is the possibility of combining these two concepts; for example, "there is a ten percent chance that the next person in the room will be short." Indeed the theory involved here has been a subject of considerable interest, but that theory is beyond the scope of this paper. The point is that the fuzzy concept deals with a dimension of uncertainty that is quite distinct from that of probability theory. This form of uncertainty is significant in its own right.

II. Fuzzy Diagnosis Methodology

In order to analyse, prevent, and diagnose an impending failure in any components or areas in complex industrial systems, steps should first be taken to find a relationship matrix between the causes and the symptoms. In order to clarify this situation consider a driver of an automobile cruising 55 mph on a highway, and assume that he at some time senses a burning smell while all pertinent logical indicators on his automobile showing no signs of impending failure. Intuitively he asserts linguistically as the followings:

(1) A temperature rise; where?; why?

(2) Something is burning; why?; where?

(3) A dangerous situation is developing;how?; when?,

and he may command linguistically as the followings:

(a) Slow down the automobile; carefully

(b) Stop the automobile; immediately

(c) Turn the engine off; as soon as possible.

Similar situations may occur for complex industrial plants and thus certain linguistic assertions and commands might be initiated by the plant operators. Furthermore, where the known relationship are vague and qualitative, a fuzzy logic diagnosis may be initiated to implement the known heuristics. Thus, in such diagnostic situations the variables are set equal to nonfuzzy universes which render the possible range of measurement or magnitudes of actions to be taken. As discussed and llustrated before, these variables take on liinguistic values which are then expressed as fuzzy subsets of the universes.

In the present paper, we intend to elaborate on the relevance of fuzzy logic in the failure-diagnosis of industrial plant as a whole or components. Below, we present basic idea for fuzzy diagnosis methodologies that have been developed. Three different solutions of the inverse probems of certain basic fuzzy relational equations are described, and then the results of the solution for sample problems are followed.

II.1. Basic Logic for Fuzzy Diagnosis

Let $U=\{x\}$ be a universe of events and let a fuzzy subset of U be F such that F is a mapping $\mu_F(x): U \rightarrow [0, 1]$ by which each x is assigned a number in a closed interval $[0, 1]$. These numbers indicate the extent to which x has the properties or characteristics that are attributed to F. Therefore, if x is the magnitude of the reactor core temperature, then very large, VL, may be considered as a particular fuzzy value such as the variable core temperature and such x is then assigned a number $\nu_{VL}(x) \in [0, 1]$ which indicates the extend to which x is considered to be very large. Basically, $\mu_A(x)$ is considered as the degree of membership of element x to set A. Thus, $\mu_A(x)$ itself is a set called the membership set accompanying the fuzzy subset A of U.

The grade of membership indicates the level of believing on specific symptom. Here, we postulate a set of prime formula of basic logic which is used to derive the basic relations between causes and symptoms. Let $\boldsymbol{A}, B$, and $\boldsymbol{C}$ be propositions with the truth values of a, b, and c, respectively. We set the following operation rules for negation, disjunction, conjunction, implication, existential, and universal quantification as;

$\boldsymbol{C}=$not $\boldsymbol{A}$: $c=1-a$

$\boldsymbol{C}=\boldsymbol{A}$ and B : $c=\min(a, b)$

$\boldsymbol{C}=\boldsymbol{A}$ or B : $c=\max(a, b)$

$\boldsymbol{C}=\boldsymbol{A}$ implies B : $c=\min(1, 1-a+b)$

$\boldsymbol{C}=\exists k\boldsymbol{A}$: $c=\sup(a')$

$\boldsymbol{C}=\forall k\boldsymbol{A}$: $c=\inf(a')$

where, a' is any substitution instead of a element of $\boldsymbol{A}$, and $\exists$ and $\forall$ signify "there exits at least one" and "for all," respectively.

The prime formula cited above are taken as a sort of quantified Lakasiewicz infinite logic. Notice that the definition of implication has the property that $c=1 \leftrightarrow a \leq b$, in other words, c is equal to 1 iff (if and only if) a is not greater than b. However, we have to deal with the case in which the implication itself is not precise. It will become apparent that this case is very important from view point of application. If the lower limit of the truth value of the implication itself as $c=1$, then the rule of inference by modus ponents is given as follows;

$$\frac{A \;\; A \subset B}{B} : \max(0, a-(1-c) \leq b,$$

where the amount of $(1-c)$ may be taken as the decreass in the truth of the consequent due to fuzziness in the implication itself. In this paper, our cconcern is to determine the truth value of antecedent from consequence and implication. This will be formally written as;

$$\frac{B \;\; A \supset B}{B} : a \leq \min(1, \; b+(1-c)),$$

where c is the lower limit of the truth value of the implication. The above rule of inference is taken as the inverse problem.

II.2. Inverse Problem of Fuzzy Relational Equation

Almost all reports concerning the inverse problem are fundamentally originated from Sanchez's study[6] on fuzzy relational equation. Before we state the inverse problem, we define following definition.

[Definition 1] $\circ$ −composition.

"Given two fuzzy relation $Q \subset \boldsymbol{U} \times \boldsymbol{V}$ and $S \subset \boldsymbol{U} \times W$, find $\boldsymbol{R} \subset \boldsymbol{V} \times W$ such that $\boldsymbol{R} \circ Q = S$" where $\circ$ denotes sup-min composition, and $\boldsymbol{U} \times \boldsymbol{V}$ denotes cartesian product of the fuzzy sets $\boldsymbol{U}$ and $\boldsymbol{V}$.

Sanchezs hows an existence condition of the solutions by giving the least upper bound, lub, of the solutions. In general, the set of all the possible solutions for the above problem forms an upper simi-lattice[7]. Therefore, the greatest lower bound, glb, does not always exist.

The inverse problem denotes "given a fuzzy relation $\boldsymbol{R} \subset U \times \boldsymbol{V}$ and a fuzzy subset $B \subset \boldsymbol{V}$, find all $\boldsymbol{A} \subset \boldsymbol{U}$ such that $\boldsymbol{A} \circ \boldsymbol{R} = \boldsymbol{B}$". Although it is a special form of Sanchez's equations, this fuzzy relational equation is widely used because of its simplicity and its usefullness in practical appications.

The problem is stated using sup-min composition as follows:

$$\boldsymbol{A} \circ \boldsymbol{R} = \boldsymbol{B}; \; b_j = \bigvee^i (a_i \wedge r_{ij}), \; \text{for } 1 \leq j \leq n, \tag{1}$$

where b_j, r_{ij} and a_i are the real numbers which take the values in the interval [0, 1], and $\boldsymbol{B}$ and $\boldsymbol{A}$ are the row vectors and $\boldsymbol{R}$ is $m \times n$ matrix. In Eq. (1) the symbols "$\vee$" and "$\wedge$" signify "disjunction of P and Q such that $v(P \vee Q) = \max(v(P), v(Q))$" and "conjunction of P and Q such that $v(P \wedge Q) = \min(v(P), v(Q))$, respectively. The problem to find a_i for $1 \leq i \leq m$ satisfying Eq. (1) giving r_{ij} and b_j for $1 \leq i \leq m$ and $1 \leq j \leq n$ is called the inverse problem of fuzzy relational equations.

We first discuss three different algorithms that solve the inverse problem, and apply these algorithms to diagnose the main coolant pump.

Algorithm-1[2)]

[Definition 2] ω-composition.

$$p\omega q \triangleq \begin{cases} q, & \text{if } p>q \\ [q, 1], & \text{if } p=q \\ \phi, & \text{if } p<q. \end{cases}$$

[Definition 3] $\tilde{\omega}$-composition.

$$p\tilde{\omega}q = \begin{cases} [0, q], & \text{if } p>q \\ \phi, & \text{if } p\leq b. \end{cases}$$

In the above two definitions, p and q are the real numbers in the interval [0, 1] and ϕ is a set characterized by the only two operations as follows;

$$\phi\cap\phi=\phi \text{ and } \phi\cap G=G$$

for an arbitrary subset G included by the interval [0, 1].

Now, let us show the algorithm to obtain the sloutions of the inverse problem. For given r_{ij} and b_j in Eq. (1), we can obtain the matrices U and V whose elements u_{ij} and v_{ij} are derived respectively, as follows;

$$u_{ij}=r_{ij}\omega\ b_j \text{ and } v_{ij}=r_{ij}\tilde{\omega}b_j \tag{2}$$

for each i and j. Let $W(k)$ be one of the matrices derived from all the different compositions such that the elements of the j-th column are defined as;

$$w_{ij}= \begin{cases} u_{ij}, & \text{for } \exists 1 i\in\{i \mid \inf(u_{ij})=b_j\} \\ v_{ij}, & \text{for other i's,} \end{cases}$$

where, in the case of $b_j=0$, simply, $w_{ij}=v_{ij}=0$ for all i. By k *is* denoted an index representing one of the different combinations. Then, the existence conditions of the solution of the inverse problem for Eq. (1) are described as follows:

$$\bigcap_i u_{ij}=\phi \text{ for all } j \tag{3}$$

$$\exists k(\bigcap_i w_{ij}(k)=\boldsymbol{\phi} \text{ for all } i, \tag{4}$$

where $\boldsymbol{\phi}$ is an empty set. Let K be the index set consisted of all the k's satisfying Eq. (4). Then, the solutions of the inverse problem are written as follows;

$$a_i(k)=\bigcap_j w_{ij}(k), \text{ for } 1\leq i\leq m, \ \forall k\in K. \tag{5}$$

In general, the above solution has an upper bound solution and a number of lower bound solutions.

The relationship between failures and symptoms could be taken as that of causes and effects. Let X and Y be the the universal sets of the kinds of failures and symptoms, respectively as follows;

$$X=\{X_i \mid i\in M\} \text{ and } Y=\{Y_j \mid j\in N\},$$

where

$$M=\{i \mid 1\leq i\leq m\} \text{ and } N=\{j \mid 1\leq j\leq n\}.$$

Let F and S be fuzzy sets characterized by membership function as;

$$h_F : X\rightarrow[0, 1] \text{ and } h_S : Y\rightarrow[0, 1]$$

where the grade of membership of X_i and Y_j are assumed to represent the intensity of the i-th kind of failure and the exactitude of the j-th kind of symptom, respectively. Further, assume a fuzzy set denoted by R which is characterized by membership function as;

$$h_R : X\times Y\rightarrow[0, 1]$$

with the understanding that the grade of membership of (X_i, Y_j) represents the degree of the *causational relation from the i-the failure to the* j-th symptom.

The values of the membership functions $h_F(X_i)$ and $h_S(Y_j)$ could also be interpreted as the grades to which the statements "the i-th item is at fault" and "the j-the symptom is observed" are true, respectively. Similarly, $h_R(X_i, Y_j)$ is corresponding to the statement "the i-th kind of fault is causationally related to the j-th symptom." Let these statements be denoted by A_i, B_j and T_{ij}, respectively, and let the truth values of the statement be a_i, b_j and t_{ij}, respectively.

Note that the statement B_j says about the observation or perception of the j-th symptom by operator, but not about the objective existence of the j-th symptom. So, the truth value of the statement "A_i implies B_j" will depend highly on the observational conditions. On the other hand, the causational relation between X_i and Y_j is considered to be less fuzzy than that of A_i and

B_j.

By t_{ij} is denoted $Tv(A_i \supset B_j)$, that is, the truth value of the implication is as follows; if the i-th item is at fault, then the j-th symptom is observed. Let us assume that t_{ij} takes the value$[0, 1]$ and r_{ij} is in the interval$[0, 1]$.

This simply means that there is possibility that a symptom will not be perceived when the fault having relation to it takes place.

Now, let us consider the following two kinds of compound propositions, P_j and P_{ij}, concerning the relationship between causes and symptoms as;

$$P_j : B_j \text{ implies}(\exists i(\tilde{R}_{ij} \text{ and } A_i)),\ 1 \leq j \leq n,$$

$$P_{ij} : A_i \text{ implies } B_j,\ 1 \leq i \leq m,\ 1 \leq j \leq n.$$

What P_j means is that if Y_j is observed, then at least one kind of fault among ones relating to Y_j has occured. Consequently, the truth value of the proposition P_j might as well be equal to 1 for each j. On the contrary, it is doubtful that the proposition P_{ij} is true. The truth value will be different depending on the various sorts of things, for instance, according as a mechanical sensor has the alarming system or not.

Under the assumption, all $Tv(P_{ij})$' s, for $i \in \boldsymbol{M}$ and $j \in N$, are specified as follows;

$$Tv(P_{ij}) \geq t_{ij}.$$

Now, we can formulate the basic relations by means of the prime formula of fuzzy logic as follows;

$$P_j \rightarrow 0 \leq b_j \leq \sup_i(\min(t_{ij}, a_i)),\quad 1 \leq j \leq n, \quad (6)$$

$$P_{ij} \rightarrow 0 \leq a_i \leq \min(1,\ b_j + 1 - r_{ij}) \quad (7)$$

for all $i \in \boldsymbol{M}$ and $j \in N$, where "$\rightarrow$" indicates one of the logical symbols of implication.[9]

The problem is to find a_i for all $i \in \boldsymbol{M}$ which satisfies the inequality relations just formulated above for given r_{ij}, t_{ij} and b_j. In order to solve this problem, the following algorithm for the inverse problem of fuzzy relational equation is applicable. The lower bound solutions of a_i are obtained by solving Eq. (6) while Eq. (7) will determine the upper bound of a_i. The solution is obtained as follows;

$$\max_j(\inf(w_{ij}(k'))) \leq a_i \leq \min_j(\sup(e_{ij})), \quad (8)$$

for

$$\exists k' \in \{k \mid \max_j(\inf(w_{ij}(k))) \leq \min_j(\sup(e_{ij}))$$
$$\text{and } k \in K\} \quad (9)$$

for $1 \leq i \leq m$. Here $e_{ij} = [0,\ \min(1,\ b_j + 1 - r_{ij})]$. The relation, Eq. (9), stands for the existence condition of the solution for this problem. In general, we will have some different set-valued solutions which bring about available information about the possible abnormal state.

Algorithm-2[3]

[Definition 4] γ-composition.

$$p\gamma q = \begin{cases} q, & \text{if } p > q \\ [q, 1], & \text{if } p = q \\ \phi, & \text{if } p < q, \end{cases}$$

for $\forall p$, $\forall q \in [0, 1]$. Where ϕ is a set defined as the following property,

$$\phi \cap \boldsymbol{E} = \boldsymbol{E} \text{ for } \boldsymbol{E} \neq \phi.$$

Given a matrix $\boldsymbol{R}$ and a vector b, we can obtain a matrix denoted by W whose element w_{ij} is given as follows;

$$w_{ij} = r_{ij}\ \gamma\ b_j,\quad i \in \boldsymbol{M} \text{ and } j \in N.$$

Let w_i and w^j be the i-th row and the j-th column of the matrix W, respectively. Let ϕ be an empty set and w_i^* be the i-th row of $W(k)$, where K is the universal set consisting of all the indices k representing one of the possible combinations. The matrix $w^*(k)$ is one of the matrices derived from all the different combinations by modifying the column of W as follows;

$$(w^j)^* = ((w^j)_1^*, \cdots, (w^j)_n^*)^T,\ \text{ for } 1 \leq j \leq n,$$

such that

$$(w^j)_i^* = (w^j)_i,\ \text{ for } \exists 1 i \in \{i \mid (w^j)_i \neq \phi\}$$

$$(w^j)_l^* = [0,\ \max((w^j)_l)],\ \text{ for } l \neq i.$$

Then, the existence conditions of the solution of the inverse operation are written as follows;

$$\bigcap_i (w^j)_i \neq \phi,\ \text{ for } \forall j \in N \quad (10)$$

$$\bigcap_j w_i^*(k)) \neq \phi,\ \text{ for } \forall i \in \boldsymbol{M} \text{ and } \exists k \in K. \quad (11)$$

When the existence conditions are satisfied, the solution of the inverse operation can be obtained as follows;

$$\max_i(\inf^j(w_i^*(k))_j) \leq a_i(k) \leq \min_i(\sup_j(w_i^*))_j),$$

$$\text{for } i \in \boldsymbol{M} \qquad (12)$$

and

$$\max_i(\inf_j(w_i^*(k))_j) \leq \min_i(\sup_j(w_i^*))_j). \qquad (13)$$

When the existence condition, Eq. (11), is not satisfied, fuzzy numbers should be introduced in order to obtain approximate solutions.

[Definition 5] A fuzzy number $\tilde{Z}$ in $L=[0, 1]$ is a fuzzy set characterized by membership function $h_{\tilde{Z}}$ as;

$h_{\tilde{Z}} : L \rightarrow [0, 1]$.

A fuzzy number $\tilde{Z}$ may be expressed as

$$\tilde{Z} \triangleq \int_{x \in [0,1]} h_{\tilde{Z}}(x)/x,$$

where $\int$ denotes the union of $h_{\tilde{Z}}(x)/x$'s.

Considering the w_{ij} given in γ-composition as a fuzzy number that is normal and convex, we can obtain an approximate solution of the inverse operation in the absence of the exact solution. Let $\tilde{w}_{ij}$ be

$$\tilde{w}_{ij} = \int^{w_{ij}}_0 (x+1-w_{ij})/x + \int^1_{w_{ij}} (-x+1+w_{ij})/x, \qquad (14)$$

then, an approximate solution denoted by $\tilde{a}_i$ is defined as follows;

$$\tilde{a}_i = \{x | \sup_{x \in L}(\bigcap_j (w_i)_j)\}, \quad \forall i \in \boldsymbol{M}. \qquad (15)$$

Moreover, we define an index representing the degree of approximation as,

$$\sigma_i = 2(1 - \sup_{x \in L}(\bigcap_j (w_i) j), \quad \forall i \in \boldsymbol{M}. \qquad (16)$$

When $\sigma_i = 0$ for all $i \in \boldsymbol{M}$, it means the case where the solution exists for the original inverse problem. On the contrary, $\sigma_i \neq 0$ means that there is some inconsistency in the calculation process of the inverse operation. Therefore, when this is the case, the obtained approximate solution $\tilde{a}_i$ should be under-estimated. In the context of fault diagnosis, this means some errors caused by the detection of symptoms. It should be noted that the introduction of fuzzy numbers is to be based on fuzzy set theory dealing with fuzzy set of type 2.[10]

In fault diagnosis problem we should consider that a human operator will very often fail to detect some of symptoms appearing.

In this case, the use of the complements of fuzzy sets is necessary to avoid an erroneous diagnosis.

Denote the complements of the fuzzy sets F and S by

$$F^c \Leftrightarrow a_i^c = 1 - a_i, \quad 1 \leq i \leq m,$$

$$S^c \Leftrightarrow b_j^c = 1 - b_j, \quad 1 \leq j \leq n,$$

respectively. We consider the following equation;

$$b_j^c = \bigvee_i (a_i^c \wedge r_{ij}), \quad \text{for } j \in N. \qquad (17)$$

Assume that at least one of the symptoms appearing can be detected when a failure is caused. Then, in order that we obtain as a solution of the inverse operation

$$a_i^c = 0, \quad \forall i \in \boldsymbol{M},$$

it is sufficient that

$$b_j^c = 0, \quad \text{for } \exists j \in \{j | r_{ij} = 0\}.$$

On the contrary, if

$$b_j^c = 0, \quad \text{for } j \in N,$$

then

$$a_i^c = 0, \quad \forall i \in \{i | r_{ij} \neq 0\}.$$

Therefore, the inverse operation of Eq. (15) will be useful to list the possible kinds of failure.

Algorithm-3[8]

[Definition 6] α-composition.

$$p\alpha q \triangleq \begin{cases} 1, & \text{if } p \leq q \\ q, & \text{if } p > q \end{cases}$$

[Definition 7] β-composition.

$$p\beta q \triangleq \begin{cases} 0, & \text{if } p < q \\ q, & \text{if } p \geq q \end{cases}$$

[Definition 8] $\boldsymbol{\Phi}$-sets.

Given a column vector $\hat{\boldsymbol{a}} = (a_1, a_2, \cdots, a_m)^T$, such that $a_i = \hat{a}$ or 0, $i = 1, \cdots, m$, the set $\boldsymbol{\Phi}(\boldsymbol{a})$

of column vectors $\phi(\boldsymbol{a})$ is defined as follows:

$$\boldsymbol{\Phi}(\boldsymbol{a})=\{\boldsymbol{\Phi}(\boldsymbol{a})\},$$

where,

$$\phi(\boldsymbol{a})=(\phi_1, \phi_2, \cdots, \phi_m)^T,$$

$$\phi_i=0 \text{ or } \hat{a},\ 1\leq i\leq m,\ \sum_{i=0}^{m}\phi_i=\hat{a}.$$

Thus, if there are k nonzero elements in $\boldsymbol{a}$, there are k vectors in $\phi(\boldsymbol{a})$. Note that $\boldsymbol{\Phi}(\boldsymbol{a})$ is defined iff $a_i=0$ or $\hat{a}$, $\forall i$.

Given an $m\times n$ matrix $\boldsymbol{R}=[r_{ij}]$, let $\boldsymbol{r}_j$ be its j-th column vector and assume that $\boldsymbol{\Phi}(\boldsymbol{r}_j)$ is defined for $1\leq j\leq n$. Then the set $\boldsymbol{\Phi}(\boldsymbol{R})$ of mat rices $\phi(\boldsymbol{R})$ is defined as follows:

$$\boldsymbol{\Phi}(\boldsymbol{R})=\phi(\boldsymbol{R}),$$

where,

$$\boldsymbol{\Phi}(\boldsymbol{R})=[\phi(\boldsymbol{r}_1),\ \phi(\boldsymbol{r}_2), \cdots, \phi(\boldsymbol{r}_n)].$$

Note that if there are z matrices in $\boldsymbol{\Phi}(\boldsymbol{R})$,

$$z=\prod_{j=1}^{n}z_j,$$

where,

$$z_j=\begin{cases}\text{number of nonzero elements in } \boldsymbol{r}_j & \text{if } \boldsymbol{r}_j\neq 0\\ 1 & \text{if } \boldsymbol{r}_j=0.\end{cases}$$

[Definition 9] δ-composition.

Given an $m\times n$ matrix $\boldsymbol{R}=[r_{ij}]$ and a row vector $\boldsymbol{b}=(b_1, \dots b_n)$, then,

$$\boldsymbol{R}\delta b=[s_{ij}],$$

$$s_{ij}=\left(\bigwedge_{k=1}^{n}(r_{ik}\alpha b_k)\right)\beta(r_{ij}\beta b_j),\quad 1\leq i\leq m \text{ and } 1\leq j\leq n.$$

With given α-, β-, and δ-compositions and $\boldsymbol{\Phi}$ set, the solutions of the inverse problem are obtained as follows:

Given the fuzzy relation $\underset{\sim}{R}$ and the fuzzy subset B, all ufzzy subset $\underset{\sim}{A}$ such that $\underset{\sim}{A}\circ\underset{\sim}{R}=B$ are given by

$$\bigvee(\phi(\boldsymbol{R}\delta\boldsymbol{b}))^T\leq\boldsymbol{a}\leq\bigwedge(\boldsymbol{R}\alpha\boldsymbol{b})^T,$$
$$\forall\phi(\boldsymbol{R}\delta\boldsymbol{b})\ni\boldsymbol{\Phi}(\boldsymbol{R}\delta\boldsymbol{b}), \tag{18}$$

provided that there exists at least one such A, where $\boldsymbol{R}$ is the matrix corresponding to R, and $\boldsymbol{a}, \boldsymbol{b}$ are the vectors corresponding to A, B, respectively.

III. Applications of Fuzzy Algorithm to Main Cooant Pump Diagnosis

In order to verify the applicability of three fuzzy logic algorithms discussed in the previous section, we present simple problem analyses on how the failure vector, $\boldsymbol{a}$, can be estimated when a symptom vector, $\boldsymbol{b}$, is given. A computer code, FUDIA (FUzzy DIagnosis Algorithm), is developed for VAX-11/780 computer based on the previous algorithms. Listed in Table 1 are the definitions of the failure and the symptom vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, respectively[4].

The truth value of the above propositions varies between 0 to 1. The operator constructs the symptom vector, $\boldsymbol{b}$, by attributing a number from 0 to 1 to each categories or components of the symptom vector, $\boldsymbol{b}$.

The observation and the relationship matrices r_{ij} and t_{ij} are constructed by the experienced plant managers and operators. In our case we assume the following structures for t_{ij} and r_{ij} matrices as follows;

Table 1. Definition of a_i and b_j for Pump Diagnosis

a_i, failure vector	b_j, symptom vector
1-blocked flow	A-high differential pressure
2-broken impeller or blades	B-low differential pressure
3-worn bearing	C-high pump noise
4-foreign objects	D-high bearing temperature
5-pressure instruments error, malfunction.	E-high power demand

$$t_{ij}=\begin{pmatrix} 1.0 & 0.0 & 0.0 & 1.0 & 1.0 \\ 0.0 & 1.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 1.0 & 1.0 \\ 0.0 & 1.0 & 1.0 & 0.0 & 1.0 \\ 1.0 & 1.0 & 0.0 & 0.0 & 0.0 \end{pmatrix}$$

$$r_{ij}=\begin{pmatrix} 0.8 & 0.0 & 0.0 & 0.6 & 0.4 \\ 0.0 & 0.4 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.4 & 0.4 & 0.2 \\ 0.0 & 0.2 & 0.4 & 0.0 & 0.2 \\ 0.4 & 0.4 & 0.0 & 0.0 & 0.0 \end{pmatrix}$$

Let us recall that $t_{ij}=1$ for some i and j implies that a definite causational relationship exists between the i-th failure and the j-th symptom. For example, a blocked flow could be responsible for a high differential pressure, *i.e.*, $t_{11}=1$, however it could not create a low differential pressure, *i.e.*, $t_{12}=0$. On the other hand the values of r_{ij} for each given set of i and j imply the possibilities or the chances of observing partially the j-th symptom if the i-th failure has occured.

In order to apply the three fuzzy algorithms, three options are available in FUDIA code. The result of developing a computer code will be reported separately[11]. We select low symptom vectors and computed causational vectors for the following two cases.

Case-1: $\boldsymbol{b}=(1.0\ 0.0\ 0.0\ 1.0\ 1.0)^T$

In this case, the operator notifies a high differential pressure, high bearing temperature, and high power demand with grade of membership of 1.0 for all symptoms. We get the results using FUDIA code as follows:

The above results show that each algorithm gives different indications. In Algorithm-1, all failure modes are possible with different grades

Algorithm-1

$$\begin{pmatrix} 1.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{pmatrix} \le \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix} \le \begin{pmatrix} 1.0 \\ 0.4 \\ 0.6 \\ 0.6 \\ 0.6 \end{pmatrix}$$

Algorithm-2

$$\begin{pmatrix} 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{pmatrix} \le \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix} \le \begin{pmatrix} 1.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{pmatrix}$$

Algorithm-3

$$\begin{pmatrix} 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{pmatrix} \le \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ {}_5a \end{pmatrix} \le \begin{pmatrix} 1.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{pmatrix}$$

of membership while a_1, blocked flow, is definite. However, in Algorithm-2 and -3, only a_1 is possible with grade of membership between 0.0 and 1.0. Therefore, combining the results, one can say that the symptoms are caused by blocked flow.

Case-2: $\boldsymbol{b}=(0.1\ 0.4\ 0.5\ 0.2\ 0.2)^T$

In this case, low differential pressure and high pump noise are identified with 0.4 and 0.5 grades of membership, respectively.

The computational results using FUDIA code are as shown below.

Algorithm-1

$$\begin{pmatrix} 0.1 \\ 0.5 \\ 0.5 \\ 0.5 \\ 0.4 \end{pmatrix} \le \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix} \le \begin{pmatrix} 0.3 \\ 0.9 \\ 0.8 \\ 1.0 \\ 0.7 \end{pmatrix}$$

Algorithm-2

$$\begin{pmatrix} 0.6 \\ 0.5 \\ 0.2 \\ 0.5 \\ 0.1 \end{pmatrix} \le \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix} \le \begin{pmatrix} 0.6 \\ 0.5 \\ 0.2 \\ 0.5 \\ 0.4 \end{pmatrix}$$

Algorithm-3

$$\begin{pmatrix} 0.1 \\ 0.4 \\ 0.0 \\ 0.2 \\ 0.0 \end{pmatrix} \le \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix} \le \begin{pmatrix} 0.1 \\ 0.5 \\ 0.1 \\ 1.0 \\ 0.1 \end{pmatrix}$$

The above results indicate that the symptoms are possibly caused by failure modes of a_2 and a_4, broken impeller and foreign objects, respectively. This conclusion is based on following facts. In Algorithm-1, failure modes of a_2, a_3, a_4 and a_5 are identified with about equal grades of membership. In Algorithm-2, a_1, a_2, a_4 and a_5 are possible while only a_2 and a_4 are highly possible in Algorithm-3. Combining the results, one can conclude that failure modes of a_2 and a_4 cause the symptoms.

IV. Conclusion

Three approaches for the inverse problem of fuzzy relational equation which can be used for complex technological system fault diagnosis are discussed. Based on the approaches, a computer code, FUDIA, is developed for present study. In order to apply these algorithms to the diagnosis of nuclear power plants some refinements of FUDIA are necessary because three algorithms give some what different results for the example problems. The set-theoretical approach for the inverse problem will be reported in the succeeding paper[11].

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