Journal of the Korean Nuclear Society
Volume 21, Number 3, September, 1989

A New Formulation of the Reconstruction Problem in Neutronics Nodal Methods Based on Maximum Entropy Principle

Won Joon Na and Nam Zin Cho
Korea Advanced Institute of Science and Technology
(Received February 17, 1989)

노달방법의 중성자속 분포 재생 문제에의 최대 엔트로피 원리에 의한 새로운 접근

나원준 • 조남진
한국과학기술원
(1989. 2. 17 접수)

Abstract

This paper develops a new method for reconstructing neutron flux distribution, that is based on the maximum entropy principle in information theory. The probability distribution that maximizes the entropy provides the most unbiased objective probability distribution within the known partial information. The partial information are the assembly volume-averaged neutron flux, the surface-averaged neutron fluxes and the surface-averaged neutron currents, that are the results of the nodal calculation. The flux distribution on the boundary of a fuel assembly, which is the boundary condition for the neutron diffusion equation, is transformed into the probability distribution in the entropy expression. The most objective boundary flux distribution is deduced using the results of the nodal calculation by the maximum entropy method. This boundary flux distribution is then used as the boundary condition in a procedure of the imbedded heterogeneous assembly calculation to provide detailed flux distribution.

The results of the new method applied to several PWR benchmark problem assemblies show that the reconstruction errors are comparable with those of the form function methods in inner region of the assembly while they are relatively large near the boundary of the assembly. The incorporation of the surface-averaged neutron currents in the constraint information (that is not done in the present study) should provide better results.

요 약

본 논문에서는 정보 이론의 maximum entropy principle을 이용하여 중성자속 분포를 재생하는 새로운 방법을 시도하였다. 어떤 대상에 대한 부분적인 정보가 있을 때, 이 정보의 한도 내에서 entropy를 최대화 시키는 확률 분포는 가장 객관적인 것이 된다. Nodal method 계산 결과인 평균 중성자속과 current의 값을 prior information으로 삼고, 핵 연료 집합체의 경계에

서의 중성자속 분포를 확률의 형태로 변환해서 확률로써 다룬다. Prior information의 한도 내에서 entropy를 최대화 시키는 경계에서의 확률 분포를 구하면 핵 연료 집합체의 경계에서의 중성자속 분포가 구해지는 데, 이것을 경계조건으로 heterogeneous assembly calculation을 행하여 세부적인 중성자속 분포를 구한다.

이 새로운 방법을 몇 개의 benchmark problem assembly에 응용해 본 결과, 노심의 안쪽 부분에서는 이 방법이 form function method에 의한 것과 비슷한 정확도를 보였고 바깥 부분에서는 다소 큰 오차를 보였다. 본 논문에서는 surface-averaged neutron current를 prior information에 포함시키지 못했는 데, 이것을 포함시키면 결과가 훨씬 개선 될 것으로 보인다.

1. Introduction

The modern coarse-mesh nodal method developed in the last fifteen years or so allows routine neutronics calculations in the reactor design and safety analysis with reasonable computing cost. The features of the nodal method are summarized as follows. The reactor core is partitioned into large (typically, size of an assembly 20cm×20cm×20cm) homogeneous nodes. The number of meshes is reduced remarkably resulting in saving of computation time. The unknown to be solved is the node-volume averaged neutron flux. In derivation of the nodal balance equation from the neutron diffusion equation, node surface-averaged neutron currents appear as additional unknowns. Thus, auxiliary spatial coupling equations are necessary. One type of the nodal method is distinguished from another by treatment of the transverse leakage terms that appear in the auxiliary spatial coupling equations. The detailed descriptions of the modern nodal method are available in Refs. 1, 2 and 3.

The nodal coarse-mesh method provides the nodal neutron fluxes and power densities in the core with both accuracy and high speed. However, its result is the volume-averaged flux in a large size of node, while local quantities are required in addition to the global ones for calculation of power peaking or for analysis of heterogeneities. [4, 5]

So, the reconstruction of the pointwise flux and the power density is necessary and several methods are available. [1, 6] The reconstruction methods are divided into two-categories : the imbedded method and the form function method. These methods reconstruct the local quantities from the information given as the results from nodal calculation.

The imbedded method is to perform heterogeneous fine-mesh assembly calculation with boundary source conditions determined by inference from the information given by the nodal calculation. [7, 8] It is expensive due to the fine-mesh calculation, although it provides good accuracy. The form function method is to determine the coefficients of the form function that represents the reconstructed flux with combination of the results from the assembly calculation. [4, 6] Its accuracy is acceptable while it is less accurate than the imbedded method. This is faster than the imbedded method because it does not require any fine-mesh calculation.

Reconstruction of the distribution of pointwise flux and power density is an inverse problem in the sense that it finds detailed pointwise quantities from the limited information given in the from of e.g., average quantities.

The maximum entropy method proposed by Jaynes [9] can be used for inverse problems. The maximum entropy method in information theory provides the most unbiased probability distribution within the given partial information. Cho [10] used it for the most objective prior probability

distribution to be used in Bayesian reliability analysis.

In this paper, the maximum entropy method is used for the reconstruction of pointwise flux distribution. The results from the nodal calculation constitute the partial information to the inverse problem of reconstruction. The boundary conditions (flux values at the boundary of the node) that maximize the entropy are found and then used in the heterogeneous fine-mesh assembly calculation in a procedure of the imbedded method.

2. Reconstruction Methods

2.1 Introduction

The nodal calculation provides only the node-averaged quantities. Sometimes, it is necessary to know the local quantities such as the pinwise power densities. In problems such as the determination of the power peaking factor, the maximum power density among the pinwise power densities is of interest rather than the volume-averaged power. Thus, a method to reconstruct the pinwise fluxes is required. The methods introduced in the literature are divided into two categories.

2.2 Imbedded Method [7, 8]

Boundary conditions for the heterogeneous diffusion calculation are deduced by using the results of the nodal calculation. From the surface-averaged fluxes and currents, the boundary conditions on the assembly is determined, for example, by curve fitting. The heterogeneous boundary source calculation is performed with the boundary conditions thus determined. If it is possible to determine accurate boundary conditions, reconstruction can be accomplished with good accuracy. Determination of the accurate boundary conditions is the most important step in this method. However, this method is expensive due to the heterogeneous fine-mesh calculation.

2.3 Form Function Method [4, 6]

A form function with unknown coefficients are assumed. The form function is designed to multiplicatively correct the assembly flux. In other words, the assembly flux reflects local heterogeneities within an assembly and the form function represents smooth flux distribution in the global core. These unknown coefficients are determined by using the results from the nodal calculation. If the coefficients are determined, the reconstruction is completed. The form functions usually used are bi-quadratic polynomials or hyperbolic functions. In the form function method, the forms of the functions must be assumed a priori.

3. The Principle of Maximum Entropy for the Reconstruction Problem

3.1 Introduction

Given partial or incomplete information about the state of a system in question, we may hope to solve following two problems. One is to determine the complete probability distribution of the system state that is the basis of the partial information. This may be considered an inverse problem in that it tries to find the cause by knowing the effect. As prior information has the information about the system state, we can disassemble this to get the cause. The other is to assess the uncertainty of the prior information whether we have missed any other prior information or not.

Jaynes [9] proposed to solve the problems above by using the concept of statistical entropy in information theory. It leads us to the way to get the prior probability distributions (causes) or to assess the uncertainty of the prior information.

Information theory [9, 10] tells us that the entropy

$$S=-\sum_{i=1}^{n} P_i \ln P_i, \qquad (3\text{–}1)$$

where P_i is the probability of outcome i and n is the total number of mutually exclusive and exhaustive outcomes, represents a measure of missing information or uncertainty and that those P's that maximize S subject to constraints of available prior information correspond to the most unbiased (objective) and noncommittal description of the state of knowledge within the information.

3.2 The Most Objective Probability Distribution

There are a number of probability distributions that match the constraints given by the prior information. However, what needed is the only one probability distribution that is closest to the true one. In other words, it is to required determine the probability distribution that is most probable and at the same most objective.

The probability distribution found by the maximum entropy method is the one that maximizes its entropy among those distributions that satisfy the prior information. The fact that it has maximum entropy means that it is the most unbiased one among those many possible distributions and that still satisfy the prior information. And it is free from other information that is not articulated. This consideration is plausible and convincing because there is tendency in nature that the entropy of a system increases as far as it is allowed by the degree of freedom.

A prior information to be used as constraints should be "testable", The prior information is testable if, given any proposed prior probability assignment $p(\theta)d\theta$, there is a procedure which will determine unambiguously whether $p(\theta)d\theta$ does or does not agree with the information.

First, the prior information is expressed in mathematical terms. The inverse problem then becomes an optimization problem. The objective function is the entropy of the probability distribution and the constraints are the mathematical representation of the prior information. The solution is the probability distribution that maximizes its entropy while satisfying the constraints.

4. Applications to the Reconstruction of Pointwise Neutron Flux

4.1 Introduction

The key step in this study is the determination of the most unbiased boundary conditions for the node to be used in solving the neutron diffusion diffusion equation by a finite difference fine-mesh method. The boundary conditions are determined using the results of the nodal coarse-mesh calculation via the maximum entropy method. The boundary conditions are transformed to the probability distribution and the results of the nodal calculation to the prior partial information. The results of the nodal calculation are the average quantities : node volume-averaged flux, node surface-averaged fluxes and currents.

Since the average quantities are "summary" descriptors of the pointwise quantities, they represent partial or incomplete information. Mathematically, specification of the boundary conditions at the node boundary for the diffusion equation is equivalent to the knowledge of the pointwise quantities, i.e., complete information. Thus, the problem of reconstructing pointwise neutron flux may be formulated as an inverse problem that is to find accurate boundary conditions that will give the average quantities obtained by the nodal calculation.

The reconstruction approach investigated in this study thus belongs to the imbedded method where accurate boundary conditions are first determined using the results of the nodal calculation and then the neutron diffusion equation is solved over the heterogeneous node with the boundary conditions determined in the first step to obtain

the pointwise neutron fluxes.

4.2 Mathematical Formulation

Since the boundary conditions, i.e., the flux distributions at the node boundary are considered as probability distributions to be determined, they are the independent variables in the optimization problem of the maximum entropy method.

Constraints given as partial information are the assembly volume-averaged neutron fluxes, the assembly surface-averaged neutron fluxes and the surface-averaged neutron net currents. These are the results of the nodal calculation.

The maximum entropy method of information theory provides the most unbiased boundary conditions (i.e., flux distributions at the boundary) that are consistent with the given constratints.

Since boundary conditions are the neutron fluxes on the boundary surfaces of a node, probability distributions are related to the surface fluxes as follows.

$$P_i^{(1,u)}=\frac{\phi_i^{(1,u)}\cdot h_i^{(u)}}{\bar{\phi}^{(1,u)}\cdot H^{(u)}\cdot ADF^{(1,u)}} \tag{4-1}$$

$$P_i^{(1,u)}=\frac{\phi_i^{(2,u)}\cdot h_i^{(u)}}{\bar{\phi}^{(2,u)}\cdot H^{(u)}\cdot ADF^{(2,u)}} \tag{4-2}$$

$$ADF^{(1,u)}=\frac{\bar{\phi}^{(1,u)het}}{\bar{\phi}^{(1,u)}}$$

$$ADF^{(2,u)}=\frac{\bar{\phi}^{(2,u)het}}{\bar{\phi}^{(2,u)}}$$

$u=l,\ r,\ t,\ b$

where u denotes each side of the assembly that is subject to reconstruction. l is for left side of the assembly, r for the right side, t for the top side and b for the bottom side. 1 designates group 1 (fast group) and 2 designates group 2 (thermal group). H denotes the assembly size. *ADF* is the assembly discontinuity factor described in Ref. 11, which represents the relationship between the homogeneous surface-averaged neutron flux and the heterogeneous surface-averaged neutron flux. *ADF* multiplied by homogeneous surface-averaged flux is the heterogeneous surface-averaged neutron flux on the boundary of the assembly.

It is easy to check that

$\Sigma P_i^{(1,u)}=1$

and

$\Sigma P_i^{(2,u)}=1.$

The terminology used up to now was that for the three dimensional case. From now on, the problem considered is of the twodimension. Axial consideration is discarded and only $x-y$ plane is considered. Thus, volume-averaged quantities refer now to surface-averaged ones, and surface-averaged quantities correspond to line-averaged quantities.

The entropy for the probability distributions defined in Eqs. (4–1) and (4–2) is as follows:

$$S=-w_1\sum_{\text{all}}\sum_{ui=1}^{n^{(u)}}P_i^{(1,u)}\ln P_i^{(1,u)} -w_2\sum_{\text{all}}\sum_{ui=1}^{n^{(u)}}P_i^{(2,u)}\ln P_i^{(2,u)} \tag{4-3}$$

where w_1 and w_2 are weighting factors for the fast and thermal groups, respectively. A trivial choice for w_1 and w_2 is

$$w_1=w_2=1.$$

Probabilities are defined for each side of the assembly because it is easy to handle them separately.

The partial information given as constraints is to be represented in mathematical terms. Formulation is straightforward from the normal definition of these quantities. Total of eighteen constraints are obtained as follows:

$$\frac{1}{H^{(u)}}\int_u\phi^{(1)}(x)dx=\bar{\phi}^{(1,u)}\cdot ADF^{(1,u)} \tag{4-4}$$

$$\frac{1}{H^{(u)}}\int_u\phi^{(2)}(x)dx=\bar{\phi}^{(2,u)}\cdot ADF^{(2,u)} \tag{4-5}$$

$$\frac{1}{H^{(u)}}\int_u J^{(1)}(x)dx=\bar{J}^{(1,u)} \tag{4-6}$$

$$\frac{1}{H^{(u)}}\int_u J^{(2)}(x)dx=\bar{J}^{(2,u)} \tag{4-7}$$

$$\frac{1}{H^{(u)}H^{(u')}}\int_V\phi^{(1)}(x,y)dx\,dy=\bar{\bar{\phi}}^{(1)} \tag{4-8}$$

$$\frac{1}{H^{(u)}H^{(u')}}\int V\phi^{(2)}(x,y)dx\,dy=\bar{\phi}^{(2)} \quad (4\text{–}8)$$

$$0\le P_i^{(1,u)}\le 1 \quad (4\text{–}10)$$

$$0\le P_i^{(2,u)}\le 1 \quad (4\text{–}11)$$

$$u=l,\ r,\ t,\ b$$

$\phi^{(g)}$ denotes the surface flux at point x of the boundary surface of the assembly, which is the boundary condition for the diffusion equation. $\bar{\phi}^{(g,u)}$ is the surface-averaged flux given by the nodal calculation where the "surface" means line on the boundary of the assembly in two dimensional problems.

$J^{(g)}(x)$ denotes the current at point x of the surface. $J^{(g,u)}$ is the surface-averaged flux given by the nodal calculation.

$\phi^{(g)}(x,y)$ is the pointwise flux within the assembly. $\bar{\phi}^{(g)}$ is the volume-averaged flux given by the nodal calculation.

The right-hand sides are given by results of nodal calculation and all are known terms. The left-hand sides are unknown terms as they contain the boundary conditions. Constraints (4–10) and (4–11) come from the definition that P_i is the probability. The constraints above are rewritten in the finite difference form as follows:

$$\frac{1}{H^{(u)}}\sum_{i=1}^{n(u)}\phi_i^{(1,u)}h_i^{(u)}=\bar{\phi}^{(1,u)}\cdot ADF^{(1,u)} \quad (4\text{–}12)$$

$$\frac{1}{H^{(u)}}\sum_{i=1}^{n(u)}\phi_i^{(2,u)}h_i^{(u)}=\bar{\phi}^{(2,u)}\cdot ADF^{(2,u)} \quad (4\text{–}13)$$

$$\frac{1}{H^{(u)}}\sum_{i=1}^{n(u)}-D_i^{(1)}\frac{\phi_i^{(1,u)}-\phi_{ij}^{(1,u)}}{h_i/2}h_i=\bar{J}^{(1,u)} \quad (4\text{–}14)$$

$$\frac{1}{H^{(u)}}\sum_{i=1}^{n(u)}-D_i^{(2)}\frac{\phi_i^{(2,u)}-\phi_{ij}^{(2,u)}}{h_i/2}h_i=\bar{J}^{(2,u)} \quad (4\text{–}15)$$

$$\frac{1}{H^{(u)}H^{(u')}}\sum_{i=1}^{n(u)}\sum_{j=1}^{n(u)}\phi_{ij}^{(1)}h_ih_j=\bar{\phi}^{(1)} \quad (4\text{–}16)$$

$$\frac{1}{H^{(u)}H^{(u')}}\sum_{i=1}^{n(u)}\sum_{j=1}^{n(u)}\phi_{ij}^{(2)}h_ih_j=\bar{\phi}^{(2)} \quad (4\text{–}17)$$

$$0\le P_i^{(1,u)}\le 1 \quad (4\text{–}18)$$

$$0\le P_i^{(2,u)}\le 1 \quad (4\text{–}19)$$

$$u=l,\ r,\ t,\ b$$

where j in Eqs. (4–14) and (4–15) denotes the next inner mesh to the boundary.

Note that Eq. (4–12), Eq. (4–13) and the right-hand side inequalities in Eqs. (4–18) and (4–19) hold trivially because of the definitions Eqs. (4.1) and (4.2).

The problem formulated above is mathematically an optimization problem of maximizing a non-linear objective function subject to linear equality constraints, i.e.,

$$Maximize\ S=F(\phi_i^{(g,u)}) \quad (4\text{–}20)$$

$$subject\ to\ G(\phi_i^{(g,u)})=0 \quad (4\text{–}21)$$

$$\phi_i^{(g,u)}\ge 0 \quad (4\text{–}22)$$

Note that Eq. (4–21) is a set of constraint functions that are linear (albeit, not written explicitly).

4.3 Solution of Neutron Diffusion Equation

During the procedures for solving the optimization problem defined by Eqs. (4–20) through (4–22), a number of feasible points are searched as candidates of the solution. If a new candidate of the solution is chosen, it is checked whether it satisfies the constraints and the gradient of the entropy at the point is zero, i.e., the entropy is maximized.

Constraints (4–14), (4–15), (4–16), and (4–17) contain ϕ_{ij} that is a function of the boundary conditions, $\phi_i^{(g,u)}$. $\phi_{ij}^{(g)}$ is determined by solving the neutron diffusion equation with boundary conditions, $\phi_i^{(g,u)}$. Thus, it is necessary to solve the neutron diffusion equation whenever $\phi_i^{(g,u)}$ is changed during the optimization procedures.

5. Computer Programs

5.1 Finite Difference Fine-Mesh Calculation

Solution of the neutron diffusion equation is required during the optimization procedure to provide the volume-averaged neutron fluxes and the surface-averaged neutron net currents whenever

the values of the constraints on the volume-averaged fluxes or surface-averaged currents are required to check whether a candidate of the optimal solution (boundary conditions) is truly the optimizer or not and also when the new candidate is to be searched. The problem was formulated in such a way that the neutron diffusion equation is solved with Dirichlet type boundary conditions.

The neutron diffusion equation is approximated by the finite difference fine-mesh method. A computer code for the finite difference fine-mesh calculation was written in this study following the box scheme [12] and with the following assumptions:

1) Two neutron energy groups
2) No upscattering
3) Fission occurs at both energy groups
4) Fission neutrons appear only in group 1

(Thermal neutron source is only the downscattering of the neutrons from group 1.)

The finite difference fine-mesh diffusion calculation with box scheme is

$$d_{ij}^{L}\,\bar{\phi}_{i-1,j}+d_{ij}^{R}\,\bar{\phi}_{i+1,j}+d_{ij}^{T}\,\bar{\phi}_{i,j+1}+d_{ij}^{B}\,\bar{\phi}_{i,j-1}+d_{ij}^{C}\,\bar{\phi}_{i,j}=S_{ij} \quad (5\text{–}1)$$

where superscripts L,R,T and B denote the direction that is left, right, top and bottom side of the nodes in fine mesh, respectively. The coefficients d_{ij}^{i} are the same with those described in Ref. 12. In incorporation of the Dirichlet type boundary conditions, the coefficients in Eq. (5–1) corresponding to the sides of the node are changed to the boundary conditions for the neutron diffusion equation. Also, the fluxes in Eq. (5–1) corresponding to the changed coefficients are changed to the surface fluxes on the boundary.

Finite difference diffusion equations for all nodes are cast into matrix form as

$$\underline{A}\,\underline{X}=\frac{1}{k_{eff}}\underline{F}\,\underline{X}+\underline{C}$$

Vector $\underline{C}$ contains the boundary conditions, which are the surface-averaged fluxes on the surfaces of the boundary nodes. It is easily seen from the appearance of the matrix equation that it is not an eigenvalue problem but a source problem which can be solved by inversion of the matrix $\underline{A}-\frac{1}{k_{eff}}\underline{F}$ in

$$\left[\underline{A}-\frac{1}{k_{eff}}\underline{F}\right]\underline{X}=\underline{C}$$

Matrix $\underline{A}$ and $\underline{F}$ are sparse and invariant. Only vector $\underline{C}$ varies along optimization procedure. $\underline{X}$ is to be determined according to the various $\underline{C}$. The value of k_{eff} is taken from the results of the nodal coarse-mesh calculation. Since the matrix in the parenthesis is large, a direct inversion is inefficient. Thus Gauss-Seidel iterative schemem is used.

15×15 nodes are used for a PWR assembly that is subject to reconstruction in this study. One node corresponds to one fuel cell. The configuration of a typical PWR fuel assembly is shown in Fig. 1.

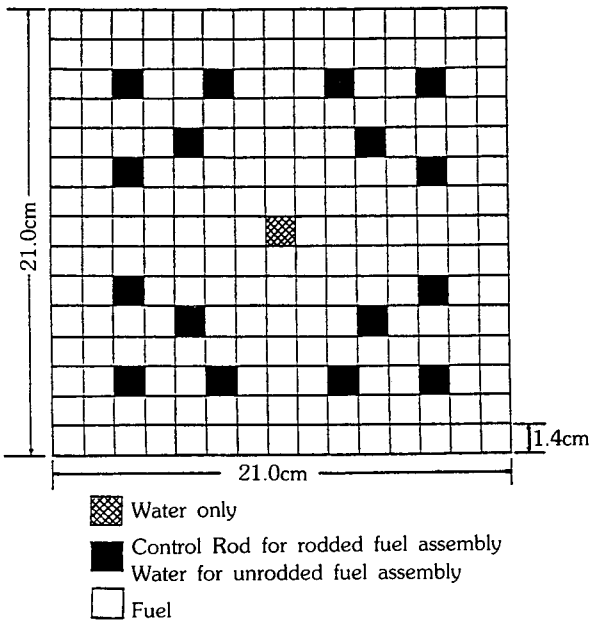


Fig. 1 Heterogeneous PWR Assembly Geometry

5.2 Optimization Program

The optimization procedure used the CONMIN algorithm. [13] The algorithm incorporates the

constraints into a modified, unconstrained objective function using the method of multipliers combined with the penalty functions. The modified objective function is then optimized by the unconstrained optimization technique of Fletcher and Powell. [14] That is, the optimization problem Eqs. (4–20) through (4–22) is transformed into

$$\textit{Maximize } \Phi = F - \sum \lambda_k G_k + B \sum G_k^2 \quad (5\text{–}2)$$

$$\textit{subject to } P_i \geq 0 \quad (5\text{–}3)$$

where λ_k are Lagrange multipliers and B is a penalty weighting parameter. These are further transformed into

$$\textit{Maximize } F = \Phi + U \times 10^{20} \quad (5\text{–}4)$$

where

$$U = \min(P_i, 0)$$

Eq. (5–4) is then optimized by the algorithm of Fletcher and Powell.

Fig.2 shows the calculational flow diagram of the maximum entropy method for reconstructing the pointwise flux distributions.

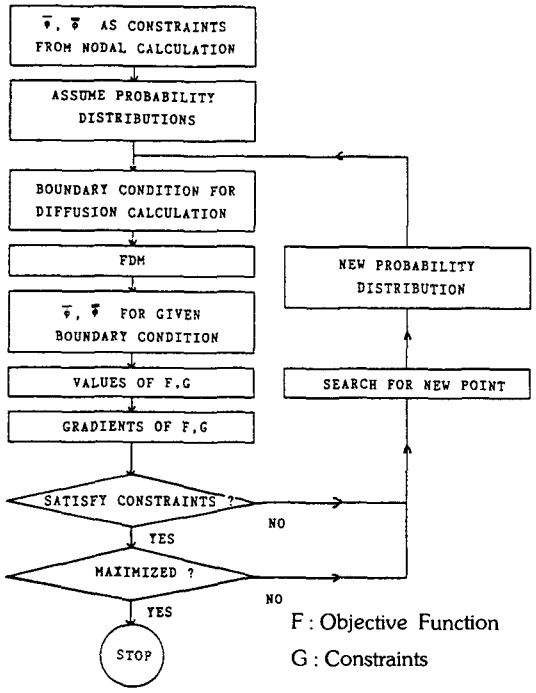


Fig. 2 Flow Chart for the Optimization Problem

6. Results and Discussions

6.1 Benchmark Problems

The reconstruction method based on the maximum entropy principle was tested [15] to Benchmark Problems 3 and 4 described in Ref. 6. Results of the nodal calculation are taken from those obtained by Jeong in his thesis. Benchmark Problems 3 and 4 were devised to simulate the inner and the peripheral region of PWR reactor cores. The configuration of a typical fuel assembly is described in Fig. 1 with dimensions. Table I shows the nuclear data for the fuel assembly in the benchmark problems. Fig. 3 shows the definition of Benchmark Problem 3.

Reference pointwise neutron fluxes are calculated by VENTURE [16] with 15×15 nodes per assembly. VENTURE solves as an option the eigenvalue problem of the neutron diffusion equation by the finite difference method.

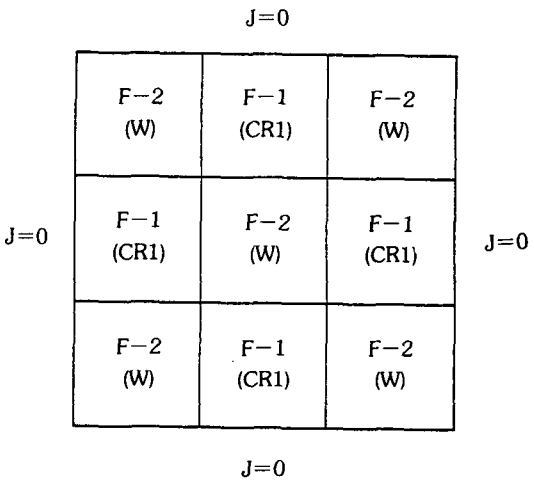


Fig. 3 Benchmark Problem 3

6.2 Results of Applications

Incorporation of the surface-averaged neutron current constraints should lead to better results than otherwise because the solution in an inverse

Table I. Heterogeneous, Pin-cell Two-group Cross Section Data for Benchmark Problems

Cross Section	g	Fuel 1	Fuel 2	Fuel 3	Fuel 4	Control Rod 1	Control Rod 2	Water
D_g	1	1,500	1.500	1.500	1.500	1.1133	1.1133	1.700
(cm)	2	0.400	0.400	0.400	0.400	0.18401	0.18401	0.350
Σ_{gg}	1	0.020	0.020	0.020	0.020	0.037529	0.0037529	0.035
(cm^{-1})	2	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Σ_{og}	1	0.013	0.010	0.010	0.011	0.049890	0.0836661	0.001
(cm^{-1})	2	0.180	0.15	0.160	0.190	0.96726	0.96726	0.05
$\nu\Sigma_{fg}$	1	0.0065	0.005	0.0065	0.0055	0.0	0.0	0.0
(cm^{-1})	2	0.240	0.180	0.240	0.2100	0.0	0.0	0.0
$\kappa\Sigma_{fg}$	1	8.850E-14	6.600E-14	8.850E-14	7.260E-14	0.0	0.0	0.0
(Ws/cm)	2	3.168E-12	2.376E-12	3.168E-12	2.772E-12	0.0	0.0	0.0

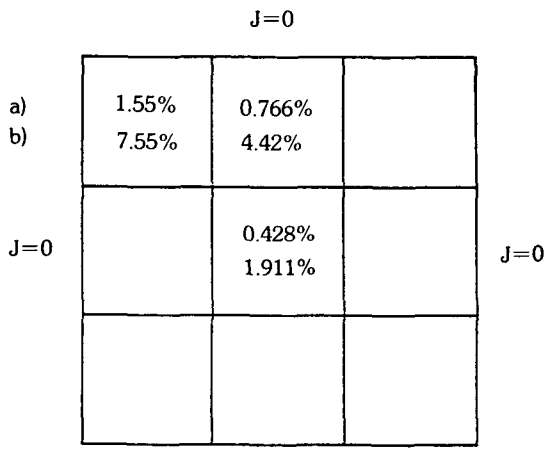
problem is closer to the true one as more information is included. However, in the present study the surface-averaged neutron currents are not included in the set of the information due to some numerical difficulties whose causes are unknown at this time.

Figs. 4 and 5 show the results of the reconstruction for each assembly in Benchmark Problem 3. The distribution of the neutron flux is symmetric to the diagonal of the problem. Results in the blank boxes can be known by the symmetric configuration of the problems. Values express the relative errors in the reconstruction:

$$Relative\ error=(\frac{value\ from\ reconstruction-value\ from\ VENTURE}{value\ from\ VENTURE}\times 100$$

In Benchmark Problem 3, the maximum error is 7.55% for the fast flux and 8.15% for the thermal flux. The larger error for the thermal flux is due to the more tilted flux than the fast flux due to its longer diffusion length. Error increases as the node is closer to the boundary. This phenomena is due to the flux tilt near the boundary and can be seen in the boundary not only of the core but also of the assembly. Similar results are obtained also for Benchmark Problems 4 and are provided in Ref. 15.

Computer time required is from 4000 to 7000 seconds in cpu time. The computer used was SUN 3.280 microsystem. Most of the time consumed is for the solution of the neutron diffusion equation by the FDM subroutine because this is called about 1000 times during the optimization procedure.



a) Average Error
b) Maximum Error

Fig. 4 Reconstruction Results for Benchmark Problem 3 (Group 1 : Fast Group)

J=0

a) b)	1.58% 8.15%	0.835% 5.59%	
J=0		0.429% 1.94%	J=0

J=0

a) Average Error

b) Maximum Error

Fig. 5 Reconstruction Results for Benchmark Problem 3 (Group 2 : Thermal Group)

6.3 Discussions

Figs. 6 and 7 show the pointwise relative errors in reconstruction for the center assembly of the Benchmark Problem 3. They show the two dimensional tendency in the distribution of errors. The "row" written on the figures means the row of the fuel cells in the assembly. Numbering starts from top and goes to bottom of the assembly. So row 1 is the top row in the assembly. The numbers in abscissa represent the column numbers of the fuel cells in the assembly. Numbering is from left to right. Node no. 1 is for the left-most node of the assembly. Row 1,2,3 and node numbers except 5 through 11 may be classified as outer region of the assembly. The other rows and nodes may be classified as inner region.

Seen from the figures is the deviation of the relative errors that become larger as the node gets to be closer to the boundary of the assembly. This tendency is common in reconstruction of pointwise quantities.[6]

The apparently excessive errors should be remedied by incorporating the surface-average current constraints which was not done in this study. The currents have information on the gradients of the fluxes. Thus, if the current information at the boundary is included in the maximum entropy method of reconstruction, the resulting neutron flux would be better shaped near the boundary of the assembly. However, the computing time will be increased accordingly as more constraints are included in the optimization procedure.

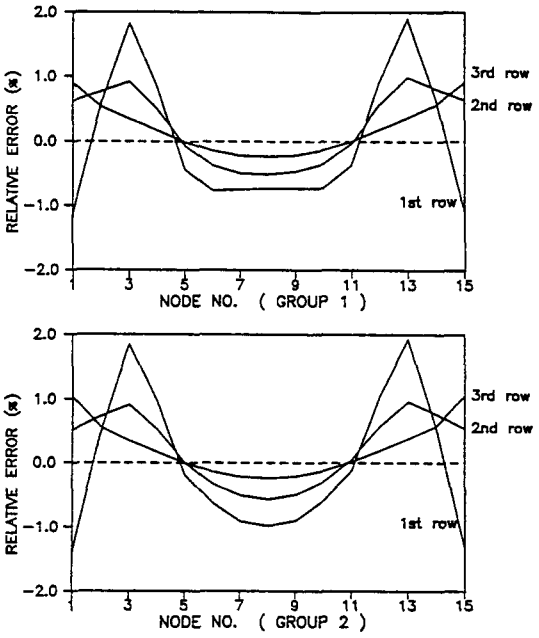
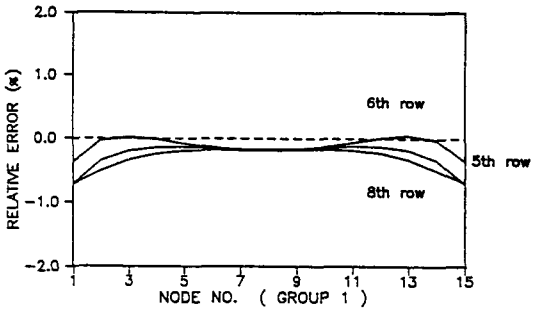


Fig. 6 Results of Reconstruction, Pointwise Errors for the Center Node of Benchmark Problem 3 (Outer Region)



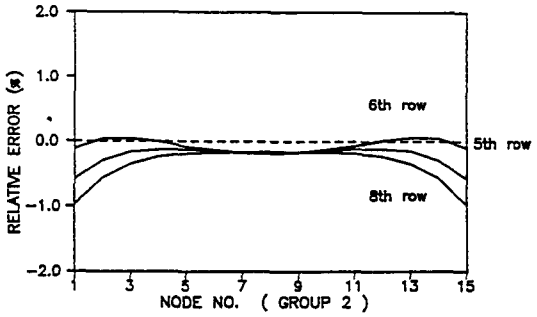


Fig. 7 Results Reconstruction, Pointwise Errors for the Center Node of Benchmark Problem 3 (Inner Region)

7. Conclusions and Recommendations

A new method based on the maximum entropy principle in information theory showed possibility of reconstruction of the distribution of pointwise neutron fluxes. The maximum entropy method works reasonably well in the pointwise flux reconstruction which is characterized as an inverse problem. The errors in reconstruction are comparable with those of the form function methods in inner region of the core or the assembly. [6] However, they are large in the boundary region of the core or the assembly.

In this study, surface-averaged neutron currents are not included in the constraints, although they are available information given by the nodal calculation. Inclusion of the surface-averaged currents into the constraints would provide better results.

The computing time was long. Most of the computing time were spent in the solution of the diffusion equation by the finite difference fine-mesh method. The FDM subroutine is called about 1000 times to obtain the solution. It is recommended to replace the expensive finite difference calculation with a computationally efficient method for solution of the neutron diffusion diffusion equation.

Acknowledgements

We like to express gratitude to Hun Young Jeong of Korea Electric Power Corporation for his valuable help during the course of the work.

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