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Observer Theory Applied to the Optimal Control of Xenon Concentration in a Nuclear Reactor

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옵저버 이론의 원자로 지논 농도 최적제어에의 응용

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Abstract

The optimal control of xenon concentration in a nuclear reactor is posed as a linear quadratic regulator problem with state feedback control. Since it is not possible to measure the state variables such as xenon and iodine concentrations directly, implementation of the optimal state feedback control law requires estimation of the unmeasurable state variables. The estimation method used is based on the Luenberger observer. The set of the reactor kinetics equations is a stiff system. This singularly perturbed system arises from the interaction of slow dynamic modes (iodine and xenon concentrations) and fast dynamic modes (neutron flux, fuel and coolant temperatures). The singular perturbation technique is used to overcome this stiffness problem. The observer-based controller of the original system is effected by separate design of the observer and controller of the reduced subsystem and the fast subsystem. In particular, since in the reactor kinetics control problem analyzed in the study the fast mode dies out quickly, we need only design the observer for the reduced slow subsystem. The results of the test problems demonstrated that the state feedback control of the xenon oscillation can be accomplished efficiently and without sacrificing accuracy by using the observer combined with the singular perturbation method.

요 약

원자로 지논 농도의 최적 제어는 Linear Quadratic Regulator Problem이다. 지논 농도와 아이오다인 농도는 측정할 수 없기 때문에 최적 제어를 수행 하기 위해서는 측정할 수 없는 상태 변수를 예측하는 것이 필요하다. 본 연구에서 사용된 예측방법은 Luenberger Observer를 기초로 했다. 원자로 상태 방정식은 빠른 상태 방정식(중성자 속, 핵연료 및 냉각재 온도)과

느린 상태 방정식(아이오다인, 지논)의 상호작용에 의해 Stiffness 문제가 발생되는데 이러한 시스템을 "Singularly Perturbed System"이라 한다. Stiffness문제를 해결하기 위해서 원 시스템을 느린 시스템과 빠른 시스템의 두 개의 모드로 나누는 Singular Perturbation Method를 사용한다. 예측기Observer를 이용한 원 시스템의 제어기는 느린 시스템과 빠른 시스템에 대한 분리된 예측기와 제어기의 설계에 의해 결정 되어진다. 특히 원자로 상태 방정식에서는 빠른 모드는 빨리 사라지게 되므로 단지 느린 시스템에 대해서만 예측기를 설계하면 된다. 콤퓨터 시뮬레이션을 통한 시험 결과는 원자로의 지논 진동은 Singular Perturbation Method와 예측기를 이용해서 거의 정확하게 효과적으로 짧은 시간내에 제어할 수 있음을 알았다.

1.Introduction

After the accidents at the Three Mile Island nuclear reactor and other facilities, there has been a growing interest in the performance prediction and control of nuclear power plants. In designing a nuclear power plant it is necessary to predict and control the performance of the plant under various dynamic conditions. In a large thermal reactor operating at a neutron flux in excess of about 10^{13} *neutrons/cm²*sec, the most important fission product poison is Xe-135 because of its exceptionally large capture cross section.

The physical explanation of how xenon can induce spatial power oscillations in a nuclear reactor is well described in textbooks on nuclear reactor physics. [1–3] Growing and serious power oscillations in a reactor can cause changes in the power-peaking factor. Thus control of the xenon oscillations is important not only for safety but also for economical operation of a nuclear power plant.

In this paper, the estimation and the optimal control of xenon concentration in a nuclear reactor is considered. The basic approach to the problem is the linear optimal control theory in state space. State-variable representation results in a model in terms of first-order differential equations.

In nuclear reactors, it is not possible to measure the state variables such as xenon concentration and iodine concentration directly. For the implementation of the optimal state feedback control law, it is thus necessary to estimate the unmeasurable state variables. [4,5] The method used in this paper to estimate the xenon and iodine concentrations is based on the Luenberger observer. [6] The optimal control of xenon concentration is posed as a linear quadratic problem with state feedback control. The control input information is obtained from the observer. In this study we are not partcularly concerned with the stability itself of the reactor, but we attempt rather to find the "best" control strategy in the context of modern optimal control theory. [7]

In numerical analysis of reactor kinetics, the system of equations is stiff. [8,9] This singularly perturbed system arises from the interaction of slow and fast dynamic modes. This problem requires expensive integration routines. As a tool to overcome the stiffness problem, the singular perturbation method is used. [10–12] The singular perturbation approach alleviates both high dimensionality and stiffness. It reduces the model order by neglecting the fast phenomena. It then improves the approximation by reintroducing their effect as "boundary layer" corrections calculated in separate time scales.

2. Reactor Kinetics Model

2.1 Reactor Kinetics [3,13]

Spatial-independence and a one-energy-group reactor are assumed in this study. The point kinetics equation used in this study is

$$\frac{1}{\upsilon}\frac{d\phi}{dt}=\frac{\rho}{\upsilon \Lambda}\phi-\sigma_X X\phi .$$

The reactivity ρ is made up of several components: Reactivity due to control rod motion, reactivities due to fuel temperature and coolant temperature. That is

$$\rho = \rho_{rod} - \alpha_f(T_f - T_{f0}) - \alpha_c(T_c - T_{c0})$$

where

ρ = reactivity

ϕ = thermal neutron flux

X = concentration of Xe^{135} nuclei per cubic centimeter

v = thermal neutron velocity

Λ = neutron generation time

α_c = coolant temperature coefficient

α_f = fuel temperature coefficient.

The fuel and coolant temperatures are governed by the following heat transfer model

$$\rho_f C_{pf} \frac{d}{dt} T_f = \varepsilon \Sigma_f \phi - h(T_f - T_c)$$

$$\rho_c C_{pc} \frac{d}{dt} T_c = h(T_f - T_c) - [\omega_v C_p]_c (T_c - T_{in})$$

where

T_f = average fuel temperature

T_c = average coolant temperature

T_{in} = inlet coolant temperature

C_{pf} = specific heat of fuel

C_{pc} = specific heat of coolant

h = heat transfer coefficient per volume

$(\omega_v)_c$ = coolant flow rate to volume ratio

ε = fission to heat energy conversion factor

ρ_f = average fuel density

ρ_c = average coolant density.

The xenon and iodine concentration are governed as follows.

$$\frac{d}{dt} I = \gamma_I \Sigma_f \phi - \lambda_I I$$

$$\frac{d}{dt} X = \gamma_X \Sigma_f \phi + \lambda_I I - \lambda_X X - \sigma_X X \phi$$

where

I = concentration of I^{135} nuclei per cubic centimeter

X = concentration of Xe^{135} nuclei per cubic centimeter

λ_I = decay constant of I^{135}

σ_I = microscopic capture cross section of I^{135} for thermal neutrons

σ_X = microscopic capture cross section of Xe^{135} for thermal neutrons

Σ_f = macroscopic fission cross section

Σ_a = macroscopic absorption cross section

γ_I = fractional yield of I^{135} from fission

γ_X = fractional yield of Xe^{135} from fission.

Initial conditions are required for the above kinetics equations to be well defined.

2.2 Linearized Equations of Reactor Kinetics

For many cases, we are interested in time behavior of the small perturbations in a nuclear reactor which is operated in a steady state condition. The steady state is obtained by setting time derivatives to zero.

$$0 = \frac{\rho_0}{\Lambda} \phi_0 - v \sigma_X X_0 \phi_0$$

$$0 = \varepsilon \Sigma_f \phi_0 - h(T_{f0} - T_{c0})$$

$$0 = h(T_{f0} - T_{c0}) - [\omega_v C_p]_c (T_{c0} - T_{in0})$$

$$0 = \gamma_I \Sigma_f \phi_0 - \lambda_I I_0$$

$$0 = \gamma_X \Sigma_f \phi_0 + \lambda_I I_0 - \lambda_X X_0 - \sigma_X X_0 \phi_0$$

The steady state condition is related to flux level as follows.

$$\rho_{rod0} = \sigma_X v \Lambda X_0$$

$$T_{f0} - T_{c0} = \frac{\varepsilon \Sigma_f \phi_0}{h}$$

$$T_{c0} - T_{in0} = \frac{\varepsilon \Sigma_f \phi_0}{[\omega_v C_p]_c}$$

$$I_0 = \frac{\gamma_I \Sigma_f \phi_0}{\lambda_I}$$

$$X_0 = \frac{(\gamma_X + \gamma_I) \Sigma_f \phi_0}{\lambda_X + \sigma_X \phi_0}$$

We then linearize the reactor equations around the steady state as follows.

$$\frac{1}{v} \frac{d}{dt} \delta \phi = \left(\frac{\rho_0}{v \Lambda} - \sigma_X X_0 \right) \delta \phi - \frac{\alpha_f}{v \Lambda} \phi_0 \delta T_f - \frac{\alpha_c \phi_0}{v \Lambda} \delta T_c - \alpha_X \phi_0 \delta X + \frac{\phi_0}{v \Lambda} \delta \rho_{rod}$$

$$\frac{d}{dt}\delta T_f=\frac{\varepsilon \Sigma}{\rho_f C_{pf}}\delta \phi-\frac{h}{\rho_f C_{pf}}(\delta T_f-\delta T_c)$$

$$\frac{d}{dt}\delta T_c=\frac{h}{\rho_c C_{pc}}(\delta T_f-\delta T_c-\frac{\omega_V}{\rho_c}(\delta T_c-\delta T_{in})$$

$$\frac{d}{dt}\delta I=\gamma_I\Sigma_f\delta \phi-\lambda_I\delta I$$

$$\frac{d}{dt}\delta X=(\gamma_X\Sigma_f-\delta_X X_0)\delta \phi+\gamma_I\delta I-(\lambda_X+\sigma_X \phi_0)\delta X$$

We can represent these equations in matrix forms as follows...

$$\dot{x}=Ax+Bu$$

$$y=Cx$$

where

$$x=\left[\frac{\delta \phi}{\upsilon}\ \delta T_f\ \delta T_c\ \delta I\ \delta X\right]^T$$

$$u=[\delta \rho_{rod}\ \delta T_{in}]^T$$

$$y=\left[\frac{\delta \phi}{\upsilon}\ \delta T_c\right]^T$$

$$A=\begin{bmatrix} \frac{\rho_0}{A}-\sigma_X X_0\upsilon & \frac{\alpha_f}{\upsilon A}\phi_0 & -\frac{\alpha_c}{\upsilon A}\phi_0 & 0 & -\sigma_X\phi_0 \\ \varepsilon\Sigma_f\frac{\upsilon}{\rho_f C_{pf}} & -\frac{h}{\rho_f C_{pf}} & \frac{h}{\rho_f C_{pf}} & 0 & 0 \\ 0 & \frac{h}{\rho_c C_{pc}} & -\frac{h}{\rho_c C_{pc}}-\frac{\omega_V}{\rho_c} & 0 & 0 \\ \gamma_I\Sigma_f\upsilon & 0 & 0 & -\lambda_I & 0 \\ \gamma_X\Sigma_f\upsilon-\sigma_X X_0\upsilon & 0 & 0 & \gamma_I & \lambda_X+\sigma_X\phi_0 \end{bmatrix}$$

$$B=\begin{bmatrix} \frac{\phi_0}{\upsilon A} & 0 \\ 0 & 0 \\ 0 & \frac{\omega_V}{\rho_c} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

with inital pertubations are provided as initial conditions $x(0)$.

3. Optimal Control Theory

The objective of optimal control theory is to determine the control inputs that will cause a process to satisfy the physical constraints and at the same time minimize (or maximize) some performance criterion. The references on the optimal control theory are rich in the literature. [4–5]

3.1 Optimal Feedback Control with Full State Measurement

Suppose the system is described by the linear state equation

$$\dot{x}=Ax+Bu, \qquad x(0)=x_0 \tag{3.1}$$

and the performance index to be minimized is

$$J=\frac{1}{2}x^T(t_f)Hx(t_f)+\frac{1}{2}\int_{t=0}^{t=t_f}(x^T(t)Q(t)x(t)+u^T(t)R(t)u(t))dt \tag{3.2}$$

where the final time t_f is fixed, H and Q are real symmetric positive semidefinite $n\times n$ matrices and R is a real symmetric positive definite $m\times m$ matrix. It is assumed that the states and controls are not bounded, and $x(t_f)$ is free. Here n is the dimension of state vector x and m is the dimension of input u.

The Hamiltonian is defined as

$$H[x(t),\ u(t),\ p(t),\ t]=\frac{1}{2}x^T(t)Q(t)x(t)+\frac{1}{2}u^T(t)R(t)u(t)+p^T(t)A(t)x(t)+p^T(t)B(t)u(t). \tag{3.3}$$

Then the necessary conditions for optimality are

$\dot{x}(t)=A(t)x(t)+B(t)u(t),\ x(0)=x_0$ (3.4)

$\dot{p}(t)=-Q(t)x(t)-A^T(t)p(t),\ p(t_f)=0$ (3.5)

$0=R(t)u(t)+B^T(t)p(t).$ (3.6)

Equation(3.6) can be solved for $u(t)$ to givge

$u(t)=-R^{-1}(t)B^T(t)p(t)$ (3.7)

where the existence of R^{-1} is assured since R is a positive definite matrix. Substituting (3.7) into (3.4) yields

$\dot{x}(t)=A(t)x(t)-B(t)R^{-1}(t)B^T(t)p(t)$ (3.8)

The augmented homogeneous differential equations can be written as

$$\begin{bmatrix} \dot{x}(t) \\ \dot{p}(t) \end{bmatrix}=\begin{bmatrix} A(t) & -B(t)R^{-1}(t)B^T(t) \\ -Q(t) & -A^T(t) \end{bmatrix}\begin{bmatrix} x(t) \\ p(t) \end{bmatrix} \quad (3.9)$$

We can represent $p(t)=K(t)x(t)$ (3.10)

where K is an $n\times n$ matrix satisfying the following Riccati equation

$\dot{K}(t)=-K(t)A(t)-A^T(t)K(t)-Q(t)+$
$K(t)B(t)R^{-1}(t)B^T K(t)$ (3.11)

with the boundary condition $K(t_f)=H$ and K is symmetric. Substituting (3.10) into (3.7), we obtain

$u(t)=-F(t)x(t)$ (3.12)

where

$F(t)=R^{-1}(t)B^T(t)K(t)$ (3.13)

Eq.(3.12) indicates that the optimal control law is a linear, albeit time-varying, combination of the system states. In addition, in order to implement the optimal control law, it is necessary that all state variables must be measured.

3.2 Optimal Control Problem with Incomplete State Measurement

When state-variable is not completely measurable, the optimal control input is obtained by

$u=-F\hat{x}$ (3.14)

where F is given in the same way as in Section 3.1 by the Eqs.(3.11) and (3.13), and $\hat{x}$ is the solution of the observer dynamics equation proposed by Luenberger[6]

$\dot{\hat{x}}A\hat{x}+Bu+G[y-C\hat{x}]$ (3.15)

G in Eq.(3.15) is called the observer gain matrix to be determined.

Using Eq.(3.14) and (3.15), the augmented system equation for x and $\hat{x}$ is represented as follows.

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix}=\begin{bmatrix} A & -BF \\ GC\ A & -BF-GC \end{bmatrix}\begin{bmatrix} x \\ \hat{x} \end{bmatrix} \quad (3.16)$$

Let

$e=x-\hat{x}.$

Then e is the difference between the true state x and the reconstructed state $\hat{x}$ estimated by the observer equation. It is easily seen that e satisfies the differential equation

$\dot{e}(t)=(A-GC)e(t)$ (3.17)

Eq.(3.17) indicates that the observer gain matrix G can be chosen such that the reconstruction error has desirable dynamic characteristics. For example, G is chosen such that the eigenvalues of $A-GC$ have large negative values. Then the reconstruction error will disappear quickly. Substitution of $\hat{x}=x-e$ into (3.1) using (3.14) yields

$\dot{x}=(A-BF)x+BFe.$ (3.18)

The augmented equation of x and e is then represented as

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix}=\begin{bmatrix} A-BF & BF \\ 0 & A-GC \end{bmatrix}\begin{bmatrix} x \\ e \end{bmatrix} \quad (3.19)$$

The characteristic equation of the system becomes

$$\det\begin{bmatrix} sI-A+BF & -BF \\ 0 & sI-A+GC \end{bmatrix}=$$
$$\det(sI-A+BF)\cdot\det(sI-A+GC) \quad (3.20)$$

Figure 1 shows the structure of an output feedback control system.

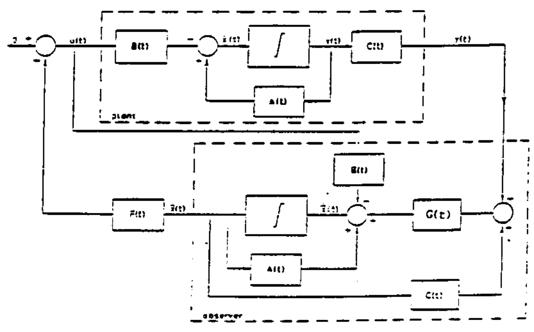

Fig.1. The Structure of an Output Feedback Control System

4. Singular Perturbation Theory

The point reactor kinetics model described in Section 2 involves characteristic time constants of widely varying magnitude due to the coexistence of slow and fast dynamic phenomena. This results in severe restriction of the time step used in numerical solution schemes directly applied to the kinetic equations.

The primary purpose of singular perturbation approach is the alleviation of the high dimensionality and ill-conditioning resulting from the coupling of slow and fast dynamic modes. [10–12]

This time-scale approach is asymptotic, that is, becomes exact in the limit as the ratio μ of the speeds of the slow versus the fast dynamics tends to zero. When μ is small, approximations are obtained from reduced-order models in separate time scales.

4.1 Two-Time-Scale System

Many systems can be modeled by the set of nonlinear differential equations as follows.

$$\dot{x}_1 = f(x_1, x_2, t),\ x_1(t_0) = x_{10} \tag{4.1a}$$

$$\dot{x}_2 = G(x_1, x_2, t),\ x_2(t_0) = x_{20} \tag{4.1b}$$

where the n_1-dimensional vector x_1 is predominantly slow and the n_2-dimensional vector x_2 contains fast transient superimposed on a slowly varying "quasi-steadystate"; that is, $\|\dot{x}_1\| \ll \|\dot{x}_2\|$. One way to express this fact is to introduce $g = \mu G$ and thus scale g to be of the same order of magnitude with f. The scaling parameter μ is the speed ratio of the slow versus fast phenomena and multiplies the derivative of x_2,

$$\dot{x}_1 = f(x_1, x_2, t),\ x_1(t_0) = x_{10} \tag{4.2a}$$

$$\mu \dot{x}_2 = g(x_1, x_2, t),\ x_2(t_0) = x_{20} \tag{4.2b}$$

This is the standard singular perturbation problem studied extensively in the literature. [10–12]

● Mode Separation of the Singularly Perturbed Linear System

Consider the singularly perturbed linear time-varying system given by

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 + B_1 u,\ x_1(t_0) = x_{10} \tag{4.3a}$$

$$\mu \dot{x}_2 = A_{21}x_1 + A_{22}x_2 + B_2 u,\ x_2(t_0) = x_{20} \tag{4.3b}$$

$$y = C_1 x_1 + C_2 x_2 \tag{4.3c}$$

where $\mu > 0$ is a small singular perturbation parameter, $x_1(t)$ and $x_2(t)$ are n_1 and n_2– dimensional state vectors respectively, $u(t)$ is an r-dimensional control vector, $y(t)$ is an m-dimensional measurement vector, and t_0 is any initial time.

The reduced subsystem of the original system (4.3) is obtained by neglecting the fast mode dynamics, i.e., setting $\dot{x}_2 = 0$:

$$\dot{x}_s = A_{11}x_s + A_{12}\bar{x}_2 + B_1 u_s,\ x_s(t_0) = x_{10} \tag{4.4a}$$

$$0 = A_{21}x_s + A_{22}\bar{x}_2 + B_2 u_s \tag{4.4b}$$

$$y_s = C_1 x_s + C_2 \bar{x}_2. \tag{4.4c}$$

If A_{22}^{-1} exists, $\bar{x}_2$ is solved from (4.4b) and then substituted in (4.4a) and (4.4c) to obtain the slow mode behavior

$$\dot{x}_s = A_0 x_s + B_0 u_s,\ x_s(t_0) = x_{10} \tag{4.5a}$$

$$y_s = C_0 x_s + D_0 u_s \tag{4.5b}$$

where

$$A_0 = A_{11} - A_{12}A_{22}^{-1}A_{21}$$

$$B_0 = B_1 - A_{12}A_{22}^{-1}B_2$$

$$C_0 = C_1 - C_2 A_{22}^{-1}A_{21}$$

$$D_0 = -C_2 A_{22}^{-1}B_2.$$

The fast mode behavior is governed by

$$\frac{d}{d\tau} x_f(\tau) = A_{22}x_f(\tau) + B_2 u_f(\tau),$$

$$x_f(0) = x_2(t_0) - \bar{x}_2(t_0) \tag{4.6a}$$

$$y_f(\tau) = C_2 x_f(\tau) \tag{4.6b}$$

where τ is the "fast time-scale" defined by

$$\tau = \frac{t - t_0}{\mu} \tag{4.6c}$$

and x_f is the "boundary layer" of the fast mode defined by

$$x_f = x_2 - \bar{x}_2. \tag{4.6d}$$

Thus the singularly perturbed perturbed system (4.3) of dimension n is decomposed into two lower-order subsystems. Thus we know that if A_{22} is a stability matrix then two-time-scale approximation of the state (4.3a) and (4.3b) is

$$x_1(t) = x_s(t) + O(\mu)$$

$$x_2(t) = \bar{x}_2(t) + x_f(\tau) + O(\mu)$$

where $x_s(t)$ and $\bar{x}_2(t)$ are the limits as $\mu \to 0$ of the exact slow and fast state of $x_1(t)$ and $x_2(t)$, respectively. $x_f(\tau)$ decays rapidly in the initial "boundary layer" interval after which the system response is essentially due to $x_s(t)$ and $\bar{x}_2(t)$.

4.2 Observer-Based Optimal Requlators for Singularly Perturbed Systems

The singularly perturbed system (4.3) can be written as

$$\dot{x}=\tilde{A}x+\tilde{B}u,\ x(t_0)=x_0 \tag{4.7a}$$

$$y=\tilde{C}x \tag{4.7b}$$

where

$$\tilde{A}=I_{\frac{1}{\mu}}A$$

$$\tilde{B}=I_{\frac{1}{\mu}}A$$

$$\tilde{C}=[C_1C_2]$$

$$x=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

and

$$I_{\frac{1}{\mu}}=\begin{bmatrix} I_{n1} & 0 \\ 0 & \frac{1}{\mu}I_{n2} \end{bmatrix}$$

$$A=\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$B=\begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

As shown in Section 3, the full-order observer can be designed to have arbitrary dynamics if the original system is completely observable. For the system described by the dynamics equations (4.7a) (4.7b) to be completely observable, it is necessary and sufficient that the following $n \times nm$ matrix has rank of n :

$$V=[\tilde{C}^T\ \tilde{A}^T\ \tilde{C}^T \cdots (\tilde{A})^{n-1}\tilde{C}^T].$$

It has been shown [10] that the state reconstruction of the original system Eq.(4.3) [or equivalently Eq.(4.7)] can be effected by separate observer design of the reduced (slow) subsystem Eq.(4.5) and the fast subsystem Eq.(4.6) in different time scales. This mode separation forms the basis of observer-based controller analysis and design in the following sections.

4.2.1 An Observer-Based Controller for the Reduced System

And observer-based controller for the reduced system is obtained from the optimal control theory as

$$u_s=-F_0\hat{x}_s \tag{4.8}$$

where $\hat{x}_s$ is an estmate of x_s and is generated by the full-order observer for the reduced (slow) subsystem

$$\dot{\hat{x}}_s=(A_0-G_0C_0)\hat{x}_s+G_0y_s+(B_0-G_0D_0)u_s. \tag{4.9}$$

The state reconstruction error defined by $e_s(t)=\hat{x}_s-x_s$ satisfies

$$\dot{e}_s=(A_0-G_0C_0)e_s,$$
$$e_s(t_0)=\hat{x}_s(t_0)-x_0(t_0) \tag{4.10}$$

where G_0 is chosen such that it is asymptotically stable.

The optimal control gain F_0 in Eq.(4.8) is given as

$$F_0=-R_s^{-1}B^T_0K_s(t) \tag{4.11}$$

where $K_s(t)$ is the solution of the Riccati equation

$$\dot{K}_s=-A^T_0K_s-K_sA_0+K_sB_0R^{-1}_sB^T{}_0K_s-Q_s,\quad K_s(t_f)=0. \tag{4.12}$$

The augmented equation of control and observer dynamics is

$$\begin{bmatrix} \dot{x}_s \\ \dot{e}_s \end{bmatrix}=\begin{bmatrix} A_0-B_0F_0 & B_0F_0 \\ 0 & A_0-G_0C_0 \end{bmatrix}\begin{bmatrix} x_s \\ e_s \end{bmatrix} \tag{4.13}$$

The reduced system is uniformly completely stabilizable by the controller, i.e.,

$$\lim_{t=0}\begin{bmatrix} x_s(t) \\ e_s(t) \end{bmatrix}=0 \tag{4.14}$$

4.2.2 An Observer-Based Controller for the Fast System

An observer-based controller for the fast system is

$$u_f(\tau)=-F_2(t)\hat{x}_f(\tau) \tag{4.15}$$

where $\dot{\hat{x}}_f(\tau)=(A_{22}-G_2C_2)\hat{x}_f+G_2y_f(\tau)+B_2u_f(\tau)$ (4.16)

The optimal control gain F_2 is given as

$$F_2(t)=-R_2^{-1}B^T_2K_f(t) \tag{4.17}$$

where $K_f(t)$ is the solution of the Riccati equation

$$\dot{K}_f=-A^T_{22}K_f-K_fA_{22}+K_fB_2R_f^{-1}B_2^TK_f-Q_f,$$

$K_f(t_f)=0.$ (4.18)

Since the state reconstruction of the fast subsystem Eq.(4.16) occurs in a $1/\mu$ faster time scale than that of the slow subsystem Eq.(4.9), the response of the observer of the original dynamics system Eq.(4.3) will be dominated by the observer Eq.(4.9) after the decay of fast observer transients. In particular, if A_{22} is uniformly asymptotically stable, we need only design an observer Eq.(4.9) for the reduced subsystem.

5. Results and Discussion

In a nuclear reactor, there is ill-conditioning in the dynamics equation which arises from coupling of fast and slow dynamics. So we partition the system equation into two subsystems of slow and fast modes using singular perturbation approach. Computing time is reduced remarkablely and the results that are obtained from the two-time-scale system are in good agreement with those of the original system solved directly. The fast mode subsystem is stable (i.e., A_{22} is stable) so the solution of the reduced subsystem is accurate to the solution of the original system, except the initial conditions. But the states of the reduced subsystem is impossible to measure. So we estimate the states using the observer.

Test problems demonstrate that the singularly perturbed system with some unmeasurable states can be controlled optimally with observer. The first test problem is an unstable system. The iodine and xenon concentrations and thus the neutron flux oscillate, which will affect safety of the reactor. The second test problem is a stable system. The settling time of the problem is, however too long so that control is needed. Only the results of the first test problem are presented in this paper. The results of the second test problem are available in Ref. 14.

The first test problem is to obtain the optimal input which minimizes the performance index J

$$J=\int_{t0}^{tf}(x_4^2+x_5^2+1.0\times10^{27}u_1^2+1.0\times10^{19}u_2^2)dt$$

where the reactor dynamics equation is represented in matrix form as(see Section 2)

$$\dot{x}=Ax+Bu$$
$$y=Cx$$

where

$$x=\left[\frac{\delta\phi}{v}\ \delta T_f\ \delta T_c\ \delta I\ \delta X\right]^T$$

$$u=[\delta\rho_{rod}\ \delta T_{in}]^T$$

$$y=\left[\frac{\delta\phi}{v}\ \delta T_c\right]^T$$

We use the equilibrium condition of the state variables in Table I, and the values of other parameters in the reactor dynamics equations are shown in Table II. The elements of the matrix A and B are obtained as

$$A=\begin{bmatrix} 0 & -0.5727\times10^{9} & -0.3682\times10^{9} & 0 & -0.1750\times10^{-3} \\ 0.1132\times10^{5} & -0.4284 & 0.4284 & 0 & 0 \\ 0 & 0.23 & -0.4836 & 0 & 0 \\ 2495 & 0 & 0 & -0.29\times10^{-4} & 0 \\ -2213 & 0 & 0 & 0.29\times10^{-4} & -0.1960\times10^{-3} \end{bmatrix}$$

$$B=\begin{bmatrix} 0.4545\times10^{14} & 0 \\ 0 & 0 \\ 0 & 4.606 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Table I. Equilibrium State Values

State Variable	Value
ϕ_0	5.0×10^{13} $/cm^2\cdot$sec
T_{f0}	1200 ^0F
T_{c0}	600 ^0F
T_{in0}	570 ^0F
I_0	1.96×10^{16} $/cm^3$
X_0	3.05×10^{15} $/cm^3$

The poles of the dynamics equation are

$\lambda_1 = -0.1424 + j0.25\times10^{+2}$

$\lambda_2 = -0.1424 - j0.25\times10^{+2}$

$\lambda_3 = -4.97$

$\lambda_4 = -0.576\times10^{-5} + j0.808\times10^{-4}$

$\lambda_5 = -0.576\times10^{-5} - j0.808\times10^{-4}$

λ_4 and λ_5 are located in the right half plane. Thus the system is unstable and the magnitudes of λ_4 and λ_5 are too small in comparison with the other values such that we can apply the two-time-scale method of the singular perturbation approach. The slow and fast modes are as the following :

slow mode $x_1 = [\delta I \;\; \delta X]^T$

fast mode $x_2 = \left[\frac{\delta\phi}{\upsilon} \;\; \delta T_f \;\; \delta T_c\right]^T$

with $\mu = 10^{-5}$.

Table II. Parameters of Neutron Kinetics

Parameter	Value	Parameter	Value
α_c	0.81×10^{-5} $\Delta k/k\cdot{}^0F$	α_f	1.26×10^{-5} $\Delta k/k\cdot{}^0F$
h	0.54 $J/cm^{3}\cdot{}^oF\cdot$sec	Λ	5×10^{-6}sec
ρ_f	10.96 g/cm^3	ρ_c	0.697 g/cm^3
C_{pf}	0.115 $J/g\cdot{}^0F$	C_{pc}	3.368 $J/g\cdot{}^0F$
ε	$3.204\times10^{-11}J/fission$	υ	2200 m/sec
γ_I	0.056	γ_X	0.003
λ_I	2.9×10^{-5}sec^{-1}	λ_X	2.1×10^{-5}sec^{-1}
σ_X	$3.5\times10^{-18}cm^2$	Σ_f	$0.2025cm^{-1}$

Table III shows the initial perturbations to the dynamics system simulated in this test problem.

Table III. Initial Perturbations

State Variable	Initial Perturbation
$\frac{\delta\phi}{\upsilon}(0)$	$0\ /cm^3$
$\delta T_f(0)$	$0\ ^0F$
$\delta T_c(0)$	$0\ ^0F$
$\delta I(0)$	$1.0\times10^8\ /cm^3$
$\delta X(0)$	$1.0\times10^9\ /cm^3$

We need focus only on the reduced system. If the state variables I and Xe concentrations are measurable, we can use them for optimal state feedback control. But it is not possible to measure these state variable, so an observer is used. The observer gain is obtained by the pole assignment method such that

$\lim e_s(t)=0$

The chosen desired poles of the observer are -65.1 and -4.9. Then the observer gain matrix becomes

$$G_0=\begin{bmatrix}-1.0\times10^{14} & 0\\ 0 & -5.0\times10^{15}\end{bmatrix}$$

The next step is to obtain the optimal feedback control gain matrix F_0 assuming the weighting matrix Q_s and R_s as

$$Q_s=\begin{bmatrix}1.0 & 0\\ 0 & 1.0\end{bmatrix},$$

$$R_s=\begin{bmatrix}1.0\times10^{27} & 0\\ 0 & 1.0\times10^{19}\end{bmatrix},$$

The steady-state solution of the Riccati equation is obtained using Kalman-Englar iterative method [4] as

$$K_s=\begin{bmatrix}30.4 & 33.9\\ 33.9 & 38.6\end{bmatrix}.$$

So the optimal feedback gain is obtained as

$$F_0=R_s^{-1}B^T{}_0K_s$$

$$=\begin{bmatrix}0.28\times10^{-3} & -0.16\times10^{-13}\\ 0.16\times10^{9} & 0.2710^{-9}\end{bmatrix}$$

Figures 2a through 2c show the results of optimal control assuming as if all the state variables are measurable. Now Figures 3a through 3c show the corresponding results of optimal control when the observer is used for the state variables (I and Xe) that are not measurable. We note in Figures 4a and 4b that the errors between the real states obtained with the observer based feedback and the setimated states provided by the observer. The observer catches up with the actual behavior of the states very quickly. Because of the slight time lag introduced by the observer however, an increased input is required and the response is different from that of the system without observer in the early phase of the transients.

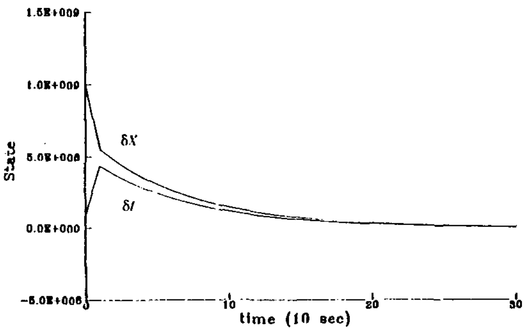

Figure 2a Response of the Controlled System Assuming States are Measurable

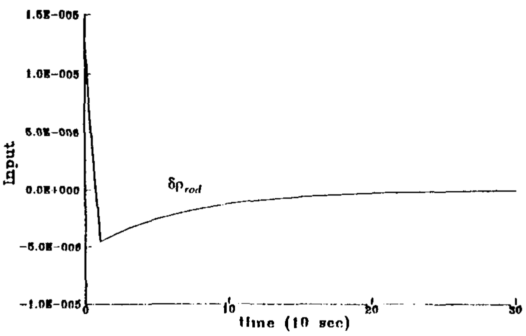

Figure 2b Optimal Control Input Obtained Assuming States are Measurable

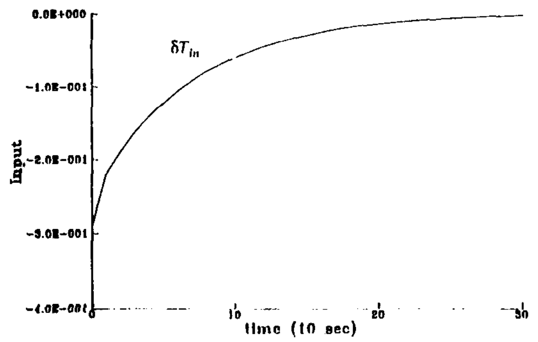

Figure 2c Optimal Control Input Obtained Assuming States are Measurable

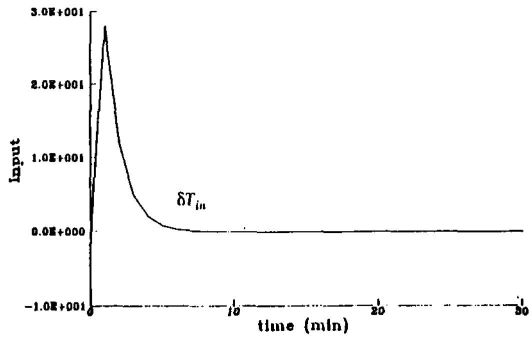

Figure 3c Optimal Input Obtained Using Observer

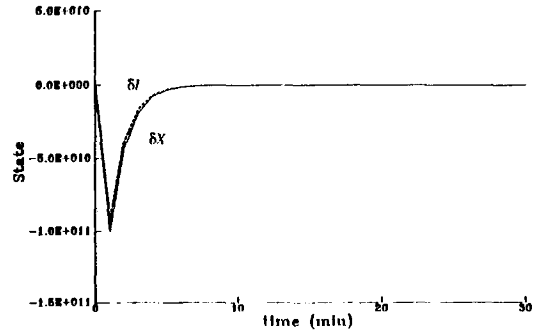

Figure 3a Response of Optimally Controlled System using Observer

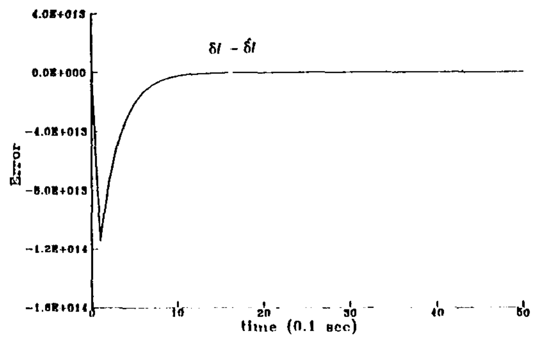

Figure 4a Error Between the Real and Estimated States

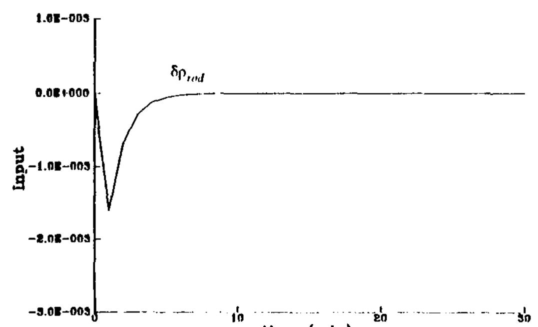

Figure 3b Optimal Input Obtained Using Observer

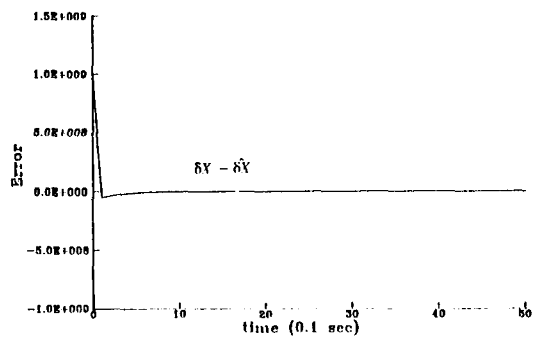

Figure 4b Error Between the Real and Estimated States

6. Conclusions

In this study, the estimation and a optimal control of xenon concentation in a nuclear power plant were investigated.

As a reactor kinetics model, we used one-energy group point reactor model with feedback effects such as xenon absorption, fuel and coolant temperatures.

The optimal control of xenon concentration is posed as a linear quadratic problem with state feedback control. It is not possible to measure the state variables such as xenon concentration and iodine concentration directly. For the implementation of the optimal state feedback control law, it is thus necessary to estimate the unmeasurable state variables. The method used in this work to estimate the xenon and iodine concentrations is based on the Luenberger observer.

In numerical analysis of reactor kinetics, the system of equation is stiff. This singularly perturbed system arises from the interaction of slow dynamic modes (iodine and xenon concentrations) and fast dynamic modes (neutron flux, fuel and coolant temperatures). As a tool to overcome the stiffness problem, the singular perturbation method is used. The singular perturbation method allows mode separation of the original stiff system into the slow reduced subsystem and the fast subsystem in different time scales.

The observer-based controller of the original system is effected by separate design of the observer and controller of the reduced subsystem and the the fast subsystem. In particular, since in the reactor kinetics control problem analyzed in the study the fast mode dies out quickly (i.e., A_{22} is uniformly asymptotically stable), we need only design the observer for the reduced slow subsystem.

The results of the test problems demonstrated that the state feedback control of the xenon oscillation can be accomplished efficiently and without sacrificing accuracy by using the observer combined with the singular perturbation method.

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