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An Approximate Determination of the Adjoint Flux by the Borresen's Coarse-Mesh Method

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Borresen의 소격해법에 의한 Adjoint속의 근사적 결정

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Abstract

A simple, approximate method for determining the two-group adjoint flux based on the Borresen's coarse-mesh 1.5 group diffusion theory scheme is proposed. With the principle of the 1.5 group diffusion theory scheme, the method describes the thermal leakage term of the adjoint flux approximately by the geomerical buckling determined from the fast adjoint flux. The accuracy of the adjoint flux is investigated by the comparison of the adjoint flux constructed from this method with a fine-mesh finite-difference KIDD computations. It is shown that the proposed method can predict the adjoint flux as good as the KIDD results. Possible applications of the present method are then suggested in conjunction with the application of the perturbation theory.

요 약

Borresen의 1.5군 소격 확산이론에 의거하여 2군 중성자 속에 대한 adjoint 함수를 근사적으로 계산할 수 있는 한가지 간단한 방법을 제안하였다. 이 방법에서는 열 중성자 속에 대한 adjoint 함수의 누설항을 1.5군 이론의 원리에 입각하여 기하학적 buckling에 의해 근사적으로 기술하게 되는데 이때 그 기하학적 buckling은 속중성자속의 adioint 함수로 부터 구하게 된다. 한편 제안된 계산 방법의 정확도를 알기 위해 adjoint함수 계산에 대한 KIDD 전산코드의 계산결과와 제안된 방법의 계산결과를 비교 하였으며 이로부터 제안된 방법이 정확도면에서 만족스런 adjoint함수를 예측 할 수 있다는 것을 보였다. 뿐만 아니라 이 방법은 섭동 이론과 관련하여 반응도 평가에 유용하게 이용될 수 있다는 것도 보였다.

1. Introduction

The purpose of this paper is to demonstrate that Borresen's coarse-mesh method[1] or its modification [2,3] can provide an approximate but very simple computational scheme for determining the two group adjoint flux. The Borresen's method is a 1.5 group diffusion theory scheme which is designed to predict the two-group flux. As such, the method has been found to be fairly accurate and efficient in analyzing both the static and the dynamic characteristics of the

LWR neutronics.[4,5] In order to develop a further application of the method, we attempt herein to show that the method can also be useful for determining the two-group adjoint flux which is needed in the various applications of the perturbation theory and the variational principle.

2. Description of the Method

The equation to be solved is the nodal balance relation for the two-group adjoint flux.

$$-\int_{vm}\nabla\cdot D_f\nabla\phi_f dV+(\Sigma_{fm}-\frac{1}{k}\Sigma_{ffm})\,\phi_{fm}V_m=\Sigma_{rm}\phi_{tm}V_m$$

$$-\int_{vm}\nabla\cdot D_t\nabla\phi_t dV+\Sigma_{tm}\phi_{tm}V_m=\frac{1}{k}\Sigma_{fm}\phi_m V_m$$

The $\bar{\phi}^{*}_{gm}(g=f,t)$ in Eq·(1) is the nodal volume-averaged adjoint flux of the node m and is defined by

$$\bar{\phi}^{*}_{gm}=\frac{1}{V_m}\int_{vm}\phi_g(r)dV \qquad (2)$$

Other notations are standard.

Eq.(1) involves no approximations in the realm of the diffusion theory. In the spirit of the Borresen's method, however, we introduce the following approximations: 1) the nodal volume-averaged fast adjoint flux is represented by the interpolation formula,

$$\bar{\phi}_{fm}=a_f\phi^{*}_{fm}+2c_f\Sigma\phi^{*}_{fm}. \qquad (3)$$

where the ϕ^{*}_{fm} and ϕ^{*}_{fm} are the fast adjoint flux at the center of the node m and that at interface between node m and j, respectively.

2) The internodal coupling confficient is given by

$$\frac{2D_{fm}D_{fj}}{D_{fm}+D_{fj}}=\overline{D_{fm}}\cdot\overline{D_{fj}} \qquad (4)$$

3) The leakage term of the thermal adjoint flux is given by

$$-\int_{vm}\nabla\cdot D_t\nabla\phi\,dV=D_{tm}B_m^2\phi_{tm}V_m \qquad (5)$$

The approximations 1) and 2) are the ones which form the basis of the Borresen's method with regard to the computation of the two-group flux. The approximation 3) leads to the so-called 1.5 group scheme in which the thermal adjoint leakage is described by an appropriate buckling of each node. It is noted that this term is discarded in the flux computation of the 1.5 group scheme.

Reformulating Eq.(1) using all these approximations, the nodal balance equation, Eq.(1), can be shown to become

$$-\sum_j\psi^{*}_j+Q_m\psi^{*}_m=\frac{\Sigma_{rm}V_m}{\sqrt{D_{fm}}\cdot(1-q_m)}\,\bar{\phi}^{*}_{tm} \qquad (6a)$$

$$(\Sigma_{tm}+B_m^2D_{tm})\,\phi^{*}_{tm}=\frac{1}{K\cdot D_{fm}}\Sigma_{ftm}\bar{\psi}^{*}_m, \qquad (6b)$$

where

$$\psi^{*}_m=\overline{D_{fm}}\,\phi_{fm};\ \bar{\psi}^{*}_m=\overline{D_{fm}}\,\bar{\phi}^{*}_{fm}. \qquad (7a)$$

$$Q_m=\frac{p_m+q_m(a_f+c_fr_m)}{1-c_fq_m}, \qquad (7b)$$

$$q_m=(\Sigma_{fm}-\frac{1}{k}\Sigma_{ffm})V_m/D_{fm}, \qquad (7c)$$

$$p_m=\overline{D_{fj}/D_{fm}},\ r_m=\overline{D_{fm}/D_{fj}}. \qquad (7d)$$

The difference relations for the adjoint flux, Eq.(6), have the same structure as those derived for the two-group flux. Therefore, the computer programs designed to solve the Borresen's coarse-mesh difference equations for the two-group flux is straightforwardly applicable for the numerical solution of Eq.(6).

The solution to Eq.(6) requires the estimation of the buckling, B_m^2 One approximation an it is the equivalent buckling derived from the balance relation of the leakage term for the fast adjoint feux,

$$-\int_{vm}\nabla\cdot D_f\nabla\phi^{*}_f\,dV=D_{fm}B_m^2\,\bar{\phi}^{*}_{fm}V_m, \qquad (8)$$

under the postulation that there is an asymptotic distribution of the adjoint flux among each computational node. The use of the above B_m^2 for the computation

Fig. 1 Octant of the Gori Unit No. 1 Core and Arrangement of Fuel Assemblies

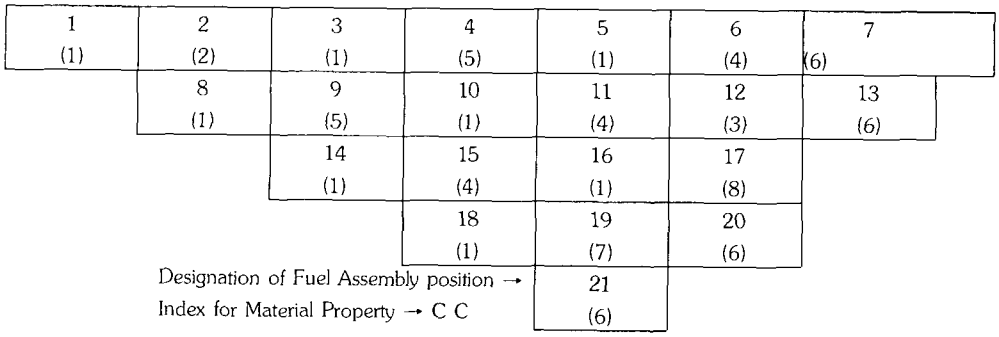

Table 1 Material Properties of Fuel Assemblies

Composition Index	Group.g	D_g(cm)	Σ_g(cm^{-1})	Σ_{fg}(cm^{-1})	Σ_r(cm^{-1})
1	1	1.45880	.026429	.005472	.017269
	2	.38224	.077837	.100680	
2	1	1.46130	.025943	.005472	.016312
	2	.38458	.087593	.102750	
3	1	1.45080	.026201	.006432	.016594
	2	.38034	.090835	.128730	
4	1	1.45700	.025432	.006430	.015152
	2	.38438	.106600	.133020	
5	1	1.45580	.025213	.006431	.014694
	2	.38619	.112330	.134870	
6	1	1.44590	.026154	.006934	.016289
	2	.37903	.097356	.142670	
7	1	1.44950	.025648	.006933	.015332
	2	.38161	.107760	.145880	
8	1	1.45080	.025397	.006932	.014857
	2	.38314	.113480	.147580	

of the thermal adjoint flux may cause a certain amount of error. since Eq · (8) does not guarantee the convergence of the B_m^2 to the desired degree in the course of iterative computatims. Yet the error will not be significant because the $D_{tm}B_m^2$ is generally much smaller than Σ_{tm} in Eq.(6).

3. Numerical Results and Discussions

In order to examine the computational accuracy of the numerical scheme just formulated, the Gori unit No.1 reactor core is chosen as a reference core for the sample calculation. Fig.1 shows the configuration of the Gori core. Table 1 specifies the material characteristics of core in terms of the homogenized two-group cross sections of the fuel assemblies.

Fig.2 shows the normalized assemblywise relative power densities of the Gori core which are computed by the fine-mesh finite difference KIDD[6] code and the modified Borresen's coarse-mesh method. Table 2. compares the present computation for the adjoint flux with the reference computation with one node per fuel pin by the KIDD code.

The core-mean relative errors of the present computation in the fast and the thermal adjoint flux shown to be about 4.5% and 3.7%, respectively. Thus two computations agree fairly well with each other.

For a further test of the present method, the adjoint

Fig. 2 Normalized Assemblwies Relative Power Densities For the Gori Core.

1.1603	1.0775	1.1471	1.1478	1.1016	1.0138	.7987
1.1280	1.0434	1.1178	1.1283	1.1254	1.0336	.7890
-2.86	-3.37	-2.62	-1.73	2.11	1.91	-1.22
	1.1539	1.1614	1.1439	1.1435	1.0824	.6712
	1.1204	1.1238	1.1410	1.1507	1.0788	.6583
	-2.99	-3.35	-.26	.63	-.34	-1.95
		1.1559	1.1830	.9986	.7623	
		1.1511	1.1976	1.0368	.7762	
		-.42	1.22	3.68	1.79	
			1.0695	1.0222	.6223	
			1.1061	1.0369	.6086	
			3.31	1.42	-2.26	

A ← Modified Borresen with 1 node/F.A. .7127
B ← The Referencce KIDD with 296 nodes .7356
C ← Relative Error % er Fuel Assm. 3.12

Table 2 Comparison of the Adjoint Flux Computations

	ϕ^{*}_{1m}		ϕ^{*}_{1m}	
Fuel Assembly	KIDD	Present Method	KIDD	Present Method
1	1.	1.	1.2608	1.2530
2	.9872	.9247	1.1780	1.2452
3	1.0251	1.0246	1.2955	1.2839
4	1.0208	.9556	1.2375	1.1825
5	1.0365	1.0271	1.3191	1.2874
6	.9207	.8877	1.1659	1.1123
7	.5943	.6595	.8376	.8669
8	1.0160	1.0160	1.2830	1.2731
9	1.0160	.9537	1.2315	1.1801
10	1.0527	1.0482	1.3366	1.3136
11	1.0105	.9752	1.2673	1.2219
12	.8786	.9580	1.2063	1.2436
13	.4780	.5385	.6664	.7078
14	1.0622	1.0512	1.3485	1.3174
15	1.0448	.9887	1.3055	1.2387
16	.9547	.9258	1.2244	1.1605
17	.6403	.6165	.8226	.7861
18	1.0159	.9895	1.3052	1.2402
19	.8392	.8378	1.1260	1.0809
20	.4509	.4836	.6233	.6354
21	.5429	.5573	.7518	.7323

flux constructed from the present method was used for estimation of the change of the core reactivity induced by a small change in the absorption cross section of the individual fuel assemblies. The figures in the second column of table 3 stand for the reactivity changes induced by an increase in the thermal absorption cross section by 1% of the indivdual fuel assemblies and are estimated from the computation of the k_{eff}

Table 3 Estimation of Reactivity by first-Order Perturbation

Perturbed Position	Modified Borresen's Method	First-Order Perturbation	
		KIDD	Present Method
1	-.83670×10^{-4}	-.80059×10^{-4}	-.82845×10^{-4}
2	-3.14908×10^{-4}	-3.00067×10^{-4}	-2.99628×10^{-4}
3	-3.38657×10^{-4}	-3.23133×10^{-4}	-3.32887×10^{-4}
4	-3.19598×10^{-4}	-3.19818×10^{-4}	-3.14152×10^{-4}
5	-3.13910×10^{-4}	-3.30841×10^{-4}	-3.20174×10^{-4}
6	-3.43470×10^{-4}	-2.66772×10^{-4}	-2.51890×10^{-4}
7	-1.24563×10^{-4}	-1.27827×10^{-4}	-1.34599×10^{-4}
8	-3.38657×10^{-4}	-3.21762×10^{-4}	-3.33026×10^{-4}
9	-6.50883×10^{-4}	-6.34186×10^{-4}	-6.34536×10^{-4}
10	-6.78541×10^{-4}	-6.79420×10^{-4}	-6.78070×10^{-4}
11	-6.19432×10^{-4}	-6.48799×10^{-4}	-6.27890×10^{-4}
12	-5.02331×10^{-4}	-5.22408×10^{-4}	-5.43970×10^{-4}
13	-1.80721×10^{-4}	-1.71472×10^{-4}	-1.86217×10^{-4}
14	-3.45342×10^{-4}	-3.45815×10^{-4}	-3.43532×10^{-4}
15	-6.62365×10^{-4}	-6.97107×10^{-4}	-6.18455×10^{-4}
16	-5.19500×10^{-4}	-5.65794×10^{-4}	-5.23050×10^{-4}
17	2.56839×10^{-4}	-2.70961×10^{-4}	-2.55144×10^{-4}
18	-2.94853×10^{-4}	-3.21934×10^{-4}	-2.99455×10^{-4}
19	-4.40548×10^{-4}	-4.40548×10^{-4}	-4.56125×10^{-4}
20	-1.55983×10^{-4}	-1.47506×10^{-4}	-1.54143×10^{-4}
21	-1.02720×10^{-4}	-1.07629×10^{-4}	-1.01788×10^{-4}

of the perturbed core by the modified Borresen's method. On the other hand, the figures in the last two columns of Table 3 denote the reactivity changes which are estimated by the first-order perturbation expression in combination with adjoint flux listed in Table 2. From comparison of these results, the adjoint flux constructed from the present method is seen to predict the reactivity changes as good as the adjoint flux from a more refined KIDD computation. It is also seen that the reactivity changes estimated from the perturbation expression are in good agreements with those from the core K_{eff} commputatons.

4. conclusions

The adjoint flux is required for determining the point kinetics parameters, eigenvalue estimates of the perturbed core, xenon spatial instability, etc.. It is usually obtained after the completion of the flux and the core eigenvalue computation. The clear advantage of the proposed method in this respect is the speedy determination of the adjoint flux due to the coarse-mesh nature of the method and the acceptable computational accuracy as demonstrated in Tables 2 and 3. What's more, there is another advantage of the proposed method in that the method makes the Borresen's 1.5 group scheme and its modification applicable for determining not only the two-group flux and the core multiplication factor but the two-group adjoint flux. This makes the Borresen's scheme very useful in solving problems in which the repeated computation of both flux and adjoint flux is required.

Nuclear fuel management optimization studies by the perturbation method belong to this kind of problem [7,8]. An immediate application of the scheme will be the optimum reload pattern search of the LWR by the perturbation method. Efforts toward this application will be presented.

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