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Verification and Sensitivity Analysis on the Elastic Stiffness of the Leaf Type Holddown Spring Assembly

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Abstract

The elastic stiffness formula of leaf type holddown spring(HDS) assembly is verified by comparing the values of elastic stiffness with the characteristic test results of the HDS's specimens. The comparisons show that the derived elastic stiffness formula is useful in reliably estimating the elastic stiffness of leaf type HDS assembly. The elastic stiffness sensitivity of leaf type HDS assembly is analyzed using the formula and its gradient vectors obtained from the mid-point formula. As a result of sensitivity analysis, the elastic stiffness sensitivity with respect to each design variable is quantified and design variables of large sensitivity are identified. Among the design variables, leaf thickness is identified as the most sensitive design variable to the elastic stiffness of leaf type HDS assembly. In addition, the elastic stiffness sensitivity, with respect to design variable, is in power-law type correlation to the base thickness of the leaf.

1. Introduction

The leaf type holddown spring(HDS) assembly, which is attached at the upper most part of the fuel assembly in pressurized water reactors, has two main functions. The first is to keep the fuel assembly firmly seated on the lower core plate during normal plant operation with enough holddown force to resist buoyancy forces and the upward hydraulic flow forces that act on the fuel assemblies due to normal reactor coolant flow. The second is to allow changes to occur in the length of the fuel assembly relative to the space between the upper and lower core plate, while still providing an acceptable holddown force[1]. These changes in relative length can occur due to differential thermal expansion between the fuel assembly structure made of Zircaloy-4 and the core support structure made of stainless steel, and irradiation-induced growth of the fuel assembly. In the design stage of the fuel assembly, maintaining these two functions during the entire residence time of the fuel assembly in the reactor core is evaluated through the analysis of the holddown force based on the force-deflection characteristic curves of the HDS assembly[1,2].

Currently, two kinds of the leaf type HDS assembly are attached to the fuel assembly: the tapered-thickness leaf type HDS assembly named TT-HDS and the tapered-width leaf type HDS assembly named TW-HDS. The leaf type HDS assembly consists of a number of leaves which are bent into design shapes and machined to have a uniformly tapered thickness or width along the leaf length. It is known to be difficult to reliably estimate the elastic stiffness which is one of the basic parameters for the holddown force analysis of the fuel assembly, because of the complicated geometric shape of each leaf. Therefore, some foreign nuclear fuel vendors have developed their own methodology to estimate the TT-HDS's elastic stiffness, and have used them only for the initial estimates of the HDS force. For example, Westinghouse (W) derived empirical formulas[2] for each leaf spring based on actual tests, and Siemens/KWU[3] derived a formula by assuming the leaf to be a horizontal cantilever and applying Euler beam theory. But the former is not appliable for estimating the holddown force reliably in cases where the leaf type HDS assembly is made by other vendors or the dimensions of the W's HDS assembly are subject to change. The latter is also not appliable for estimating the holddown force owing to much simplifications. Consequently, both vendors directly use the force-deflection characteristic curves obtained by testing production springs for design purposes.

Some research attempting to assess the stiffness characteristics of the leaf type HDS assembly has been successful. Kim et al.[4] carried out a spring characteristic analysis on the Korean Fuel Assemblies (KOFAs) type TT-HDS using ADINA code, and Yim and Sohn[5] carried out a stiffness characteristic analysis and design optimization on the KOFA type TT-HDS using ANSYS code. Recently, Song et al.[6] extended the previous methodology[7] to estimate the TT-HDS's elastic stiffness considering all of strain energy and the point of taper runout in the leaf, and reported that the extended formula could be applied to reliably estimate the elastic stiffness of both the KOFA type TT-HDS and the W's type TT-HDS. And

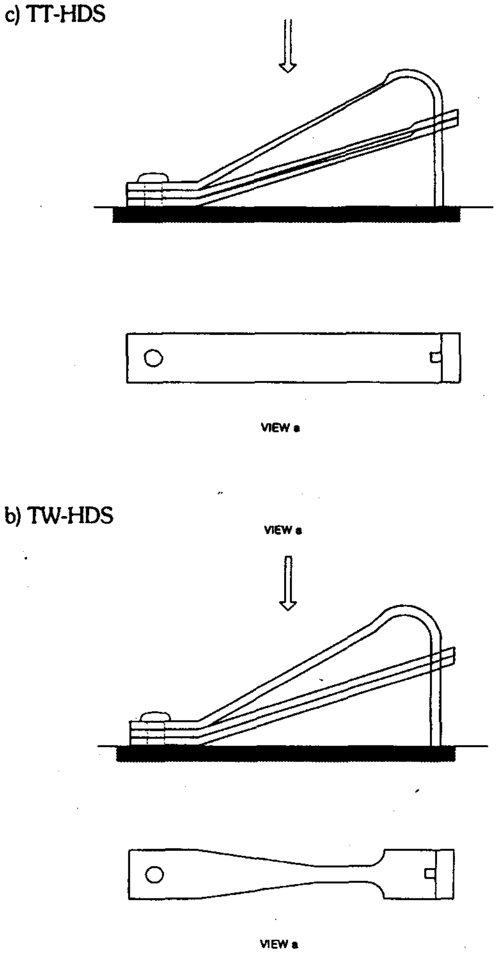

Fig. 1. Leaf Type Holddown Spring Assembly : a) TT-HDS b) TW-HDS

Song et al. derived a formula[8] to estimate the elastic stiffness of the TW-HDS and carried out a stiffness analysis on the TW-HDS which were conceived in the dimensional design spaces of the KOFA type TT-HDS.

This paper first focuses on the verification of the derived elastic stiffness formula of the leaf type HDS assembly. Then, sensitivity analysis on the elastic stiffness are carried out to identify design variables of large sensitivity.

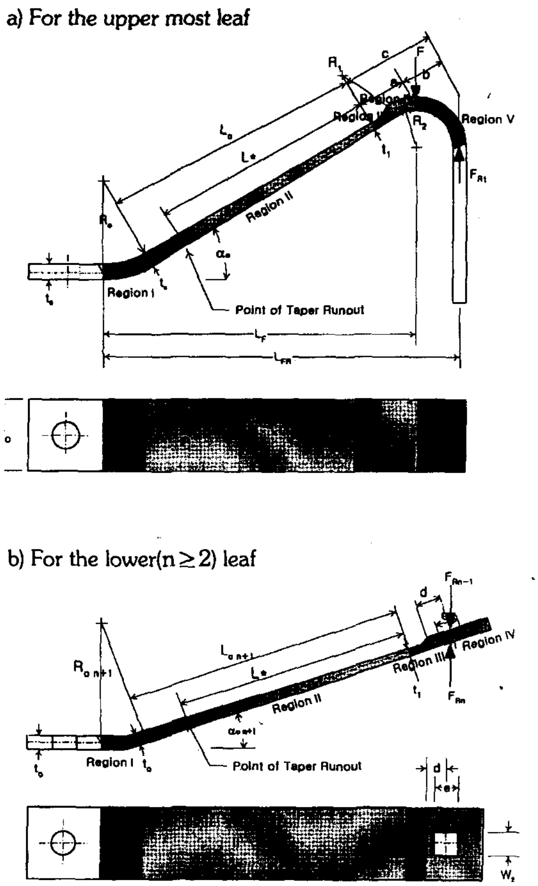

Fig. 2. Design Variables for Each Leaf of TT-HDS

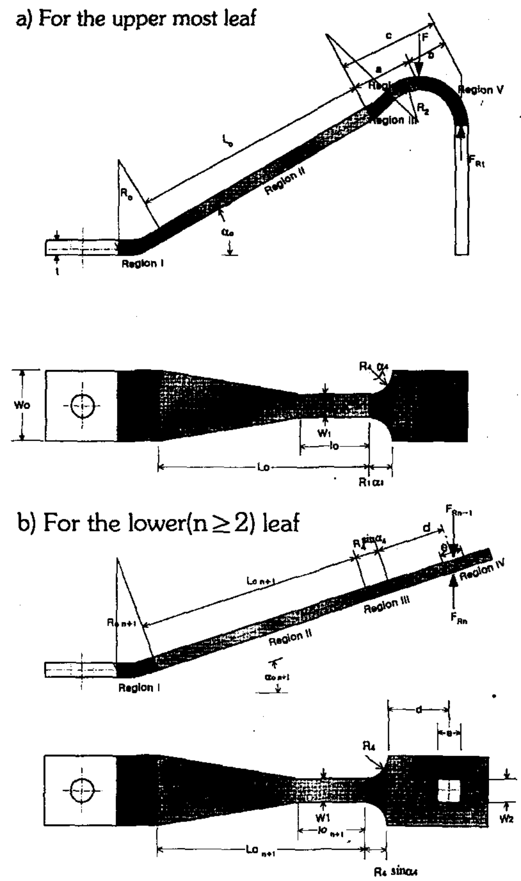

Fig. 3. Design Variables for Each Leaf of TW-HDS

2. Derivation of the Elastic Stiffness Formula

The procedure[6,8] to derive the elastic stiffness formula of leaf type HDS assembly as shown in Fig. 1 is summarized in the following three stages; first, the bending moments, axial and shear forces were obtained from free-body diagrams in each region of the leaf spring as designated in Figs 2 and 3 and these values were put to use to calculate the total strain energy in each leaf; second, in-line deflections at the loading and reaction points were obtained by applying Castigliano's theorem; third, the elastic stiffness formula was obtained by imposing conditions on the in-line deflections of each leaf. In the following subsections, the above three stages are summarized.

2.1. Total Strain Energy in Each Leaf

When the leaf type HDS assembly are deformed, the total strain energy(U_n) in each leaf is expressed as[9]:

$$U_n = \sum_{i=1}^{V} \left(\int \frac{M_i^2}{2E_iI_i} ds + \int \frac{P_i^2}{2A_iE_i} ds + \int \frac{\tau^2}{2G_i} dV \right) \quad (1)$$

where,

U_n : Total strain energy in *n-th* leaf
M_i : Bending moment
E_i : Elastic modulus

A_i : Cross-sectional area
P_i : Axial force
G_i : Shear modulus
I_i : Second moment of the beam cross-sectional area
τ : Shear stress
dV : Element of volume (or Differential volume)
i=I, II, III, IV, V : Region number of the leaf

Assuming that the shear stresses(τ) are distributed uniformly across the width and solving the equilibrium equations for the plane stress condition[10], we can obtain the shear-stress distribution in a beam of rectangular cross section as follows:

$$\tau = \frac{V_i}{2I_i}\left[\left(\frac{t_x}{2}\right)^2 - y_1^2 \right] \tag{2}$$

where,

V_i : Shear force on the beam cross section
t_x : Full thickness of the beam
y_1 : Distance from the neutral axis on the beam cross section

2.2. In-line Deflections at Loading(F) and Reaction(F_R) Points

In-line deflections at loading and reaction points are obtained by differentiating the total strain energy(U_n) with respect to the load at that point. (Castigliano's theorem[11])

2.2.1. For the Top Leaf

$$\delta_{1F} = \frac{\partial U_1}{\partial F} = AA_1F - AB_1F_{R1} \tag{3}$$

$$\delta_{1F_R} = \frac{\partial U_1}{\partial F_R} = -AB_1F + BB_1F_{R1} \tag{4}$$

2.2.2. For the Lower (n th ; n≥2) Leaf

$$\delta_{2F_R} = \frac{\partial U_2}{\partial F_R} = BB_2(F_{R1} - F_{R2}), \text{ for the 2nd leaf} \tag{5-a}$$

$$\delta_{3F_{R2}} = \frac{\partial U_3}{\partial F_{R2}} = BB_3(F_{R2} - F_{R3}), \text{ for the 3rd leaf} \tag{5-b}$$

$$\delta_{4F_{R3}} = \frac{\partial U_4}{\partial F_{R3}} = BB_4\, F_{R3} \ , \text{ for the 4th leaf} \tag{5-c}$$

AA_1, AB_1, BB_1, BB_2, BB_3, and BB_4 are coefficients expressed as a function of design variables. And F_R, F_{R_2}, and F_{R_3} are the reactions at the reaction points of each leaf as shown in Figs 2 and 3.

2.3. Constraint Conditions on the In-line Deflections at the Reaction Points

Assuming that the in-line deflections at the reaction points between leaves are equal, then constraint conditions are as follows:

$$\delta_{1F_R} = -\delta_{2F_R}, \text{ for the top and 2nd leaf} \tag{6-a}$$

$$\delta_{2F_R} = \delta_{3\,F_{R2}}, \text{ for the 2nd and 3rd leaf} \tag{6-b}$$

$$\delta_{3F_{R2}} = \delta_{4\,F_{R3}} \ , \text{ for the 3rd and 4th leaf} \tag{6-c}$$

2.4. Elastic Stiffness Formula

From the in-line deflections of equations (3), (4), (5-a, b, c) and constraint conditions of equations (6-a, b, c), the elastic stiffness formula of the leaf type HDS assembly is obtained as follows:

$$K_{ass} = \frac{1}{\delta_{1F}} = \cfrac{1}{AA_1 - \cfrac{AB_1^2}{BB_1 + \cfrac{1}{\sum_{i=2}^{n} \frac{1}{BB_i}}}} \tag{7}$$

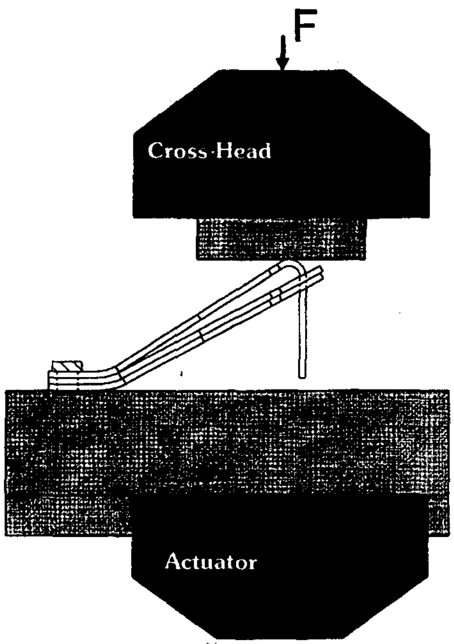

Fig. 4. Experimental Equipment

Because of different dimensions and the shape of the leaf, the coefficients in equation (7) for TW-HDS are expressed differently than those for TT-HDS.

3. Characteristic Test of the TW-HDS Specimen

Two kinds of the leaf type HDS assembly are used in the KOFAs. One is called a 14×14 type TT-HDS which is composed of one top leaf and two lower leaves, the other is called 17×17 type TT-HDS which consists of one top leaf and three lower leaves. For the TT-HDSs, characteristic tests were performed and it was found[6] that the test results were in agreement with the elastic stiffness from equation (7). With a view to examining the reliability of the elastic stiffness formula of the TW-HDS, characteristic tests on the TW-HDS specimens, which were made of aluminum plate of 3.0mm thickness instead of Inconel 718, were carried out.

Experimental equipment for the characteristic test is schematically illustrated in Fig. 4. The characteristic test of the specimens was performed until the specimens had been deflected vertically up to 30*mm* and the test results were input to the on-line delayed time personal computer in which the characteristic curve and the values of the elastic stiffness are generated by ORIGIN program[12]. Figures 5 and 6 represent the force-deflection characteristic curves generated by ORIGIN program for the 14×14 type and the 17×17 type TW-HDS specimens, respectively.

4. Elastic Stiffness Sensitivity of the Leaf Type HDS

Since the above formula (7) is expressed by coefficients, AA_1, AB_1, BB_1, BB_l which are complicated functions of the design variables, it is difficult to get the gradient vectors of the elastic stiffness (K_{ass}) at design points (b_o). Therefore, in this study a numerical differentiation method is used to get the gradient vectors instead. The mid-point formula which is known to be more accurate than the end-point formula was used to get the gradient vectors[13, 14].

4.1. Numerical Differentiation

To get the gradient vectors of the elastic stiffness at design points, the mid-point formula is used as follows:

$$\frac{\partial K_{ass}(b_o)}{\partial b_i} = \frac{K_{ass}(b_o + h) - K_{ass}(b_o - h)}{2h}$$

In the numerical differentiation, two kinds of intervals of length h of 0.01 and 0.001 are used to verify the accuracy of the gradient vectors. Since there have been no differences in significant digits to the 4*th* decimal place for the derivatives

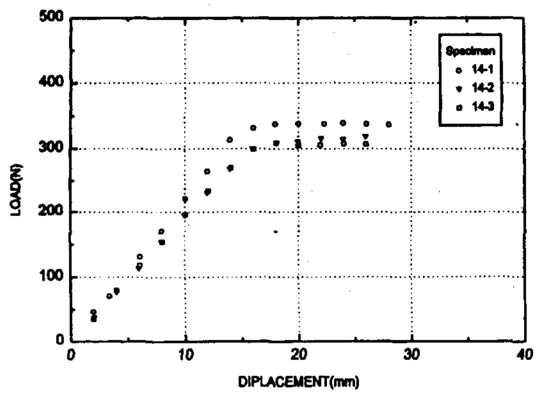

Fig. 5. Force-deflection Characteristic Curve for 14×14 Type Specimen (TW-HDS)

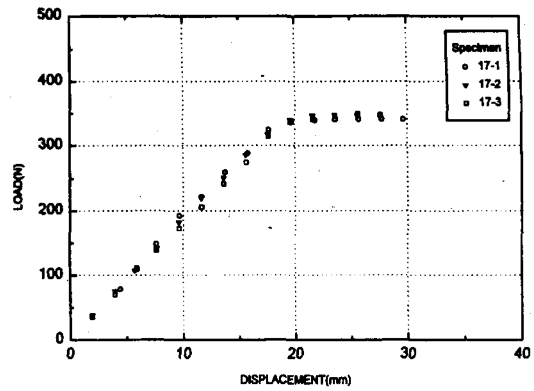

Fig. 6. Force-deflection Characteristic Curve for 17×17 Type Specimen (TW-HDS)

with h of 0.01 and 0.001, the gradient vectors are calculated with h of 0.01 in this study.

4.2. Sensitivity of the Elastic Stiffness

The sensitivity of the elastic stiffness due to the infinitesimal variation of design variables from each design point is obtained from the following:

$$\delta K_{ass}(b_o) = \frac{\partial K_{ass}(b_o)}{\partial b_i} h_i \tag{9}$$

5. Results and Discussion

5.1. Validity of the Elastic Stiffness Formula

Figures 5 and 6, which represent the characteristic test results of the TW-HDS specimens, show that the values of the elastic stiffness were 19.09~22.23N/mm for the 14× 14 type specimens and 17.79~18.89N/mm for the 17×17 type specimens. Table 1 and Fig. 7 represent the comparisons of the test results with values of elastic stiffness estimated from equation (7) using measured dimensions of the specimens. Table 1 and Fig. 7 show that the characteristic test results are somewhat lower than the analytical results: i.e. the test results are 3.0~9.5%, 0.9~4.8% lower than the analytical results for the 14× 14 type and the 17× 17 type TW-HDS, respectively. These discrepancies are assumed to be attributed to the imposition of different boundary conditions at the root part of the leaf springs and to the non-uniformity of the actual dimensions of the specimens: i.e. on the boundary conditions, the test specimens are fixed at the root part of the leaf springs by the screw as shown in Fig. 1, which allows the leaf springs to rotate and leads to more deflections, while in the analytical method all displacements are constrained: on the non-uniformity, while the actual dimensions of the specimens are not uniform along the length, some representative dimensions have been used as the input in the analytical method.

For the KOFA type TT-HDS, characteristic test had been carried out before by KAERI and it was reported[6] that the test results were in good agreement with values of the elastic stiffness from equation (7) within a maximum error range of 9.5%. And also for the W type TT-HDS, it was reported[6] that values of the elastic stiffness from equation (7) were in agreement with those from W's empirical formulas within a maximum error range of 5%.

Table 1. Comparison of Values of Elastic Stiffness from the Formula and Test Results

		Elastic Stiffness *(N/mm)*		Ratio
		from the formula (A)	test results (B)	A/B
14×14 Type	specimen #1	22.904	22.23	1.030
	specimen #2	21.117	19.28	1.095
	specimen #3	20.740	19.09	1.086
17×17 Type	specimen #1	19.052	18.89	1.009
	specimen #2	19.116	18.29	1.048
	specimen #3	18.326	17.79	1.030

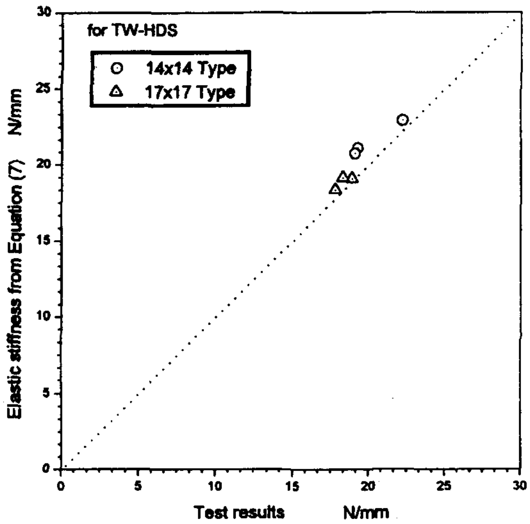

Fig. 7. Comparison of Test Results and Elastic Stiffness from Equation (7)

5.2. Sensitivity Analysis on the Elastic Stiffness

A sensitivity analysis on the elastic stiffness was carried out with respect to design variable. The elastic stiffness sensitivity of each design variable was defined by equation (9). Table 2 represents the elastic stiffness sensitivity of the 14×14 and the 17×17 type KOFA TT-HDS and Fig. 8 schematically represents the magnitude of the sensitivity with respect to design variable. Table 2 and Fig. 8 show that design variables of large sensitivity are as t_0, t_1, w_o, L_F, L-L^*, R_o, α_o and L in sequence of absolute values of sensitivity, and that sensitivity of the other design variables is negligible. Of the design variables of large sensitivity, sensitivity of t_0, t_1, w_o, L-L^* and R_o have been found to be positive, which means that increase of the dimension of the design variables leads to an increase of the elastic stiffness. On the other hand, sensitivity of L_F and α_o have been found to be negative, which means that increase of the dimension of the design variables leads to a decrease of the elastic stiffness. And for the TT-HDS, the elastic stiffness sensitivity of t_0 and t_1 is at least ten and six times as large as that of w_o, respectively. This fact shows that both t_o and t_1 are the most sensitive design variables on the elastic stiffness of the TT-HDS. Figure 9 represents correlations between the elastic stiffness sensitivity (δK) and the base thickness of the leaf (t_o). Figure 9 shows that the elastic stiffness sensitivity of design variables, t_o and t_1, for the TT-HDS is in power-law type correlation to the base thickness of the leaf (t_o) as follows.

$$\frac{\delta K_1}{\delta K_2}=\left\{\frac{t_{o\,1}}{t_{n\,2}}\right\}^n : n = 1.218 \text{ (17×17 type) and } 1.265 \text{ (14×14 type) for design variable } t_o$$

$$\frac{\delta K_1}{\delta K_2}=\left\{\frac{t_{o\,1}}{t_{n\,2}}\right\}^n : n = 2.508 \text{ (17×17 type) and } 2.554 \text{ (14×14 type) for design variable } t_1$$

Table 2. Sensitivity of the KOFA TT-HDS with Respect to Design Variable

Design Variables , b_i		Sensitivity of Elastic Stiffness, δK(N/mm)	
i	notations	14×14 type	17×17 type
1	α_o	-.12855E-01	-.9215 E-02
2	R_o	.17410E-01	.10815E-01
3	L	.11460E-01	.7190E-02
4	w_o	.10310E+00	.78490E-01
5	t_o	.10601E+01	.79146E+00
6	L_F	-.75055E-01	-.46135E-01
7	L_{FR}	-.6565 E-02	-.5195E-02
8	c	0.0	0.0
9	R_1	-.395E-03	-.225E-03
10	R_2	-.610E-03	-.470E-03
11	α_2	-.10E-04	-.15E-04
12	R_4	0.0	0.0
13	t_1	.60859E+00	.49544E+00
14	w_2	0.0	0.0
15	$L-L^*$	.39675E-01	.26940E-01
16	d	-.810E-03	-.415E-03
17	e	0.0	0.0

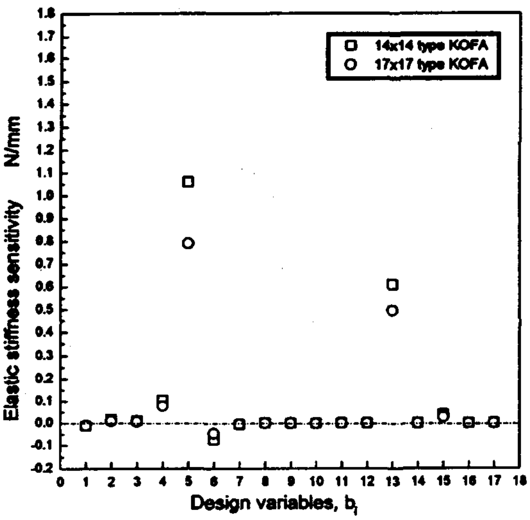

Fig. 8. Elastic Stiffness Sensitivity of TT-HDS vs. Design Variables

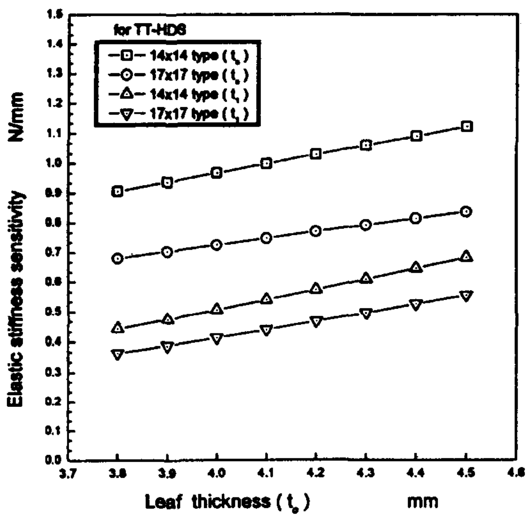

Fig. 9. Elastic Stiffness Sensitivity of t_0, t_1 vs. Leaf Thickness(t_0)

Table 3 represents the elastic stiffness sensitivity of the 14×14 and the 17×17 type TW-HDS which were conceived[8] in the dimensional design spaces of the KOFA type TT-HDS, including the case that the number of leaves composing a HDS assembly are reduced by one compared with the KOFA type TT-HDS. Figure 10 schematically represents the magnitude of the sensitivity with

Table 3. Sensitivity of the KOFA TW–HDS with Respect to Design Variable

Design Variables, b_i		Sensitivity of Elastic Stiffness, δK (N/mm)			
i	notations	14×14 type		17×17 type	
		N leaves	N−1 leaves	N leaves	N−1 leaves
1	α_o	.21755E−01	.16675E−01	.15925E−01	.13560E−01
2	R_o	−.32475E−01	−.21180E−01	−.18825E−01	−.13845E−01
3	L	−.70070E−01	−.46990E−01	−.44525E−01	−.33410E−01
4	w_o	.10175E+00	.67735E−01	.74970E−01	.56015E−01
5	t	.16150E+01	.10789E+01	.12225E+01	.91630E+00
6	a	−.69225E−01	−.47945E−01	−.43720E−01	−.33620E−01
7	b	0.0	0.0	0.0	0.0
8	c	−.4495E−02	−.1940E−02	−.2950E−02	−.1615E−02
9	R_1	−.5E−05	0.0	−.5E−05	−.5E−05
10	R_2	−.985E−03	−.215E−03	−.925E−03	−.365E−03
11	α_2	−.15E−04	0.0	−.15E−04	−.5E−05
12	R_4	.4545E−02	.3325E−02	.2900E−02	.2300E−02
13	w_1	.40255E−01	.27410E−01	.34410E−01	.26160E−01
14	w_2	−.15E−04	−.10E−04	−.5E−05	−.5E−05
15	l_o	0.0	0.0	0.0	0.0
16	d	−.1175E−02	−.600E−03	−.965E−03	−.655E−03
17	e	−.15E−04	−.5E−05	−.5E−05	−.5E−05

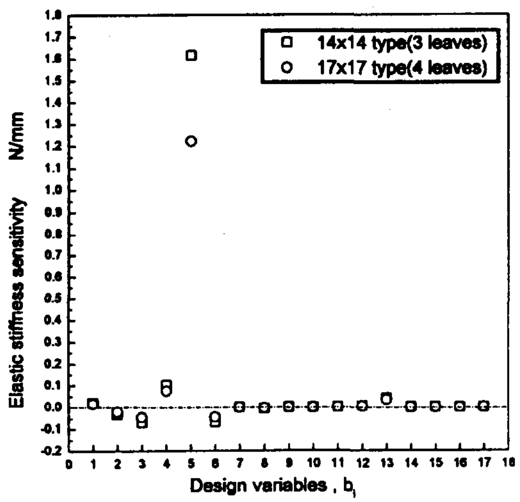

Fig. 10. Elastic Stiffness Sensitivity of TW-HDS vs. Design Variables

respect to design variables. Table 3 and Fig. 10 show that the design variables of large sensitivity are as t, w_o, L, a, w_1, R_o and α_o in sequence of absolute values of sensitivity, and that sensitivity of the other design variables is negligible. Of the design variables of large sensitivity, sensitivity of t, w_o, w_1 and α_o were found to be positive, which means that increase of the dimension of the design variables leads to an increase of the elastic stiffness. On the other hand, sensitivity of L, a and R_o were found to be negative, which means that increase of the dimension of the design variables leads to decrease of the elastic stiffness. And for the TW-HDS, the elastic stiffness sensitivity of t is at least fifteen times as large as that of w_o. This fact shows that the leaf thickness is the most sensitive design variable on the elastic stiffness for the TW-HDS. Figure 11 represents correlations between the elastic stiffness sensitivity and the leaf thickness. Figure 11 shows that the elastic stiffness sensitivity of t for the TW-HDS is in power-law type correlation to the base thickness of the leaf (t) as follows.

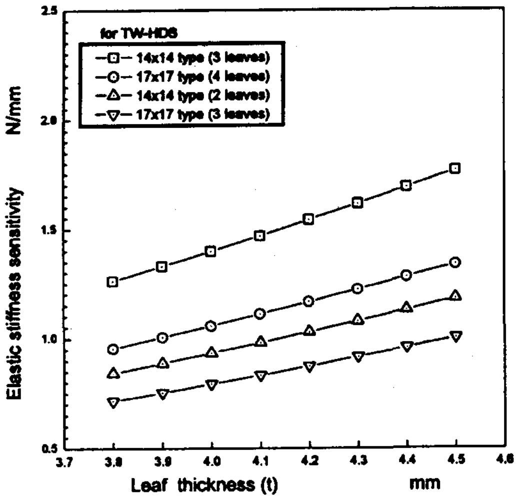

Fig. 11. Elastic Stiffness Sensitivity of t vs. Leaf Thickness

$\frac{\delta K_1}{\delta K_2}=\left(\frac{t_1}{t_2}\right)^n$: n = 1.996 (17×17 type) and 1.994 (14 ×14 type) for design variable t

6. Conclusions

In this study the derived elastic stiffness formula of leaf type HDS assembly, based on Euler beam theory and Castigliano's theorem, were verified by comparing the values of elastic stiffness with the characteristic test results. And sensitivity analysis on the elastic stiffness of the leaf type HDS assembly with respect to design variable was carried out using the elastic stiffness formula and its gradient vectors. The results from this study are listed as follows.

1. Since the values of elastic stiffness from the derived formula were in agreement with test results of both the TT-HDS and the TW-HDS, the derived formula could be useful to reliably estimate the elastic stiffness of leaf type HDS and the holddown force of nuclear fuel assemblies.
2. Design variables of large elastic stiffness sensitivity are identified as t, w_o, L, a, w_1, R_o, and α_o for the TW-HDS and t_o, t_1, w_o, L_F, $L\text{-}L_F$, R_o, α_o, and L for the TT-HDS.
3. For the TT-HDS, the most sensitive design variables to the elastic stiffness are identified as the leaf thickness, t_o and t_1, and the sensitivity of t_o and t_1 is at least ten and six times as large as that of the leaf width, w_o, respectively.
4. For the TW-HDS, the most sensitive design variable to the elastic stiffness is identified as the leaf thickness, t, and the sensitivity of t is at least fifteen times as large as that of the leaf width, w_o.
5. For both the TW-HDS and the TT-HDS, the elastic stiffness sensitivity of the leaf thickness is in power-law type correlation to the base thickness of the leaf.

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