Journal of the Korean Nuclear Society
Volume 33, Number 4, pp.409～422, August, 2001

A Dispersion and Characteristic Analysis for the One-dimensional Two-fluid Mode with Momentum Flux Parameters

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(Received March 20, 2001)

Abstract

The dynamic character of a system of the governing differential equations for the one-dimensional two-fluid model, where the momentum flux parameters are employed to consider the velocity and void fraction distribution in a flow channel, is investigated. In response to a perturbation in the form of a traveling wave, a linear stability analysis is performed for the governing differential equations. The expression for the growth factor as a function of wave number and various flow parameters is analytically derived. It provides the necessary and sufficient conditions for the stability of the one-dimensional two-fluid model in terms of momentum flux parameters. It is demonstrated that the one-dimensional two-fluid model employing the physical momentum flux parameters for the whole range of dispersed flow regime, which are determined from the simplified velocity and void fraction profiles constructed from the available experimental data and C_o correlation, is stable to the linear perturbations in all wave-lengths. As the basic form of the governing differential equations for the conventional one-dimensional two-fluid model is mathematically ill posed, it is suggested that the velocity and void distributions should be properly accounted for in the one-dimensional two-fluid model by use of momentum flux parameters.

Key Words : two-fluid model, stability analysis, momentum flux parameters, dispersed flow

1. Introduction

The one-dimensional two-fluid model is the state of the art model widely used to describe the complex two-phase flow system. As an example, it is employed in the thermal-hydraulic analysis computer codes, such as, RELAP5/MOD3[1] and CATHARE[2]. However, it is well recognized that the basic form of the governing differential equations for the one-dimensional two-fluid model is mathematically ill posed as an initial value problem as discussed by Lyczkowski, Gidaspow, and

Hughes[3], Ramshaw and Trapp[4], and Stuhmiller[5]. There have been efforts to improve the one-dimensional two-fluid model stable by considering the virtual mass force[1, 6], the phase-to-interface pressure difference [2, 7,8,9], and the bulk stress tensor for dilute dispersions of bubbles[10]. The issue is still controversial. Recently, Song and Ishii [11,12] proposed to consider the void fraction and velocity distribution in the flow channel by use of the momentum flux parameters. By performing a characteristic analysis of the governing differential equations, they showed that the one-dimensional two-fluid model is well posed for the whole range of flow regime, when physical momentum flux parameters are considered.

As the characteristic analysis only gives information on the limiting short wavelength behavior, it is necessary to perform a dispersion analysis to investigate the dynamic character of a differential model. The effect of algebraic terms on the right hand side of the governing differential equations on the intermediate wavelengths can be estimated, as these terms do not affect the characteristic analysis. While the characteristic analysis gives necessary condition for the stability, the dispersion analysis provides sufficient condition for the stability. Jones and Prosperetti [13] performed a linear stability analysis for the steady uniform flow by employing a general one-dimensional two-fluid model and considered the effect of virtual mass force term. Pokharna, Mori and Ransom[14] performed a dispersion analysis and showed that the basic one-dimensional two-fluid model results in a positive growth factor in proportional to the wave number.

In the present paper a dispersion analysis is performed for the one-dimensional two-fluid model to derive the necessary and sufficient conditions for the stability and discuss the role of the momentum flux parameters.

2. Dispersion Analysis of the One-Fimensional Two-fluid Model

By defining an area average and void fraction weighted average as below

$$<F>=1/A \int F\, dA \tag{1}$$

$$<<F_k>> = <\alpha_k F_k>/<\alpha_k> \tag{2}$$

The velocity for each phase is defined as below

$$<<u_k>>=<\alpha_k u_k>/<\alpha_k> \tag{3}$$

Let us denote $\alpha=<\ _k>$, $u_k=<<u_k>>$ and assume that the density of each phase is uniform such that $\rho_k=<<\rho_k>>$. Then the governing differential equations for the two-phase flow described by the incompressible one-dimensional two-fluid model is written as follows [11, 12, 15, 16, 17].

$$\alpha\rho_g\partial u_g/\partial z + \rho_g\partial\alpha/\partial t + \rho_g u_g\partial\alpha/\partial z = 0 \tag{4}$$

$$\alpha_f\rho_f\partial u_f/\partial z - \rho_f\partial\alpha/\partial t - \rho_f u_f\partial\alpha/\partial z = 0 \tag{5}$$

$$\begin{aligned}\alpha\rho_g\partial u_g/\partial t + \alpha\rho_g(2C_{vg}-1)u_g\partial u_g/\partial z + \rho_g(C_{vg}-1)u_g^2\partial\alpha/\partial z \\ = -\alpha\partial p/\partial z + \alpha\rho_g g\cos\theta - F_I + M_{ig}\end{aligned} \tag{6}$$

$$\begin{aligned}\alpha_f\rho_f\partial u_f/\partial t + \alpha_f\rho_f(2C_{vf}-1)u_f\partial u_f/\partial z - \rho_f(C_{vf}-1)u_f^2\partial\alpha/\partial z \\ = -\alpha_f\partial p/\partial z + \alpha_f\rho_f g\cos\theta + F_I + M_{if}\end{aligned} \tag{7}$$

where the $\alpha_f=1-\alpha$, θ is the inclination angle from the vertical direction, F_I is the inter-phase drag, and M_{ig} and M_{if} are the generalized drag force including the transient forces and wall drag. It is assumed that there is no mass transfer between phases and the phasic pressures are the same. The C_{vf} and C_{vg} are momentum flux parameters [16, 17] defined as

$$C_{vk}=\langle\alpha_k u_k^2\rangle/\langle\alpha_k\rangle\langle\langle u_k\rangle\rangle^2 \tag{8}$$

By including the momentum flux parameters, we can preserve the information on the two-phase flow structure of void fraction and velocity distributions in a flow channel. In the the conventional one-diemensional two-fluid model, this information on the flow structure is lost by assuming that the value is unity. The momentum flux parameter is a function of void fraction and velocities. However, by noting the fact that the flow regime does not change along the flow direction, we neglected the derivatives of momentum flux parameters in the derivation of the above governing differential equations for mathematical simplicity.

As previous research indicates that the generalized drag force terms tend to stabilize the system, we neglect the effect of those terms to focus our analysis on the role of interfacial drag and momentum flux parameters. By defining a vector $\mathbf{x} = (\alpha, u_g, u_f, p)$. The system of continuity and momentum equations can be written as

$$\mathbf{A}\ \partial \mathbf{x}/\partial t + \mathbf{B}\ \partial \mathbf{x}/\partial z + \mathbf{C} = 0 \tag{9}$$

where **A**, **B**, and **C** are coefficient matrices. The dependence of the solution on the prescribed initial data can be reduced to an investigation of the roots of equation

$$\text{Determinant of } (\mathbf{A}\,\lambda - \mathbf{B}) = f(\lambda) = 0 \tag{10}$$

where we have introduced the characteristic root λ. If we have real roots of λ for satisfying $f(\lambda) = 0$, then the set of differential equation is hyperbolic. If we have complex conjugate roots of λ, then the set of differential equations becomes elliptic. If the system is elliptic, the above set of equations becomes ill posed as an initial value problem. By using equations (4), (5), (6), and (7), the matrix $\mathbf{A}\ \lambda - \mathbf{B}$ is determined as

$$\begin{matrix} \rho_g(\lambda-u_g) & -\alpha\rho_g & 0 & 0 \\ -\rho_f(\lambda-u_f) & 0 & -\alpha_f\rho_f & 0 \\ -\rho_g(C_{vg}-1)u_g^2 & \alpha\rho_g[\lambda-(2C_{vg}-1)u_g] & 0 & -\alpha \\ \rho_f(C_{vf}-1)u_f^2 & 0 & \alpha_f\rho_f[\lambda-(2C_{vf}-1)u_f] & -\alpha_f \end{matrix} \tag{11}$$

The determinant of this matrix is calculated as

$$\begin{aligned} f(\lambda)= &-\alpha(1-\alpha)\rho_g\rho_f[(1-\alpha)\rho_g(\lambda^2-2\lambda C_{vg}u_g+C_{vg}u_g^2) \\ &+ \alpha\rho_f(\lambda^2-2C_{vf}u_f\lambda+C_{vf}u_f^2)] \end{aligned} \tag{12}$$

For the conventional two-fluid model, the momentum flux distribution parameters are equal to 1. The above equation then becomes,

$$f(\lambda)= -\alpha(1-\alpha)\ \rho_g\rho_f\ [(1-\alpha)\ \rho_g\ (\lambda - u_g)^2 + \alpha\ \rho_f\ (\lambda - u_f)^2] \tag{13}$$

The equation $f(\lambda)= 0$ can have real roots only if $\lambda = u_g = u_f$, which indicates that the basic form of the two-fluid model becomes ill posed. On the other hand, we can have real roots for λ for the equation $f(\lambda)= 0$, if the momentum flux parameters satisfy the following equation [11]

$$\begin{aligned} P = &(\alpha_f\rho_g\,C_{vg}u_g+\alpha\rho_f\,C_{vf}u_f)^2 \\ &- (\alpha_f\rho_g+\alpha\rho_f)(\alpha_f\rho_g C_{vg}u_g^2 +\alpha\rho_f C_{vf}u_f^2\,) \ \ \geq \ \ 0 \end{aligned} \tag{14}$$

It suggests that the one-dimensional two-fluid model might allow two real characteristic roots for λ, if we use appropriate momentum flux parameters for the two-phase flow system in various flow regimes.

As the characteristic analysis gives information on the system in response to the short wave length disturbance only, a dispersion analysis is necessary to establish the dynamic character of a differential model over the frequency range of interest [11,12]. The dispersion relation is obtained for a system of quasi-linear partial differential equations by linearizing the system about an initial state and using a general Fourier representation for each solution component. The linear differential equation for the perturbation, $\delta\varphi = \varphi - \varphi^{o}$ is

$$\mathbf{A}_o \partial\delta\varphi/\partial t + \mathbf{B}_o \partial\delta\varphi/\partial t + [(\partial\mathbf{A}/\partial\varphi)_o(\partial\varphi/\partial t)_o + (\partial\mathbf{B}/\partial\varphi)_o(\partial\varphi/\partial x)_o + (\partial\mathbf{C}/\partial\varphi)_o]\delta\varphi = 0 \quad (15)$$

Equation (15) describes the behavior of a small perturbation about the unperturbed solution. A perturbation in the form of a traveling wave is considered

$$\delta\varphi = \delta\varphi^o \exp[i(kx-\omega t)] \quad (16)$$

where k is the wave-number, and ω is the frequency in a complex number. $\lambda = \lambda_R + i\lambda_I = \omega_R/k + i\omega_I/k$. The imaginary part of (will govern the growth or decay of the perturbation and the real part determines the speed of propagation for the Fourier component corresponding to each k. If we have positive ω_I, the perturbation increases with time. Then the system becomes unstable. It we have a negative growth factor, the perturbation decays and the system is stable. $\delta\varphi^o$ denotes the initial amplitude of the perturbation. On substitution of equation (16) into equation (15), a compatibility condition for $\delta\varphi^o$ is obtained

$$-i\omega\mathbf{A}_o\delta\varphi^o + ik\mathbf{B}_o\delta\varphi^o + [(\partial\mathbf{A}/\partial\varphi)_o(\partial\varphi/\partial t)_o + (\partial\mathbf{B}/\partial\varphi)_o(\partial\varphi/\partial x)_o + (\partial\mathbf{C}/\partial\varphi)_o]\delta\varphi^o = 0 \quad (17)$$

Equation (17) is a homogeneous linear system of equations which determines the components of $\delta\varphi^o$. The condition for the existence of a non-trivial solution for $\delta\varphi^o$ is that the determinant of the coefficient matrix must vanish; i.e.,

$$\det(-i\omega\mathbf{A} + ik\mathbf{B} + \mathbf{D}) = 0 \quad (18)$$

$$\mathbf{D} = [(\partial\mathbf{A}/\partial\varphi)_o(\partial\varphi/\partial t)_o + (\partial\mathbf{B}/\partial\varphi)_o(\partial\varphi/\partial x)_o + (\partial\mathbf{C}/\partial\varphi)_o]^T \quad (19)$$

For nonzero ω, equation (18) can be written in a meaningful form as

$$\det(\mathbf{A}\lambda - \mathbf{B} + i/k\,\mathbf{D}) = 0 \quad (20)$$

Note that for finite k/ω in the limit $k \rightarrow \infty$, equation (20) reduces to the characteristic equation. And the λ value corresponds to the characteristic eigen-value.

For the case of a perturbation wave-length much smaller than the length-scale of the initial steady state or for an initial uniform steady state, $(\partial\varphi/\partial t)_o$ and $(\partial\lambda/\partial x)_o$ will be negligible or zero so that **D** becomes

$$\mathbf{D} = (\partial\mathbf{C}/\partial\varphi)_o \quad (21)$$

The drag forces consist of wall drag, inter-phase drag and transient drag forces. For highly dispersed air-water flow at low liquid velocity, the inter-phase drag force is dominant. We consider only the interfacial drag force to simplify the analysis. It also enables us to compare our analysis results with those of Pokharna et al. [14] who used the simple Darcy model suggested by Ishii and Zuber[18] for the interfacial drag

$$F_I = 1/2 C_D \rho_f (u_g - u_f)|u_g - u_f| A_p/V_b \quad (22)$$

For a mono-dispersed bubbly flow, A_p/V becomes $3/4r_b$. We assumed that the drag coefficient C_D is constant for convenience. Assume that gas velocity is bigger than liquid velocity, then the matrix **C** becomes

$$\mathbf{C} = [0,\ 0,\ -\alpha\rho_g g + K\alpha\rho_f u_r^2,\ -\alpha_f\rho_f g - K\alpha\rho_f u_r^2] \quad (23)$$

where $\alpha_f = 1-\alpha$ is liquid void fraction, $K = 0.5C_D A_p/V_b$, and $u_r = u_g - u_f$ is relative velocity.

Let's consider the highly dispersed bubbly flow in the horizontal pipe, then the body force term due to gravity can be omitted. The matrix **D** becomes

$$D_{11} = D_{12} = D_{13} = D_{14} = 0. \quad (24)$$

$$D_{21} = D_{22} = D_{23} = D_{24} = 0. \quad (25)$$

$$D_{31} = K\rho_f u_r^2,\ D_{32} = 2K\alpha\rho_f u_r,\ D_{33} = -2K\alpha\rho_f u_r,\ D_{34} = 0 \quad (26)$$

$$D_{41} = -D_{31},\ D_{42} = -D_{32},\ D_{43} = -D_{33},\ D_{44} = 0. \quad (27)$$

Let $\Phi^* = iD_{31}/k$, $\Omega_1^* = iD_{32}/k$, then the matrix $(\mathbf{A}\lambda - \mathbf{B} + i/k\,\mathbf{D})$ becomes

$$\begin{pmatrix} \rho_g(\lambda-u_g) & -\alpha\rho_g & 0 & 0 \\ -\rho_f(\lambda-u_f) & 0 & -\alpha_f\rho_f & 0 \\ -\rho_g(C_{vg}-1)u_g^2+\Phi^* & \alpha\rho_g[\lambda-(2C_{vg}-1)u_g]+\Omega^* & -\Omega^* & -\alpha \\ \rho_f(C_{vf}-1)u_f^2-\Phi^* & -\Omega^* & \alpha_f\rho_f[\lambda-(2C_{vf}-1)u_f]+\Omega^* & -\alpha_f \end{pmatrix} \quad (28)$$

3. Dispersion Relation and Stability Criteria

The determinant of the matrix $(\mathbf{A}\lambda - \mathbf{B} + i/k\,\mathbf{D})$ is calculated as below

$$\begin{aligned} f(\lambda,k) = {} & -\alpha\rho_g\alpha_f\rho_f[\rho_g\alpha_f(\lambda^2-2C_{vg}u_g\lambda+C_{vg}u_g^2) \\ & +\alpha\rho_f(\lambda^2-2C_{vf}u_f\lambda+C_{vf}u_f^2)] \\ & -\rho_g\rho_f[\Omega^*\alpha_f(\lambda-u_g)+\Omega^*\alpha(\lambda-u_f)]-\alpha\rho_g\alpha_f\rho_f\Phi^* \end{aligned} \quad (29)$$

The relation $f(\lambda,k)=0$ gives the dispersion relation. Let $\Omega = D_{32}/(\alpha\alpha_f) = 2K\rho_f u_r/\alpha_f$ and $\Phi = D_{32}$. By introducing the relation $\lambda = \lambda_R + i\lambda_I = \omega_R/k + i\omega_I/k$, the solution for $f(\lambda,k)=0$ is determined from the real and imaginary parts of equation (29) as below

$$\begin{aligned} & \rho_g\alpha_f(\lambda_R^2-2C_{vg}u_g\lambda_R+C_{vg}u_g^2-\lambda_I^2) \\ & +\alpha\rho_f(\lambda_R^2-2C_{vf}u_f\lambda_R+C_{vf}u_f^2-\lambda_I^2)-\Omega/k\,\lambda_I=0 \end{aligned} \quad (30)$$

$$\begin{aligned} & 2\rho_g\alpha_f\lambda_I(\lambda_R-C_{vg}u_g)+2\alpha\rho_f\lambda_I(\lambda_R-C_{vf}u_f) \\ & +[\lambda_R\Omega-\Omega\alpha_f u_g-\Omega\alpha u_f]/k+\Phi/k=0 \end{aligned} \quad (31)$$

From these two equations we can eliminate λ_R and determine the relation between growth factor ω_I and other flow paramerters including the wave number k. For convenience, let $\underline{\rho\alpha} = \rho_g\alpha_f+\alpha\rho_f$, $\underline{\rho\alpha u} = u_g\alpha_f+\alpha u_f$, and $\underline{\rho\alpha u} = \rho_g C_{vg}u_g\alpha_f+\rho_f C_{vf}\alpha u_f$. Then the equation (31) is rearranged as

$$\lambda_R(2\underline{\rho\alpha}\lambda_I+\Omega/k) = \underline{\alpha u}\Omega/k - \Phi/k + 2\underline{\rho\alpha u}\lambda_I \quad (32)$$

We obtain the dispersion relation expressed as equation (33) below from the equation (30) by eliminating (R using equation (32)

$$\begin{aligned} g(\omega_I,k) = {} & 4\underline{\rho\alpha}^3\omega_I^4+8\underline{\rho\alpha}^2\Omega\omega_I^3+5\underline{\rho\alpha}\Omega^2\omega_I^2+\Omega^3\omega_I \\ & +k^2(4\underline{\rho\alpha}\omega_I^2P+4\Omega\omega_I P-Q)=0 \end{aligned} \quad (33)$$

where P is the same one as that in equation (14) and Q is presented as

$$\begin{aligned} Q = {} & \alpha_f\rho_g[(\Omega\underline{\alpha u}-\Phi)^2-2C_{vg}u_g(\Omega\underline{\alpha u}-\Phi)\Omega+\Omega^2C_{vg}u_g^2] \\ & +\alpha\rho_f[(\Omega\underline{\alpha u}-\Phi)^2-2C_{vf}u_f(\Omega\underline{\alpha u}-\Phi)\Omega+\Omega^2C_{vf}u_f^2] \end{aligned} \quad (34)$$

If we have positive real solution of ω_I at a given wave number for equation (33), the disturbace will grow expoentailly with time. So the system is unstable. If we have neagative real solution, the disturbace will decay exponentially with time and the system is stable.

From equation (33), we can see that the solution of ω_I for $g(\omega_I,k)=0$ at given k is determined by the intersection of two curves

$$g1(\omega_I) = \omega_I\,[4\underline{\rho\alpha}^3\omega_I^3+8\underline{\rho\alpha}^2\Omega\omega_I^2+5\underline{\rho\alpha}\Omega^2\omega_I+\Omega^3] \quad (35)$$

$$g2(\omega_I,k) = -k^2(4\underline{\rho\alpha}\omega_I^2P+4\Omega\omega_I P-Q) \quad (36)$$

The fact that the four roots of the $g1(\omega_I)=0$ can be analytically determined from the following equation, greatly helps us to understand the behavior of fucntion g1.

$$\begin{aligned} g1(\omega_I) & = 4\Omega^4/\underline{\rho\alpha}\; x(x^3+2x^2+5/4x+1/4) \\ & = 4\Omega^4/\underline{\rho\alpha}(x+1/2)^2(x+1)x = 0 \end{aligned} \quad (37)$$

where $x = \omega_I\underline{\rho\alpha}/\Omega$. The function $g1(\omega_I)=0$ has roots of $\omega_I = 0$, $\omega_I = -\omega/\underline{\rho\alpha}$, and double roots at $\omega_I = -0.5\Omega/\underline{\rho\alpha}$ The minimum of the function $g1(\omega_I)$ is $-1/16\,\Omega^4/\rho\alpha$ at $x = (-2+\sqrt{2})/4$.

The curve in equation (36) is a parabolic curve and can be rearranged as below.

$$g2(\omega_I,k) = -k^2\{4P\Omega^2/\underline{\rho\alpha}(x+1/2)^2-P\Omega^2/\langle\rho\alpha\rangle-Q\} \quad (38)$$

In the section below, we will compare the case of the basic form of the one-dimensiona two-fluid model with the case with the momentum flux parameters. As indicated before, we neglected the effect of virtual masss force terms and phasic pressure difference to focus our analysis on the role of momentum flux parameters.

3.1. Basic form of the One-dimensional Two-fluid Model

The basic one-dimensional two-fluid model assumes that the value of momentum flux parameters is unity. Then P is presented as

$$P = - \alpha_f\rho_g\alpha\rho_f(u_g - u_f)^2 \tag{39}$$

By using the fact that P is always less than zero, let' s determine the growth factor for the case with no interfacial drag and the case with interfacial drag.

For the case with no interfacial drag, as Q becomes zero, equation (33) is simplified as below

$$g(\omega_I,k) = 4\underline{\rho\alpha}^3\omega_I^4 - 4k^2\underline{\rho\alpha}\omega_I^2\alpha_f\rho_g\alpha\rho_f(u_g-u_f)^2 = 0 \tag{40}$$

Then the growth factor is easily determined as

$$\omega_I = k/(\rho_g\alpha_f + \alpha\rho_f)u_r(\rho_g\alpha_f\alpha\rho_f)^{1/2} \tag{41}$$

This result is the same as that of Pokharna and et. al. [14] and Ramshaw and Trapp[4], which shows that the growth factor is proportional to the wave number and the relative velocity.

When the interfacial drag is present, Q is determined as $Q=\alpha_f\rho_g[(\underline{\Omega\alpha u}-\Phi)-\Omega u_g]^2+\rho\alpha_f[(\underline{\Omega\rho u}\ \Phi)-\Omega u_f]^2$ and it is always positive. So, we have negative P and positive Q. With this information, the general shape of curves in equation (37) and (38) is determined and they can be utilized to find the location of intersections of those two curves, where the growth factor ω_I of the system is determined.

We assumed that k=0.5 or 1.0, P=-1.0, Ω = 1.0, <ρα>=1.0, Q=1.0 for convenience to figure

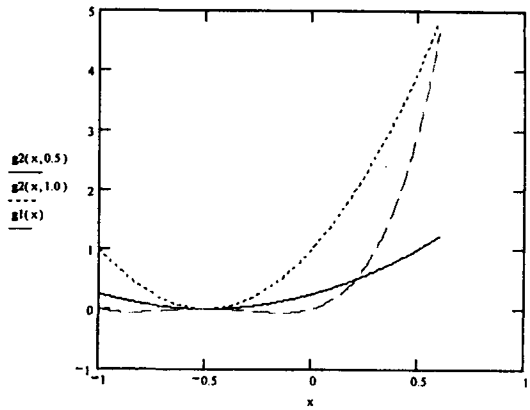

Fig.1(a). Plots of Functions g1(x) and g2(x,k) with (=1 and k=0.5 or 1.0

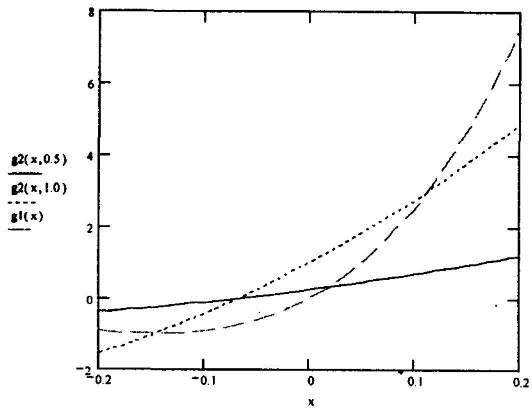

Fig.1(b). Plots of Functions g1(x) and g2(x,k) with Ω = 2 and k=0.5 or 1.0

out the typical shape of those two curves. It is to be noted that positive Ω has physical meaning. The curves are shown in Fig. 1(a).

The function g1(x) has zeros at x=0, -1/2, -1.0 and is always positive for the positive value of x, as it has the minimum value of 1/16 at $-0.5+\sqrt{2}/4$. The parabolic curve g2(x) has a positive value of k^2Q at x=0. So, it can be seen that the function g1(x) and g2(x) always intersect at the positive value of x. The general shape of the curve indicates that this conclusion does not change, even though the parameters k, Ω, and $<\rho\alpha>$ change. So, it is cleary demonstrated that the basic form of the conventional one-dimensional two-fluid model always results in a positive growth factor. From Fig. 1(a), we can also notice that the growth factor increases as the wave number increases.

Figure 1(b) shows the curves for the case with $\Omega=2$ to illustrate the effect of interfacial drag. As the function g1(x) is proportional to Ω^4, while g2 is proportional to Ω^2, the growth factor decreases as Ω^2 increases. This trend is the same as that discussed by Pokharna et. al.[14].

While previous researches[13, 14] calculated the growth factor at specific values of flow parameters by numerical analysis, the present analysis has an advantage of being able to show the general picture by the analytically derived dispersion relation in equation (33). It clearly illustrates that that the basic form of the conventional one-dimensional two-fluid model is unstable to the disturbances at all wave numbers and the growth factor is proportional to the wave number and inversely proportional to the interefacial drag.

3.2. When the Momentum Flux Parameter is Considered

Firstly, consider the case with negligible interfacial drag force. As Ω and Q becomes zero, equation (33) becomes

$$g(\omega_I,k)= 4\underline{\rho\alpha}\omega_I^2\ [\underline{\rho\alpha}^2\omega_I^2+ k^2 P\ ] =0 \qquad (42)$$

As P is positive, it has the solution of $\omega_I=0$. It means that the perturbation does not grow and is stationary.

Let's consider the case with the interfacial drag. If we employ the physcial momentum flux parameters that suggested by Song and Ishii [12], the qunatity P in equation (14) is expected to be positive for the whole range of flow regime. As P

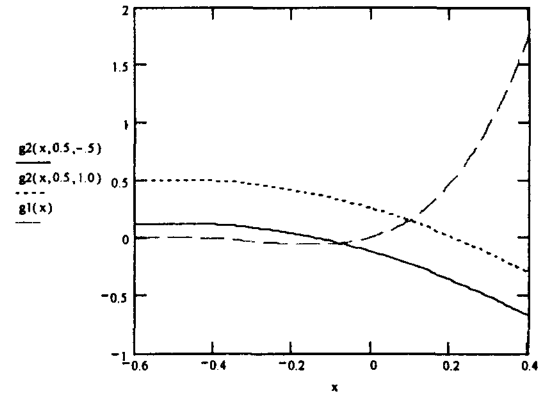

Fig.2(a). Plots of Functions g1(x), g2(x,k,Q) at k=0.5, Q=1.0 and Q=-0.5

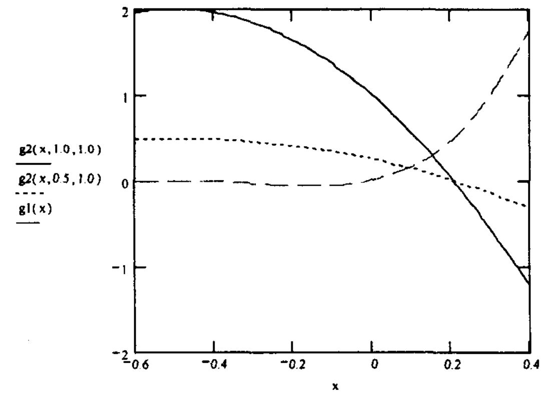

Fig.2(b). Plots of functions g1(x), g2(x,k,Q) at Q=1.0 , k=0.5 and k=1.0

is positive, the parabolic curve in equation (38) has maximum value of $k^2[P\Omega^2/\rho\alpha+Q]$ at $-\Omega/(2\underline{\rho\alpha})$, while the curve in equation (37) maintains the same shape as in the previous section.

Figure 2(a) below illustrates the general shape of the curves represented by equation (37) and (38). We assumed that k=0.5, P=1.0, Ω=1.0, $<\rho\alpha>$=1.0, Q=1.0 or -0.5. From the shape of these two curves, we can easility see that the curve g1(x) and g2(x) will intersect at the positive value of x if Q is positive On the other hand, if Q is negative, the two curves would not intersect at the positive value of x, because the curve g2(x) is always negative at the positive value of x. In this case the growth factor cannot be positive and the disturbance does not grow with time. The general shape of the curve indicates that this conclusion does not change, even though the parameters k, Ω, and $<\rho\alpha>$ change. From this we can conclude that the disturbaces at all wave length does not grow if Q is positive.

Figure 2(b) shows the comparison of the cases at different wave numbers of k=0.5 and 1.0. It shows that the growth factor will increase as the wave number increases.

3.3. Stability Criteria

From the discussions above, we can determine the conditions for the stability of the one-dimensional two-fluid model by the following two inequalities

$$P = (\alpha_f\rho_g C_{vg}u_g+\alpha\rho_f C_{vf}u_f)^2 - (\alpha_f\rho_g+\alpha\rho_f)(\alpha_f\rho_g C_{vg}u_g^2+\alpha\rho_f C_{vf}u_f^2) \geq 0 \qquad (14)$$

$$Q = \alpha_f\rho_g[(\Omega\underline{\alpha u}-\Phi)^2-2C_{vg}u_g(\Omega\underline{\alpha u}-\Phi)\Omega+\Omega^2 C_{vg}u_g^2] + \alpha\rho_f[(\Omega\underline{\alpha u}-\Phi)^2-2C_{vf}u_f(\Omega\underline{\alpha u}-\Phi)\Omega+\Omega^2 C_{vf}u_f^2] \leq 0 \qquad (43)$$

If these two inequalities are met, the one-dimensional two-fluid model is stable to the linear disturbances. The first criterion in equation (14) makes the system of governing differential equations hyperbolic. The second criterion expressed in terms of momentum flux parameter, interfacial drag, densities, and velocities makes the flow stable to the perturbations in all wave-lengths. From these observations, we suggest that the stability criteria proposed by the dispersion analysis are necessary and sufficient conditions for the stable two-phase flow.

The dispersion analysis performed in this paper shows that the basic form of the one-dimensional two-fluid model is always unstable to the linear distuirbances, which is contradictory with the existence of various flow regimes. This dilemma is solved by considering the velocity and void fraction profiles by use of momentum flux parameters. Song and Ishii [11] demonstrated that the first criterion of positive P in equation (14) is satisfied by the momentum flux papramemeters of the simplified two-phase flow configuration in various flow regime. So, if the second criterion could also be met for the typical two-phase flow by use of thes momentum flux parameters, it can be suggested that the nature is such that the flow and void distributions adjust to make the flow stable. In other words, the two-phase flow configuration always has distributions of void fraction and velocity such that the flow is stable. Otherwise, the distributions should change immediately.

4. Application of the Proposed Arguments for the Bubbly Flow and Slug Flow

To investigate the feasibility of the proposed arguments, we consider the typical steam-water two-phase flow system in bubbly and slug flow regime. We limited the discussion to the dispersed flow regime of bubbly and slug flow. Because, the annular flow can be treated by seprated flow model, which does not encounter the problem of mathematical ill-posedness.

The first stability criteria in equation (14) can be written in a non-dimensional form by introducing the parameters $S=u_g/u_f$ and $R=\rho\alpha_f/((1-\alpha)\rho_g)$ as discussed in Song and Ishii[11].

$$C_{vg} \geq 0.5(1+R) + 0.5[(1+R)^2-4(1+R)RC_{vf}1/S^2(S-1)]^{1/2} - RC_{vf}1/S \qquad (44)$$

$$C_{vf} \geq ¼\,[1/R+1]\,S^2/(S-1), \quad S>1 \qquad (45)$$

If either of equation (44) or (45) is met, the equation (16) is met. By using the relation of $\Phi=\alpha\alpha_f\,\Omega$, the equation (43) can be simplified in a non-dimensional form as below

$$S[1-\alpha(S-1)]C_{vg} \geq -[S+\alpha(1-S)]RC_{vf} + 0.25(1+R)(1+\alpha+\alpha_f S)^2 \quad (46)$$

To demonstrate the applicability of the proposed argument, consider a typical steam water flow at 460K and 1.17 MPa. The properties are determined as ρ_g=5.9795 kg/m^3, ρ_f=879.55 kg/m^3.

Before evaluating the momentum flux parameters, we need to determine the parameters R and S. The modified density ratio $R=\rho\alpha_f/((1-\alpha)\rho_g)$ can be determined from the average void fraction and densities of two phases. To determine the slip ratio $S = u_g/u_f$, we neglected local slip for mathematical simplicity. As the global slip has a dominant effect than the local slip, this is a good first order approximation. Then, the slip ratio S can be determined by using the Ishii correlation [17] for C_o.

$$C_o = (1.2-0.2\sqrt{(\rho_g/\rho_f)})(1-e^{-18\alpha}), \quad 0<\alpha<0.7 \quad (47)$$

$$S=(1-\alpha)C_o/(1-C_o\alpha), \quad (48)$$

4.1. Bubbly Flow and Slug Flow

Consider a bubbly and slug flow with the void fraction in the range of 0.2-0.7. When the void fraction is very low between 0.0-0.2, the local void fraction tends to be in wall-peaked profile [19,20]. It will be discussed in the next section.

We determine the void fraction and velocity profile and corresponding momentum flux parameters for the simplified bubbly flow at typical steam-water two-phase system according to the procedure described in Song and Ishii[11]. Then, the parameters of n and m of power law profiles

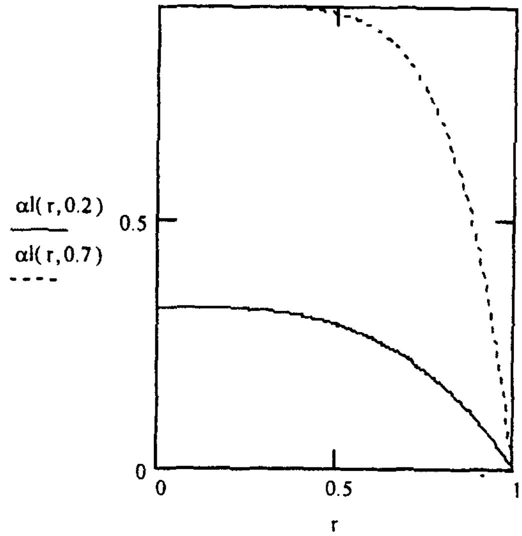

Figure 3(a) Local Void Fraction Profile

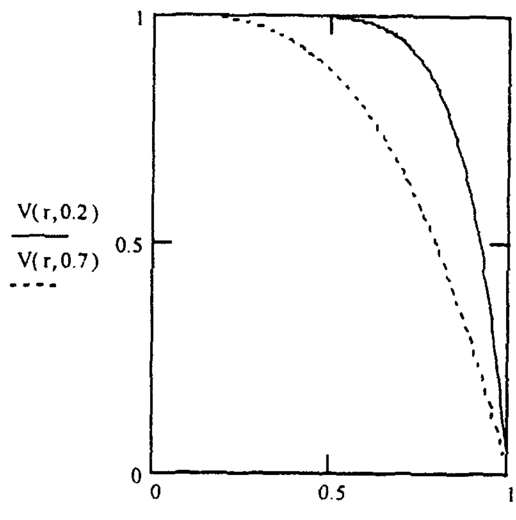

Fig. 3(b) Local Velocity Profile

are determined as

$$m=10(1-\alpha) \quad (49)$$

$$n=0.5(C_o-1)-m-2 \quad (50)$$

and corresponding velocity and void fraction profiles are determined as shown in Fig. 3(a) and

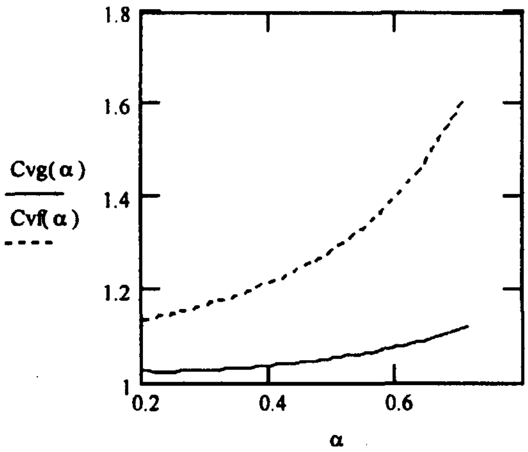

Figure 4. Gas and Liquid Momentum Flux Parameters

3(b), where α_l indicates the local void fraction. They are consistent with the typical velocity and void fraction profile observed in the experiments [19,20]. The momentum flux parameters are determined as

$$C_{vg} = (m+2)/(m+1)[1+(m+n+2)(C_0-1)I(m,n)]/C_0^2 \quad (51)$$

$$C_{vf} = (1-\alpha)(m+2)/(m+1)[1-\alpha-\alpha(m+n+2)(C_0-1)I(m,n)]/(1-C_0\alpha)^2 \quad (52)$$

where

$$I(m,n)=[1+(2m+2)/(m+n+2)]/(2m+2+n) \quad (53)$$

The calculated liquid and gas momentum flux parameters for simplified bubbly flow are shown in Fig. 4.

At given liquid momentum flux parameter, we can compare the calculated momentum flux parameter with that of the criterion (44) and (46) respectivley to evaluate whether they are located in the stable region or located in the unstable region.

The comparisons are shown in Fig. 5(a). The stability boundary of equation (44) and (46) at

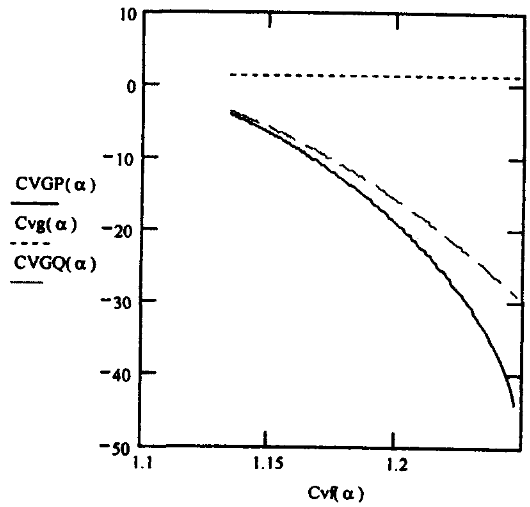

Fig. 5(a). Comparison of Calculated Gas Momentum Flux Parameter Cvg and Stability Boundaries (CVGP, CVGQ) at Given Liquid Momentum Flux Parameters Cvf

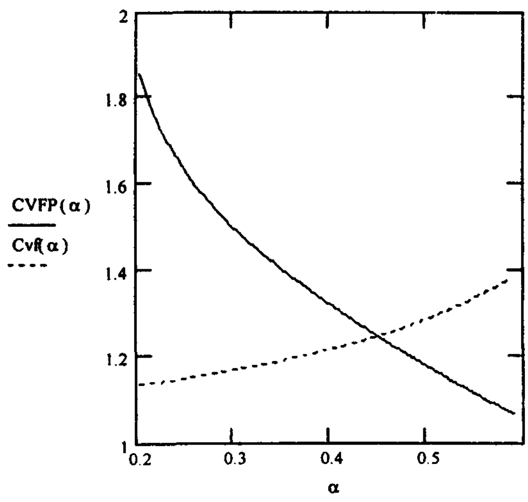

Fig. 5(b). Comparison of Calculated Gas Momentum Flux Parameter Cvf and Stability Boundary CVFP at Given Void Fraction

given liquid momentum flux parameter is denoted as CVGP and CVGQ respectively. It is shown that the momentum flux parameters of the simplified bubbly flow is bigger than the boundary CVGP and CVGQ. It demonstrates that the simplified bubbly flow satisfies the criteria of positive P in equation (14) and negative Q in equation (43).

Figure 5(b) shows the comparison of calculated momentum flux parameter and stability boundary in equation (45) at given void fraction. However, as the condition of positive P is already satisfied for the proposed momentum flux parameters by satisfying equation (44), this criterion is not a necessary condition. It is redundent.

4.2. Wall-peaked Bubbly Flow

When the void fraction is very low, the local void fraction tends to be in wall-peaked [19,20]. By following the same procedure described in Song and Ishii [11], we can determine the void fraction and the velocity profiles for the simplified wall-peaked bubbly flow at void fraction below 0.2.

The parameters of m, n, and wall void fraction (w are determined from the following relation

$$m=8 \tag{54}$$

$$n= 0.4836/\alpha \tag{55}$$

$$\alpha_w = \alpha[1 + 0.5(m+n+2)(1- C_0)] \tag{56}$$

The local void fraction is shown in Fig. 6. Note that the velocity profile maintains the same shape as that at void fraction of 0.2.

The momentum flux parameters are determined from equation (51) and (52). The calculated liquid and gas momentum flux parameters for the wall-peaked bubbly flow are shown in Fig. 7.

At given liquid momentum flux parameter, we can compare the calculated momentum flux parameters with those criteria (44) or (45), and (46) to evaluate the stability of proposed simplified velocity and void fraction profile. The comparisons are shown in Figure 8(a) and 8(b).

The stability boundary of equation (44) and (46) at given liquid momentum flux parameter is denoted as CVGP and CVGQ respectively. It is

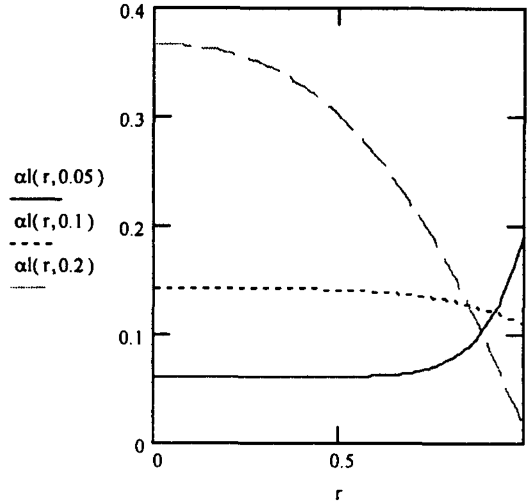

Fig. 6. Local Void Fraction Profile

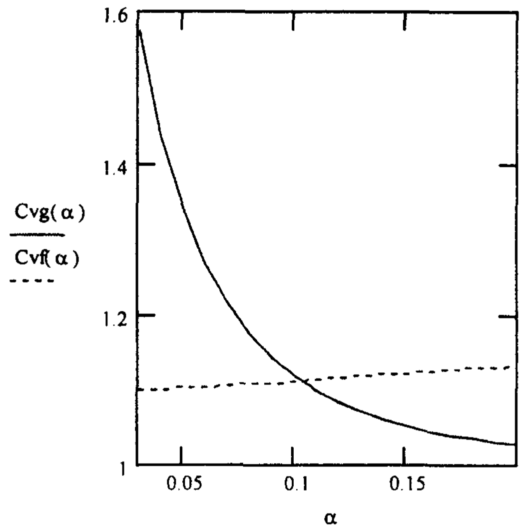

Fig. 7. Gas and Liquid Momentum Flux Parameters

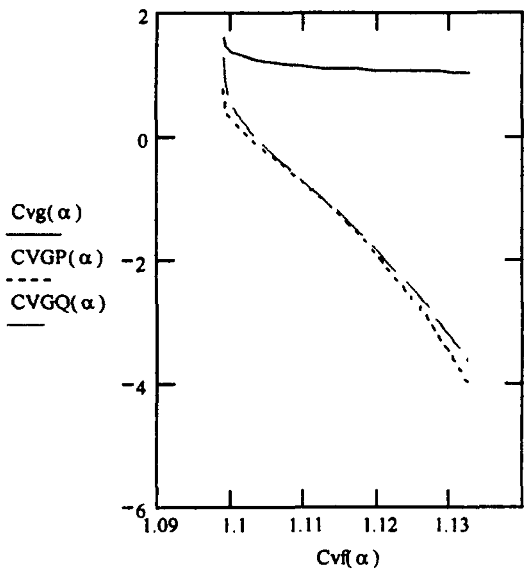

Fig. 8(a). Comparison of Calculated Gas Momentum Flux Parameter Cvg and Stability Boundary CVGP at Given Liquid Momentum Flux Parameters Cvf

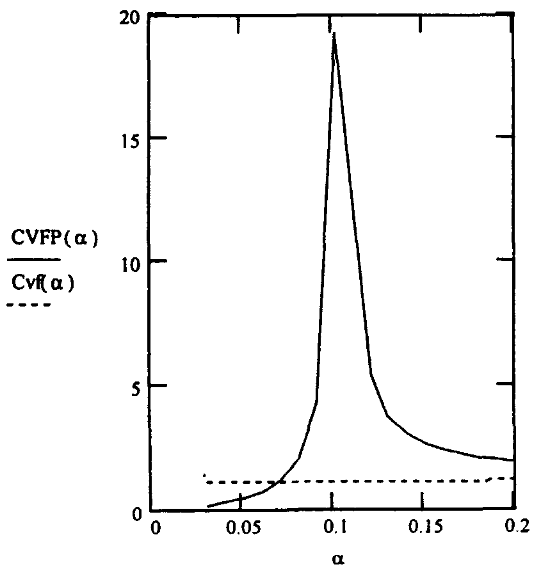

Fig. 8(b). Comparison of Calculated Gas Momentum Flux Parameter Cvf and Stability Boundary CVFP at Given Void Fraction

shown that the momentum flux parameters of the wall-peaked bubbly flow is bigger than the boundary CVGP and CVGQ. Figure 9(a) shows that the wall-peaked bubbly flow is stable with respect to the equation (16) by satisfying either equation (44) or (45).

Figure 9(b) shows the comparison of calculated momentum flux parameter and stability boundary in equation (45) at given void fraction. As the condition of positive P is already satisfied for the proposed momentum flux parameters by satisfying equation (44), this criterion is redundent.

It can be concluded that the wall-peaked bubbly flow considered in the present study satisfies the stability criteria in equation (14) and (43).

5. Summary and Conclusions

A dispersion analysis is performed for the one-dimensional two-fluid model with momentum flux parameters. The necessary and sufficient conditions for the stability are analytically derived for the dispersed flow including wall-peaked bubbly flow, bubbly flow, and slug flow. It is shown that the one-dimensional two-fluid model is mathematically well posed by use of physical momentum flux parameters, which are calculated from the simplified bubbly and slug flow constructed from available experimental data, while the basic form of the conventional one-dimensional two-fluid model is unconditionally unstable. It suggests that the nature might be such that the flow and void distributions adjust to make the flow stable in each flow regime stable. Though the present analysis took a simplified approach for mathematical clarity, the concept can be extended to cover a more general case by a numerical analysis. We could consider a local slip, generalized drag forces, and separated flows. Therefore, we propose the use of one-dimensional two-fluid model with momentum flux parameter for the analysis of complex two-phase flow system.

Acknowledgement

The authors appreciate the support from the Ministry of Science and Technology of the Korean Government.

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