

Analysis of Core Disruptive Accident Energetics for Liquid Metal Reactor

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Abstract

Core disruptive accidents have been investigated at Korea Atomic Energy Research Institute(KAERI) as part of the work to demonstrate the inherent and ultimate safety of conceptual design of the Korea Advanced Liquid Metal Reactor(KALIMER), a 150 MWe pool-type sodium cooled prototype fast reactor that uses U-Pu-Zr metallic fuel. In this study, a simple method and associated computer program, SCHAMBETA, was developed using a modified Bethe-Tait method to simulate the kinetics and thermodynamic behavior of a homogeneous spherical core over the period of the super-prompt critical power excursion induced by the ramp reactivity insertion.

Calculations of the energy release during excursions in the sodium-voided core of the KALIMER were subsequently performed using the SCHAMBETA code for various reactivity insertion rates up to 100 \$/s, which has been widely considered to be the upper limit of ramp rates due to fuel compaction. Benchmark calculations were made to compare with the results of more detailed analysis for core meltdown energetics of the oxide fuelled fast reactor. A set of parametric studies were also performed to investigate the sensitivity of the results on the various thermodynamics and reactor parameters.

Key Words : core disruptive accident, liquid metal reactor, energy release

1. Introduction

In early safety studies of small uranium metal reactors like EBR-II [1] and the Fermi Reactor[2], a sequence of super-prompt critical accident caused by fuel slumping in the sodium voided core, which is eventually terminated by disassembly of the core, was assumed to set the upper-bound design limits

of containment systems. The analytic method used in the evaluation of this type of super-prompt critical core disruptive accident(CDA) in a fast reactor was originally developed by Bethe and Tait [3], and further elaborated by Jankus [4].

The two most essential assumptions that characterize the method are the following ; first, the power distribution is independent of time and

the reactivity changes during the excursion are estimated by first-order perturbation theory, and, secondly, the material density remains constant, independent of time, in the hydrodynamic equations for disassembly and, therefore, pressure wave propagation is ignored. These two assumptions had been shown to result in little error for most of the super-prompt critical accidents. This is because the maximum reactivity attained above prompt critical is small and thus the total movement of material required to compensate this amount of reactivity is small[5]. They assumed a simple relation between pressure generation and energy density, taking the vapor pressure to be negligible until the energy density reaches a threshold value of single-phase liquid and increasing linearly thereafter. Other simplifications of the original method include restriction of the analysis to uniform spherical geometry, and the neglect of delayed neutrons, among others.

Many improvements and modifications had subsequently been made on the basic method by a number of authors and they are often classified as *Modified Bethe-Tait Methods* as long as they adopt the two assumptions above. The two main modifications made to the original method are the inclusion of the Doppler reactivity effect and the use of a more realistic equation of state of the fuel. It had been suggested in the studies of Wolfe et al.[6], Nicholson [5,7], Hicks and Menzies[8] and Meyer et al.[9] that the vapor pressure becomes significant while the power is varying much less rapidly, and core dispersion is then due to much lower pressures acting for a much longer time. The difference was particularly marked with large oxide-fuelled power reactors having a large Doppler constant. Calculations were performed in these studies using the spherical and cylindrical models of *Modified Bethe-Tait Methods*, and the results showed good agreement between the two

models[7,9]. It was also shown that the influence of the delayed neutrons was generally insignificant except for the excursion initiated by low rate of reactivity insertion[7]. Meanwhile, a more rigorous approach was initially taken by Okrent *et al.* for the computer code AX-1[10], in which the reactivity is calculated periodically by a modified S_n method rather than perturbation theory, and the core hydrodynamics are treated more rigorously. Subsequent studies[11], using the QX-1 code[12,13], confirmed that such a quasistatic space-time treatment is an accurate approximation to treating local effects of reactivity feedback for delayed supercritical excursions in fast reactors.

Recognition that the arbitrary assumption of coherent core collapse gave excessively conservative results led to the development of a mechanistic approach to the analysis of core disruptive accidents over the 1970s and early 1980s. Instead of postulating arbitrary conditions that lead to core disassembly, the mechanistic approach attempts to analyze accident sequences from a given initiating event up to the conclusion of the accident. A large number of codes have been developed for mechanistically analyzing comprehensive phenomena of accident sequences for the oxide-fuelled fast reactors. It would not be possible, however, in the current state of the art to mechanistically trace an accident sequence through a generalized meltdown sequence from the initiator to a final, stable, coolable geometry, particularly for the metal-fuelled reactors like KALIMER.

Under the circumstances, an effort has been made to evaluate the inherent safety of a conceptual design of the KALIMER for core meltdown accidents utilizing *Modified Bethe-Tait method*. Modifications were made to the original method mainly in the use of a more realistic equation of state of the fuel. The equations of state of the pressure-energy density relationship

were derived for the saturated vapor as well as the solid liquid of metallic uranium fuel, and implemented into the formulations of the disassembly reactivity. Mathematical formulations and a computer code called SCHAMBETA (Scoping Code for HCDA Analysis using Modified Bethe-Tait Method) [14] were developed in a form relevant to utilize the improved equations of state as well as to consider the Doppler effects. Calculations of the energy release during excursions in the sodium-voided core of the KALIMER were subsequently performed using the scoping code for various reactivity insertion rates up to 100 \$/s, which has been traditionally set as the upper limit of ramp rate.

To test the accuracy of calculations with the simple method developed, a number of calculations were carried out and compared with more detailed analysis results given in the work by Hicks and Menzies for oxide fuelled fast reactor[8]. Our method results in a conservative estimate of the core energy density relative to that of Hicks and Menzies. Various parametric studies were also performed to investigate the sensitivity of the results on the equation of state for pressure and energy, and other thermodynamics and reactor parameters. A scoping code like SCHAMBETA proved very useful for sensitivity studies of various parameters of uncertainties. Sensitivity studies are in need particularly for the fast reactor core loaded with metal fuel, for which our experience and knowledge are limited relative to the oxide-fuelled core.

2. Method of Analysis

2.1. Basic Approach

It is assumed that the power excursion begins with the reactor prompt critical at time zero and the energy density generated during the excursion

is governed by the reactor kinetics equation with no delayed neutrons and the source[4,5,6,15],

$$\frac{d^2 Q}{dt^2} = \frac{k-1-\beta}{\ell} \frac{dQ}{dt} \quad (1)$$

where $Q(t)$ is the time dependence of the energy generation density $E(\vec{r},t)=N(\vec{r})Q(t)$. $N(\vec{r})$ is the normalized spatial power distribution. The other quantities in Eq.(1) are expressed in standard notation; k for multiplication constant, ℓ for prompt neutron lifetime, and β delayed neutron fraction.

The neutron multiplication constant as a function of time may be expressed in the form

$$k(t) = k_0 + k_I(t) + k_d(t) + k_D(t) \quad (2)$$

where k_0 is the initial multiplication constant, $k_I(t)$ is the reactivity insertion responsible for initiating the excursion, $k_d(t)$ is the reactivity feedback resulting from material displacement during disassembly process, and $k_D(t)$ is the feedback from Doppler effect. The initial multiplication constant at prompt critical is by definition $k_0=1+\beta$. Initial energy content $Q(0)$, initial power level $\dot{Q}(0)$, and $k(0)$ are the initial conditions to be specified for a set of the coupled equations in the above to have a unique solution. Starting with the initial conditions, the coupled equations (1) and (2) can be numerically solved by iteration.

2.2. Reactivity Insertion and Initial Conditions

The rate of reactivity insertion initiating the excursion is assumed constant and $k_I(t)$ may be written as; $k_I(t)=[dk/dt]t=\alpha t$. In the case that a ramp insertion of reactivity initiates the accident, an equivalent step insertion is frequently used in the Bethe-Tait type of analysis. For the purpose of determining the equivalent step insertion, it is convenient to divide the power excursion into two

phases. During the first phase, reactivity is added at an assumed constant rate and power rises until time t_1 , when the total energy generated becomes sufficiently large to produce pressures that bring about significant material movement. Once the core begins to disassemble it goes very rapidly, and it is found that one can safely neglect any further addition of reactivity afterward. An asymptotic representation of the time t_1 may be obtained by solving Eq.(1) without reactivity feedback. The result is given as [5,7,15],

$$t_1 \approx \sqrt{\frac{\ell}{\alpha}} \sqrt{\ln X + \ln(\ln X)} \quad (3)$$

where

$$X \equiv \frac{\alpha \rho^2(t_1)}{\ell} [\dot{\rho}(0)]^{-2} \quad (4)$$

Total reactivity inserted by the ramp prior to the large pressure is then given by

$$k_1(t_1) = \alpha t_1 = \sqrt{\alpha \ell} \sqrt{\ln X + \ln(\ln X)} \quad (5)$$

It is assumed that the step reactivity, equivalent to the total reactivity inserted by the ramp during the excursion, is initially introduced beyond prompt critical. The initial multiplication constant is then defined as

$$K(0) = k_0 + k_1(t_1) = 1 + \beta + \alpha t_1 \quad (6)$$

Since the net reactivity is initially at its maximum and reduced with negative reactivity feedback from the Doppler effect and/or core disassembly during the excursion, $k_1(t_1)$ is termed k_{\max} in the following for clarity as well as for convenience. It is assumed in our study that t_1 comes when the fuel boiling occurs at the peak power location of the core.

2.3. Disassembly Reactivity

2.3.1. Bethe-Tait Approach

The reactivity feedback due to a change in density, $\rho(\vec{r}, t)$, of the reactor material caused by the material displacement, $\vec{u}(\vec{r}, t)$, can be written as [4]

$$k_d(t) = \int \rho(\vec{r}, t) \vec{u}(\vec{r}, t) \cdot \nabla w(\vec{r}) dV \quad (7)$$

where $w(\vec{r})$ is the reactivity change due to removal of unit mass of material at position \vec{r} . Under the assumption that the density in the hydrodynamic equations is constant in time, Eq.(7) is differentiated twice in time, holding $\rho(\vec{r}, t)$ constant at its initial value $\rho(\vec{r})$, and then displacement is related to the pressure by equation of motion to give

$$\frac{d^2 k_d(t)}{dt^2} = - \int \nabla \rho(\vec{r}, t) \cdot \nabla w(\vec{r}) dV \quad (8)$$

Applying the first-order perturbation theory to the one group diffusion equation, we obtain for a spherical reactor [4],

$$\ddot{k}_d = \frac{48 q^2 F / \rho_c}{4\pi \Sigma_{tr} \Sigma_f b^2 [1 - (6 q^2 / 5) + (3 q^2 / 7)]} \int \rho dV \quad (9)$$

where Σ_{tr} , Σ_f are the transport and fission cross-sections, b is the core radius, F is fraction of fission in the core, ρ_c and p are density and pressure of the core, respectively. It is assumed that the flux can be approximated by a parabola in the core, $\Phi = 1 - q(r^2/b^2)$. Thus \ddot{k}_d is proportional to the pressure integrated over the volume of the core. The pressure-energy relations for the core during the power excursion are among the key parameters to be provided for the core disassembly process. Bethe and Tait assumed a particularly simple relation between pressure

generation and energy density, taking the pressure to be negligible until the energy density reaches a threshold value Q^* and increasing linearly thereafter without significant expansion.

2.3.2. Development of Equations of State

At the initiation of the super-prompt critical excursion, liquid uranium is assumed interspersed with void spaces left in the core when the coolant is expelled. As the temperature rises, the voids are filled with the expanded liquid producing saturated vapor pressure. If the liquid reaches the threshold energy to fill the voids completely, the pressure begins to rise rapidly thereupon. In this context, therefore, the equations of state of pressure-energy density relationship are derived in this study for the saturated-vapor as well as the single-phase liquid of metallic uranium fuel. Mathematical formulations for the disassembly reactivity are then developed to utilize the improved equations of state.

A vapor pressure equation for uranium is given by Raugh and Thorn[16] as,

$$\log p = 5.702 - \left(\frac{23,300}{T} \right). \quad (10)$$

where pressure is in atmosphere and temperature in K. This equation has been shown to provide the vapor pressure in reasonable accuracy from the melting point to the critical point. We need an expression relating pressure to energy rather than to temperature. Assuming 0.1J/g-K as a reference value of the specific heat of the vaporized uranium core, the pressure-temperature relation was converted to that of pressure and energy density, which was then curve-fitted to a fourth-order polynomial ,

$$p = \sum_{i=0}^4 B_i E^i \quad (11)$$

$$\begin{aligned} \text{with } B_0 &= 1.297 \times 10^3, \quad B_1 = -6.018 \times 10^3, \\ B_2 &= 10.495 \times 10^3, \quad B_3 = -8.182 \times 10^3, \\ B_4 &= 2.416 \times 10^3 \end{aligned} \quad (12)$$

where the pressure is measured in MPa and the liquid energy in KJ/g.

Meanwhile, for the single-phase liquid region, an equation of state is developed in a linear threshold type. Use is made of the equation-of-state data calculated by Brout for the uranium density of 9.92g/cm³, which is close to the density of the sodium-voided core of the KALIMER. The Brout's data are listed in Table 1[7], where T^* and p^* denote the reduced temperature and pressure, respectively; $T = 14,000T^*(K)$, $p = 8,650p^*$ (atmospheres). T_c is the critical temperature of uranium and R is gas constant. The result of our fitting is

$$p = 11,000(E - 1.10) \quad (13)$$

where the pressure is measured in MPa and the liquid energy in kJ/g. (See Figure 1).

Table 1. Data for Equation of State

T^*	p^*	E/RT_c
0.96	-	2.0
1.05	2.0	2.1
1.11	3.0	2.2
1.21	5.0	2.5
1.33	7.0	2.8
1.60	10.0	3.4
1.90	15.0	4.0
2.40	20.0	5.0

2.3.3. Equations for Disassembly Reactivity

The equations of state developed in the above can be utilized to obtain the expressions for the disassembly reactivity. For the single-phase liquid region, we may substitute Eq.(13) into Eq.(9), the Bethe-Tait form of expressions for the disassembly

reactivity. In the saturated vapor region, the curve-fitted equation of state Eq.(11) is used and we get the second derivatives of the reactivity in time as follows[17];

$$\begin{aligned}\ddot{k}_q &= 0, \quad \text{for } Q < Q_b \\ \ddot{k}_q &= -\left(\frac{1}{x} \frac{k_{\max}^3}{\ell^2}\right) \sum_{i=0}^4 A_i(z) Q^i, \quad \text{for } Q_b < Q < \frac{Q_b}{1-q} \quad (14) \\ \ddot{k}_q &= -\left(\frac{1}{x} \frac{k_{\max}^3}{\ell^2}\right) \sum_{i=0}^4 A_i(q) Q^i, \quad \text{for } \frac{Q_b}{1-q} < Q < Q_b\end{aligned}$$

where X is a constant parameter characterizing the reactor core, q is a power shape factor, Q_b is the energy density at the boiling temperature of the core, and

$$\begin{aligned}A_0(z) &= \frac{2}{3} z^{2/3} B_0 \\ A_1(z) &= \left(\frac{2}{3} z^{2/3} - \frac{2}{5} z^{2/5}\right) B_1 \\ A_2(z) &= \left(\frac{2}{3} z^{2/3} - \frac{4}{5} z^{2/5} + \frac{2}{7} z^{2/7}\right) B_2 \quad (15) \\ A_3(z) &= \left(\frac{2}{3} z^{2/3} - \frac{6}{5} z^{2/5} + \frac{6}{7} z^{2/7} - \frac{2}{9} z^{2/9}\right) B_3 \\ A_4(z) &= \left(\frac{2}{3} z^{2/3} - \frac{8}{5} z^{2/5} + \frac{12}{7} z^{2/7} - \frac{8}{9} z^{2/9} + \frac{2}{11} z^{11/2}\right)\end{aligned}$$

Here, $z = 1 - Q_0/Q$ and Q_0 denotes the initial energy density of the core. The coefficients $A_i(q)$ are obtained replacing z with the power shape factor q in Eq.(15).

It is convenient for numerical analysis to define the following dimensionless variables[5,7,15]

$$y = \frac{Q - Q_0}{Q_0} \quad (16)$$

$$\kappa(t) = \frac{k(t) - 1 - \beta}{k_{\max} - 1 - \beta} \quad (17)$$

$$\tau = \frac{k_{\max} - 1 - \beta}{\ell} t \quad (18)$$

Eq.(14) may then be reduced to a simple set of differential equations in terms of the above

dimensionless variables[17],

$$\begin{aligned}\frac{d^2 \kappa}{d\tau^2} &= 0, \quad \text{for } y < \frac{Q_b - Q_0}{Q_0} \\ \frac{d^2 \kappa}{d\tau^2} &= -\frac{1}{x} \sum_{i=0}^4 A_i\left(\frac{y}{y+1}\right) Q_0^i (y+1)^i, \\ &\quad \text{for } \frac{Q_b - Q_0}{Q_0} < y < \frac{Q_b/Q_0}{1-q} - 1 \quad (19) \\ \frac{d^2 \kappa}{d\tau^2} &= -\frac{1}{x} \sum_{i=0}^4 A_i(q) Q_0^i (y+1)^i, \\ &\quad \text{for } \frac{Q_b/Q_0}{1-q} - 1 < y < \frac{Q^* - Q_0}{Q_0}\end{aligned}$$

where Q^* is the threshold energy of the single-phase liquid region. Likewise, it can be written for the single-phase liquid region[17].

$$\begin{aligned}\frac{d^2 \kappa}{d\tau^2} &= -\frac{y+1}{x} \left(\frac{Q_0}{Q^*}\right) \left(\frac{y+1 - Q^*/Q_0}{y+1}\right)^{5/2}, \\ &\quad \text{for } \frac{Q_b - Q_0}{Q_0} < y < \frac{Q^*/Q_0}{1-q} - 1 \quad (20) \\ \frac{d^2 \kappa}{d\tau^2} &= \frac{5q^{3/2}}{2x} \left[\left(1 - \frac{3}{5}q\right) \left(\frac{Q_0}{Q^*}\right) (y+1) - 1\right], \\ &\quad \text{for } y > \frac{Q^*/Q_0}{1-q} - 1\end{aligned}$$

2.4. Doppler Reactivity Feedback and Numerical Analysis

Following the approach taken by Wolfe et al.[6], the time rate change of the Doppler effect can be written as

$$\frac{d\Gamma}{d\tau} = \frac{-\alpha_D}{k_{\max}} \frac{(1 - 0.6q)\theta^{n-1}}{[\theta + (1 - 0.6q)(y+1) - 1]^n} \frac{dy}{d\tau} \quad (21)$$

where $\Gamma = \frac{k_D}{k_{\max}}$, $\theta = \frac{C_D T_0}{Q_0}$, and $\alpha_D = -T_0 \left(\frac{dk_D}{dT}\right)_{T_0}$.

Here $(dk_D/dT)_{T_0}$ is the Doppler temperature coefficient at temperature T_0 , at which the energy density Q_0 is achieved. The Doppler effect is

assumed to decrease in magnitude inversely as the n^{th} power of the temperature T , measured from absolute zero. C_v is a constant value of the heat capacity at constant volume of reactor core.

Rewriting Eq.(1) into the dimensionless form likewise, we obtain the equation for energy density

$$\frac{d^2 y}{dt^2} - (\kappa + \Gamma) \frac{dy}{dt} = 0 \quad (22)$$

with the initial conditions

$$\begin{aligned} y(0) &= 0, \quad \kappa(0) = 1, \quad \Gamma(0) = 0, \\ \frac{dy(0)}{dt} &= 1, \quad \frac{d\kappa(0)}{dt} = 0 \end{aligned} \quad (23)$$

which, together with Eqs.(19), (20) and (21), constitute a complete set of equations to have a unique solution of the problem. Starting with the initial conditions, the above equations can be numerically integrated using the Runge-Kutta method on a digital computer. Iterations are continued until the reactor power falls below a preset value.

3. Analysis of Core Meltdown Accident in KALIMER

3.1. Reactor Model

The KALIMER core system is designed to generate 392MWt of power. The reference core utilizes a heterogeneous core configuration with driver fuel and internal blanket zones alternately loaded in the radial direction. The core consists of 48 driver fuel assemblies, 18 internal blankets, 6 control rods, 1 ultimate shutdown system(USS) assembly, 6 gas expansion modules (GEMs), and is surrounded by layers of radial blankets, reflectors, shield assemblies, and in-vessel storage of fuel assemblies, in an annular configuration. There are no upper or lower axial blankets

surrounding the core. The reference core has an active core height of 120 cm and a radial equivalent diameter(including control rods) of 172 cm[17,18].

The driver fuel assembly includes 271 fuel pins. The fuel pins are made of sealed HT-9 tubing containing metal fuel slug of U-Pu-10%Zr in columns. The driver fuel and blanket have smeared densities of 75% and 85%, respectively. The power fractions of the internal blankets significantly increase with burnup and, consequently, the location of the peak linear power shifts from the inner driver fuel zone to the innermost internal blanket region. The peaking factor is close to 1.5, which provides a basis for using the power-shape factor q of 0.6 in this study. The peak linear power is 286.5 W/cm, which is equivalent to a specific power of about 60 W per gram of fuel. Table 2 lists the KALIMER reactor parameters used in this study for the base cases.

The fuel temperature (Doppler) coefficients are evaluated for sodium-flooded/voided cases. It is estimated to vary as $0.11 T^{-1.49}$ for the sodium-voided case, whereas it varies as $0.10 T^{-1.43}$ in the case of the sodium-flooded core. The Doppler

Table 2. Reactor Parameters

Reactor Parameters	Values
$\Sigma_{lr} v \Sigma_f$	1.030×10^{-3}
q	0.6
l (sec)	2.65×10^{-7}
β (cm)	0.035
Volume Fraction(%)	
Fuel Slug	29.75
Coolant	42.91
Structure	27.34
Core Density	
$\rho_{hyd}(\text{g/cm}^3)$	
$\rho_c(\text{g/cm}^3)$	4.70
Fuel Loading(MT)	9.23

coefficient does not show any substantial change with burnup. Taking into account some uncertainty in the correlation for Doppler coefficients, -0.002 is taken as the best-estimate value of the Doppler constant for subsequent analyses, for the sake of conservatism [19].

3.2. Initial Conditions and Thermal Properties

The core is assumed to be initially at prompt critical in molten state. Initial energy content of the core, Q_0 , is therefore taken to be 0.25 KJ/g, the internal energy needed to heat uranium from room temperature to the melting point(1,400 K). The boiling temperature of the core is set at around 4,500 K and the corresponding energy Q_b at 0.8 KJ/g. The specific heat of metallic fuel is assumed to be close to 0.2 J/g-K just above the melting point and assumed to stay constant beyond.

Another initial condition to be specified is k_{max} , the amount of step reactivity equivalent to the total reactivity inserted by the ramp during the excursion, as given in Eq.(6). In addition to what are given in the above, we need power at the prompt critical state, $\dot{Q}(0)$. A simple formula for $\dot{Q}(0)$, brought by introducing the reactivity at the constant rate of a dollars per second to an initially delayed critical reactor of the power level, \dot{Q}_{ss} , may be derived by solving the one-group point kinetics equations without reactivity feedback[20] ;

$$\dot{\alpha}(0) = \dot{Q}_{ss} \left(\frac{\pi\beta}{2\lambda} \right)^{1/2}. \quad (24)$$

Table 3 shows the ratio of the power at prompt critical to a steady-state power, time of boiling t_1 , and k_{max} for each of the assumed reactivity insertion rates, estimated using Eqs.(3),(4),(5) and (24).

Table 3. Parameters for Initial Conditions

$a(\$/s)$	$\dot{Q}(0)/\dot{Q}_{ss}$	$t_1(ms)$	$k_{max}(\$)$
10	45.548	9.71E-03	9.71E-02
20	32.207	7.27E-03	1.45E-01
30	26.297	6.12E-03	1.84E-01
50	20.369	4.92E-03	2.46E-01
60	18.594	4.54E-03	2.73E-01
80	16.103	4.01E-03	3.21E-01
100	14.403	3.63E-03	3.63E-01

3.3. Analysis Results

Results of the reference case are listed in Table 4 for the peak values of energy generation density, temperature and pressure for various reactivity insertion rates and Doppler constants. It can be observed from the table that the Doppler effect significantly affects the power excursion.. Without the Doppler effect considered, the excursions are terminated with fairly large energy releases accompanying strong pressure rises, in the range of a few tens of thousands atmosphere, over quite a short period of time, typically a few hundred microseconds. The Doppler effects are more pronounced with the excursions initiated by low rates of reactivity insertion. For the Doppler constant of -0.002 taken as the reference value for KALIMER in this study, the power excursions are terminated even before the core reaches the assumed energy density of the boiling point(0.8KJ/g) for reactivity insertion rates up to 50 \$/s. And reactor would shutdown without any significant pressure rise or energy release. For the reactivity insertion rate of 100 \$/s, the energy density at the peak location of the core goes over the boiling point but stays around the threshold value of the solid liquid region(1.10 KJ/g). Only the peak spot of the core would boil, however, whereas most area of the core would be in the pre-boiling liquid state. As the fuel vapor

Table 4. Results of Energy, Temperature and Pressure

Ramp	Doppler Constant	Peak Values at Core Center		
Rate (\$/s)	Constant (α_D)	Energy Density(KJ/g)	Temperature (K)	Pressure (bar)
10	0.0	1.46	7,440	4,030
	-0.001	0.68	3,560	38
	-0.002	0.48	2,570	0
20	0.0	1.58	8,030	52,400
	-0.001	0.91	4,720	70
	-0.002	0.58	3,040	0
50	0.0	1.79	9,090	75,700
	-0.001	1.32	6,770	1,700
	-0.002	0.80	4,160	0
100	0.0	2.01	10,220	100,700
	-0.001	1.58	8,030	5,250
	-0.002	1.12	5,770	360

generated at the peak spot of the core fills some of the voids left out of sodium coolant, the pressure gradually rises, while the power continues to be in decline under the influence of Doppler feedback. The core dispersion would be then with the fuel of low energy density driven by relatively low pressure.

4. Benchmark Analysis of Oxide-Fuelled Core

A number of simulations were performed for the cases studied by Hicks and Menzies[8], as a means of checking the extent of the accuracy or conservatism of our method, particularly the assumption of step reactivity insertion equivalent to ramp rate. Hicks and Menzies investigated various aspects of the course of events during a super-prompt critical excursion for a spherically symmetric sodium-voided core using the PHOENIX program. An extensive set of density-dependent equations of state for temperature and energy density as well as pressure and energy density was developed for the fuel assumed to be

UO₂. The Doppler constant was estimated to be -0.24 %. In our calculations of energy release, information available in the report by Hicks and Menzies or typical values of oxide-fuelled core were assumed for the reactor parameters.

The results of these calculations are summarized for several reactivity insertion rates in Table 5, which compares the peak values of the energy density at the core center with those given by Hicks and Menzies. It may be noted from the fourth column of the table that our method, using the asymptotic values of t_1 and k_{\max} given in Eqs.(3), (4), and (5), consistently predicts higher values of energy release,(about two times on the average.) than those of Hicks and Menzies. This trend is more pronounced as the excursion gets stronger.

The trend of our method to overestimate energy release mostly comes from the conservatism put into estimating the amounts of step reactivity equivalent to the ramp rates. It was observed in the course of our calculations that the asymptotic values of t_1 , are much larger than the actual values of time of boiling t_b (, which resulted from our

Table 5. Comparison of Energy Densities of Oxide-Fuelled Core

Ramp Rate (\$/s)	Doppler Constant (α_D)	Energy Density at Core Center(KJ/g)		
		Hicks & Menzies	Our Methods	
			Asymptotic k_{max}	Adjusted k_{max}
75	0.0	3.52	7.25	3.47
	-0.001	1.81	4.10	1.63
	-0.002	1.71	2.42	1.57
150	0.0	4.11	8.82	3.96
	-0.001	1.99	5.30	1.76
	-0.002	1.90	3.55	1.70

analyses of the excursions), about 50 times depending on the ramp rates and Doppler constants assumed for the study. Being converted into the inserted reactivity by Eq. (5), this gives rise to overestimation of k_{max} , which comes to drive power excursions much more severely in our calculations. The values of t_1 were subsequently adjusted so that the resulting values of t_b come close to for each case of excursions. The results are listed in the last column of Table 5. As shown, the results are in good agreement with those of Hicks and Menzies, being within about 10 % in cases of ramp rates of 75 and 150 \$/s, which are in the range of our design-basis ramp rates. Such agreement appears fairly remarkable, considering the uncertainties involved in these kinds of hypothetical accidents, including the high-temperature material properties, equations of state and reactor parameters, among others.

Calculations were repeated for the KALIMER with the amounts of the maximum step reactivity k_{max} adjusted as described in the above. All the power excursions studied were terminated before the core gets to its boiling point, given the reference value of Doppler constant. It was also noted that the accidents are terminated without any significant energy yield even with a lower value of Doppler constant ($\alpha_D = -0.001$).

5. Sensitivity Study

5.1. Equation of State

The equation of state plays an important role in calculations of the course of a hypothetical fast reactor excursion, for it serves as the link between the neutronic relations and dynamic behavior of a core which leads to ultimate shutdown. There exist, however, considerable uncertainties in our knowledge of the equation of state as well as material properties at extreme conditions of the temperature and pressure, occurring during the power excursion of fast reactors. Resort has therefore been made to theory and correlation for estimation of these physical properties at extreme conditions. For instance, R.H.Brout applied the law of corresponding states and developed empirical relations data for pressure and energy for constant-volume, single-phase conditions of uranium[7]. A set of his data, as listed in Table 1, was used to generate the reference equation of state for the single-phase region of sodium-voided core. Another equation of state is generated based on the Brout's data for lower core density ($\rho = 7.44\text{g/cc}$), to see the effect on the energy yield of higher value of threshold energy and slower

Table 6. Calculation of Energy Densities for Various Equations of State

Ramp Rate (\$/s)	Doppler Constant (α_D)	Peak Energy Densities(kJ/g)			
		Case 1 (Ref.)	Case 2	Case 3	Case 4
50	0.0	1.79	1.79	2.30	2.41
	-0.001	1.32	1.34	1.38	1.53
	-0.002	0.80	0.80	0.80	0.80
100	0.0	2.01	2.02	2.67	2.74
	-0.001	1.58	1.58	1.85	1.98
	-0.002	1.12	1.14	1.12	1.14

pressure rise. The equation is

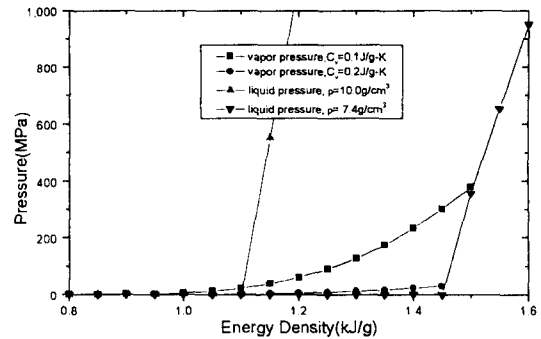
$$p = 5,940(E - 1.44) \quad (25)$$

where the pressure is measured in MPa and the liquid energy in KJ/g.

The vapor pressure equation, given by Rau and Thorn in Eq.(10), has been shown to provide the vapor pressure in reasonable accuracy from the melting point to the critical point. The framework of methods in our study, however, requires an expression relating pressure to energy rather than to temperature. The specific heat of uranium is not well known in the high temperature region, particularly so above the vaporization temperature. As described in Section 2.3.2, the reference value of the specific heat constant of uranium was set to be 0.1 J/g-K at the vapor region, in line with the works by Brout and Nicholson[7]. Sensitive studies were carried out assuming 0.2 J/g-K as the specific heat, which was expected to yield higher pressure as well as energy release. The resulting coefficients for the pressure as function of energy, curve-fitted to a fourth-order power series are $p = \sum_{i=0}^4 B_i E^i$;

$$\text{with } B_0 = -87.25, B_1 = 104.7, B_2 = -1.530, B_3 = -36.25, B_4 = 11.07 \quad (26)$$

where the pressure is measured in MPa and the

**Fig. 1. Equations of State for Uranium**

liquid energy in KJ/g. Figure 1 illustrates various equations of state selected for the sensitivity studies.

Two sets of the pressure-energy relationship for each phase then make four cases of sensitivity calculation in this study ;

- 1) Case 1(Reference Case) : Eq.(12) for saturated vapor($C_v \approx 0.1$ J/g-K), Eq.(13) for single-phase liquid ($\rho = 9.92$ g/cc),
- 2) Case 2 : Eq.(26) for saturated vapor($C_v = 0.2$ J/g-K), Eq.(13) for single-phase liquid ($\rho = 9.92$ g/cc),
- 3) Case 3 : Eq.(12) for saturated vapor($C_v = 0.1$ J/g-K), Eq.(25) for single-phase liquid ($\rho = 7.44$ g/cc),
- 4) Case 4 : Eq.(26) for saturated vapor($C_v = 0.2$ J/g-K), Eq.(25) for single-phase liquid ($\rho = 7.44$ g/cc),

Results of the energy densities at the peak spot

Table 7. Energy Densities with Various Specific Heat Constants

Ramp Rate (\$/s)	Doppler Constant (α_D)	Peak Energy at Core Center (kJ/g)		
		$C_v = 0.10$ (J/g-K)	$C_v=0.15$ (J/g-K)	$C_v=0.20$ (J/g-K)
50	0.0	1.71	1.79	1.79
	-0.001	0.64	1.18	1.32
	-0.002	0.44	0.66	0.80
100	0.0	2.01	2.01	2.01
	-0.001	0.80	1.48	1.58
	-0.002	0.53	0.90	1.12

of the core are listed in Table 6 for each of the four cases described in the above. There are essentially no differences in the results between the first two cases, meaning that the results are insensitive to the particular equation of state for the saturated vapor pressure. Meanwhile, we can see by comparing Cases 1 and 3 (and Cases 2 and 4 as well) that the results are rather sensitive to the equation of state for the liquid. The differences are more pronounced with a smaller value of Doppler constant, reaching as much as 35 % for the case of no Doppler feedback. It was observed that the value of threshold energy affects the results more than the gradient of the linear curve. For the Doppler constant of -0.002, however, the results essentially remain the same upon changing the linear threshold equation, simply because the core is not heated up much above the threshold energy of 1.10 kJ/g.

An observation to be drawn from this study is then that the results of energy release are not sensitive at all to the equation of state for vapor pressure. The threshold energy of the single-phase liquid of uranium affects the results but only when Doppler effects are rather small.

5.2. Specific Heat of the Metal Fuel

We have seen that the Doppler reactivity

feedback effect plays a crucial role in determining the core behavior during the accidents. One of the parameters of importance for the effect is the specific heat of the metal fuel. There had been considerable disagreement about its value for uranium at a high temperature above its melting point. Some measurement has been made just above the melting point, the values ranging from 0.1 to 0.2 J/g-K[7]. More recent measurement indicates toward close to 0.2J/g-K [21,22]. There exist, however, large uncertainties about the behavior of the specific heat far above the melting point. A parametric study was performed, therefore, to look into the sensitivity of our calculation with three values of the specific heat at the melting temperature and beyond; 0.1, 0.15 and 0.2 J/g-K. As described in the above, 0.2 J/g-K was taken as the reference value above the melting temperature in our study.

Table 7 lists the peak-spot energy densities of the core calculated for each of the three values of specific heat. It can be seen that, as the specific heat gets lower, energy densities decrease. The extent of the effect is more pronounced as the Doppler constant gets larger. Assuming the specific heat to be 0.15 J/g-K, for instance, gives the energy density of 0.9 kJ/g, which is about 20% less than what we get with the reference value of specific heat for the reference case. As

Table 8. Energy Densities for Various Neutron Lifetimes

Ramp Rate (\$/s)	Doppler Constant (α_D)	Peak Energy at Core Center (kJ/g)		
		1.0×10^{-7} s	2.65×10^{-7} s (Reference)	5.0×10^{-7} s
50	0.0	1.87	1.79	1.74
	-0.001	0.95	1.32	1.43
	-0.002	0.59	0.80	1.04
100	0.0	2.13	2.01	1.95
	-0.001	1.33	1.58	1.65
	-0.002	0.75	1.12	1.37

the specific heat is further reduced to the lower extreme of 0.1 J/g-K, the energy densities decrease by about 50 % for the Doppler constant of -0.001 to -0.002. It may be summarized then that the effect of the specific heat of the fuel on the Doppler reactivity feedback would not be that important as long as it stays in the range of 0.15 to 0.2 J/g-K above the melting point.

5.3. Reactor Parameters

There are a number of reactor parameters known to potentially influence the energy yield, which include the prompt neutron lifetime, the power distribution in the core, and neutron cross sections, among others. It was shown in our parametric study that the results of energy yield are not sensitive to neutron cross sections of the core. In this section, the results of sensitivity studies with the remaining two parameters are described.

In this scoping study, the power distribution in the assumed spherical core is represented by the power-shape factor q in the normalized distribution; $N(r) = 1 - q(r^2/b^2)$, where b is the core radius. At the time a core disassembly takes place, only the central part of the core goes beyond the boiling point and vapor pressure is built up there. When the core power is flattened, much larger

portion of the core may go beyond the boiling point resulting in the increase of energy yield. A parametric study was performed with three values of q ; 0.6, 0.4 and 0.2 in the increasing order of flattening, for various reactivity insertion rates and Doppler constants. When the value of q was changed from the reference value of 0.6 to 0.2, the peak energy density was increased by about 10% from the reference value, for the case of ramp rate of 100\$/s and Doppler constant - 0.002. If the Doppler effect is not considered, the amount of increase reached 20 % or so for the same case. The effect of the core flattening is shown to become smaller for the lower ramp rates.

Table 8 lists the peak energy density calculated for three values of prompt neutron lifetime, 10^{-7} s, 2.65×10^{-7} s (design value) and 5×10^{-7} s, for the two different ramp rates and various Doppler constants. It may be noted that, in the case of no Doppler effect, the energy release decreases with increase in neutron lifetime. With the Doppler effect considered, however, the energy density increases with increasing neutron lifetime. With the increase of the neutron lifetime to about two times the reference value, the energy yield increases about 20% for the reference case. The extent of the increase gets smaller for lower ramp rates and Doppler constants.

6. Conclusions

Analysis of the behavior of the sodium-voided core of the KALIMER during super prompt-critical excursions was performed for various reactivity insertion rates up to 100 \$/s, using the SCHAMBETA code developed in this study. The result shows that there exist significant influences of Doppler effect on the power excursions in the metallic core of the KALIMER. For the best-estimate value -0.2 % for the KALIMER, the power excursion was terminated without an energetic disassembly even for extremely large reactivity insertion rates of 100\$/s,

Benchmark calculations showed that our method predicts higher values of energy release, about two times on average, than that of Hicks and Menzies on the oxide fuelled core of a fast reactor. It was found that the trend of our method to overestimate energy release mostly comes from the conservatism put into estimating the amounts of step reactivity equivalent to the ramp rates. With a parametric adjustment of the maximum reactivity inserted into the core, the results came to be in good agreement with those of Hicks and Menzies, being within about 10 % in the range of our design-basis ramp rates. The current scoping method should be useful for first-time conservative estimate of core disruptive accident energetics. However, simulating the ramp insertion of reactivity may well improve the accuracy of the results.

Finally, sensitivity studies were performed to look into the influences of various parameters on the consequences of the power excursions. Parameters investigated in this study include equations of state for pressure and energy, specific heat of uranium, and such reactor parameters as neutron lifetime, power distribution and neutron cross sections. It turned out that the results of energy release were insensitive to the equation of

state for vapor pressure. The threshold energy of the single-phase liquid of uranium affects the results to some degree but only when Doppler effects are rather small. For the KALIMER core, in which the Doppler constant is in the range of -0.002, equations of state for pressure should not be critical to a scoping analysis like this one. On the other hand, the specific heat of the fuel may significantly affect the consequence via changing the Doppler reactivity feedback effect if its value is out of the range between about 0.15 and 0.2 J/g-K at the melting point and beyond. The influence of the reactor parameters was not significant in terms of the peak energy density of the core as long as they remain within a reasonable range of the design value.

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