< Technical Notes >

EQUIVALENT MATERIAL PROPERTIES OF PERFORATED PLATE WITH TRIANGULAR OR SQUARE PENETRATION PATTERN FOR DYNAMIC ANALYSIS

MYUNG JO JHUNG* and JONG CHULL JO
Korea Institute of Nuclear Safety
19 Guseong-dong, Yuseong-gu, Daejeon, 305-338 Korea
*Corresponding author. E-mail : mjj@kins.re.kr

Received October 20, 2005
Accepted for Publication July 18, 2006

For a perforated plate, it is challenging to develop a finite element model due to the necessity of the fine meshing of the plate, especially if it is submerged in fluid. This necessitates the use of a solid plate with equivalent material properties. Unfortunately, the effective elastic constants suggested by the ASME code are deemed not valid for a modal analysis. Therefore, in this study the equivalent material properties of a perforated plate are suggested by performing several finite element analyses with respect to the ligament efficiencies.

KEYWORDS : Perforated Circular Plate, Equivalent Material Property, Ligament Efficiency, Modal Analysis

1. INTRODUCTION

The analysis of multiholed plates has attracted the attention of many engineers and designers due to the widespread use of tubesheet heat exchangers and other related equipment. A stress analysis of a plate perforated with a large number of holes, by the finite element method for instance, was a very costly and time-consuming technique. However, the rapid development of computers and software makes it possible to model and analyze a perforated plate, and it is no longer time-consuming. On the other hand, if a perforated plate is submerged in fluid it is nearly impossible to model and analyze it and the fluid simultaneously in order to investigate the effect of the fluid-structure interaction. The simplest way to avoid a time-consuming and costly analysis of a perforated plate submerged in fluid is to replace the perforated plate with an equivalent solid plate considering the weakening effect of the holes.

In the past, a general method was required that could replace an actual drilled plate with an equivalent undrilled plate of the same dimensions in which the classical solid plate theory, in the elastic range, would be applicable. The equivalent plate must have the appropriate elastic constants so as to show the same behavior as an actual plate when subjected to the same loading.

These effective elastic constants must be correctly evaluated, especially in fixed tubesheet heat exchangers. If they are too low, the stresses at the junction of the shell and head will be lower than they are in realty. If they are too high, the stress at the center of the plate, which may be at the maximum, will be too low. Thus, a proper design can only be obtained if the analysis is performed using the best possible estimation of the effective elastic constants.

Many authors have proposed experimental or theoretical methods to solve this problem. Slot and O'Donnell [1] determined the effective elastic constants for thick perforated plates by equating the strains in an equivalent solid material to the average strains in the perforated material. O'Donnell [2] also presented those of thin perforated plates. These results are implemented in Article A-8000 of Appendix A to the ASME code Section III [3], which contains a method of analysis for flat perforated plates when subjected to directly applied loads or loadings resulting from structural interactions with adjacent members. This method applies to perforated plates that satisfy the conditions (a) through (e) below:

(a) The holes are in an array of equilateral triangles.
(b) The holes are circular.
(c) There are 19 or more holes.
(d) The ligament efficiency is greater than 5%.
(e) The plate is thicker than twice the hole pitch. If only in-plane loads or thermal skin stresses are considered, this limitation does not apply.

The equivalent material properties presented in the ASME code were originally determined based on the deflection due to the loading, which considers only the first mode from a modal characteristic point of view. Therefore, it is necessary to redefine the equivalent material properties of a perforated plate for a modal analysis.

This study deals with the free vibration characteristics of a circular perforated plate with rectangular and square penetration patterns. The natural frequencies are obtained by theoretical calculations and three dimensional finite element analyses. The effect of holes on the modal characteristics is investigated; in addition, new equivalent elastic constants which can be used for the modal analysis of a perforated plate are proposed.

2. ANALYSIS

2.1 Theoretical Analysis

On the basis of the theoretical analysis developed by Jhung et al. [4, 5], the non-dimensional Galerkin equation is numerically solved using MathCAD in order to find the natural frequencies of a circular plate. In order to check the validity and accuracy of the results from the theoretical study, finite element analyses are also performed and frequency comparisons between the two sets of data are carried out.

The model of the structure simulates the lower control element guide plate of an integral reactor, which is a circular perforated plate with a single or double hole pattern connected to a fixed closed-type container of carbon steel. The plate is made of 321 SS with a radius of 1384 mm and a thickness of 30 mm and 3 mm (Fig. 1). The physical properties of this material at 310ºC are as follows: Young's modulus = 173.0 GPa, Poisson's ratio = 0.3 and mass density = 8027 kg/m^3.

The frequency equations derived by Jhung et al. [4, 5] involve an infinite series of algebraic terms. Before exploring

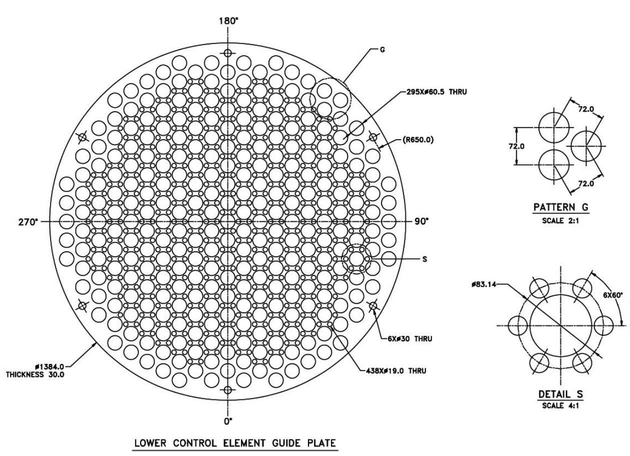
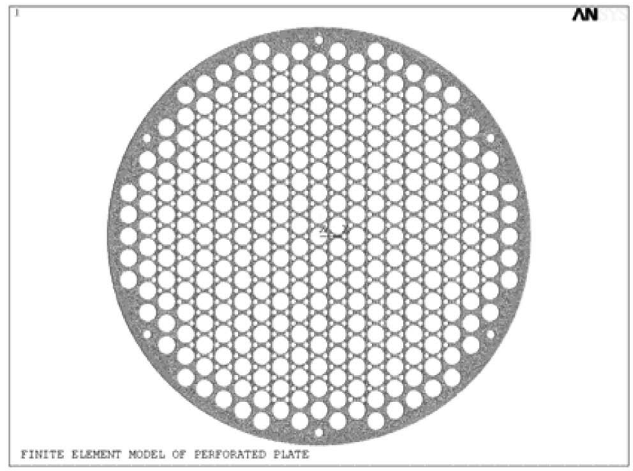
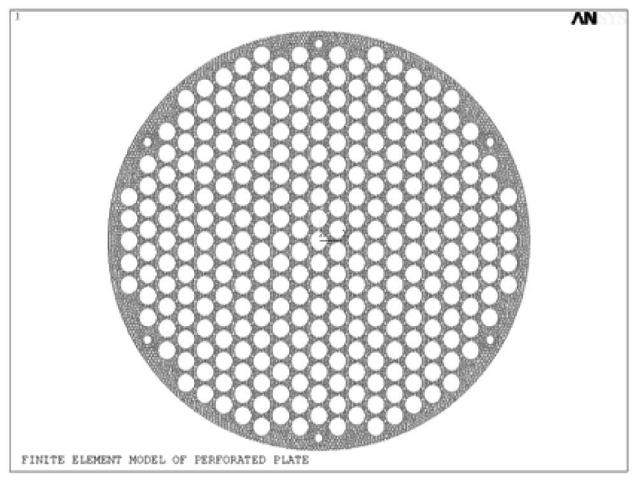


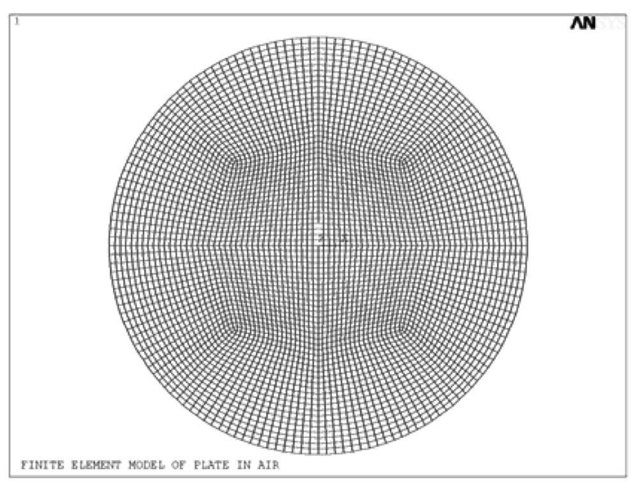
Fig. 1. Perforated Plate



(a) Perforated plate with double hole



(b) Perforated plate with single hole



(c) Solid plate

Fig. 2. Finite Element Models

the analytical method to obtain the natural frequencies of the fluid-coupled plate, it is necessary to conduct convergence studies and establish the number of terms required in the series expansions involved. In the numerical calculation, the Bessel-Fourier expansion term s is set to 200 and the expanding term m (or M) for the admissible function is set to 40, which gives a sufficiently accurate solution by convergence.

2.2 Finite Element Analysis

Finite element analyses using the commercial computer code ANSYS 8.1 [6] were performed to verify the analytical results for the theoretical study. The results from the finite element method were used as the baseline data. Three different three-dimensional models were developed for perforated plates, with a single pattern, a double hole pattern and solid plate, as shown in Fig. 2. The circular plate is modeled as an elastic shell element (SHELL63) with four nodes. The perimeter nodes of the plate are fixed with six degrees of freedom.

Several finite element analyses were performed depending on the temperature, plate thickness or plate modeling. The Block Lanczos method was used for the eigenvalue and eigenvector extractions to calculate 300 frequencies including fluid modes [7]. It uses the Lanczos algorithm in which the Lanczos recursion is performed with a block of vectors. This method is as accurate as the subspace method, but faster. The Block Lanczos method is especially powerful when searching for eigenfrequencies in a given part of the eigenvalue spectrum of a given system. The convergence rate of the eigenfrequencies will be nearly identical when extracting modes in the midrange and higher end of the spectrum as when extracting the lowest modes.

3. RESULTS AND DISCUSSION

The frequency comparisons between the analytical solution and finite element method are shown in Fig. 3 for plates with a thickness of 30 mm in air and the frequency differences between them are practically negligible. The symbol m in the tables represents the number of nodal circles of the mode and the symbol n denotes the number of the nodal diameter. A comparison of frequencies between a thickness of 30 mm and 3 mm shows that the frequencies are proportional to the thickness. This is also shown from the theory, as follows:

Introducing the Rayleigh quotient of the circular plate in air, the natural frequency ω becomes [4, 5]

$$\omega^2 = \frac{V_d}{T_d} \tag{1}$$

where V_d and T_d are the maximum potential energy and reference kinetic energy, respectively. By insertion of energy terms into Eq. (1), the relationship between the natural frequency and the thickness of the plate results in

$$\omega \propto \sqrt{\frac{D}{\rho t}} = t\sqrt{\frac{E}{12\rho(1-\mu^2)}} \tag{2}$$

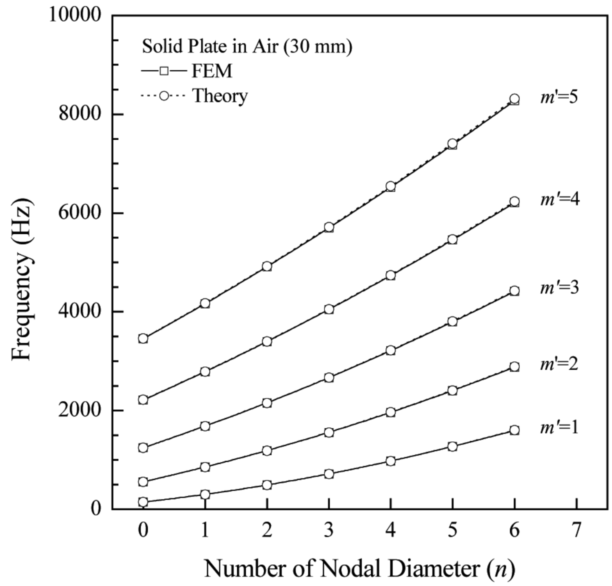


Fig. 3. Frequency Comparisons of Solid Plate Between FEM and Theory (thickness = 30 mm)

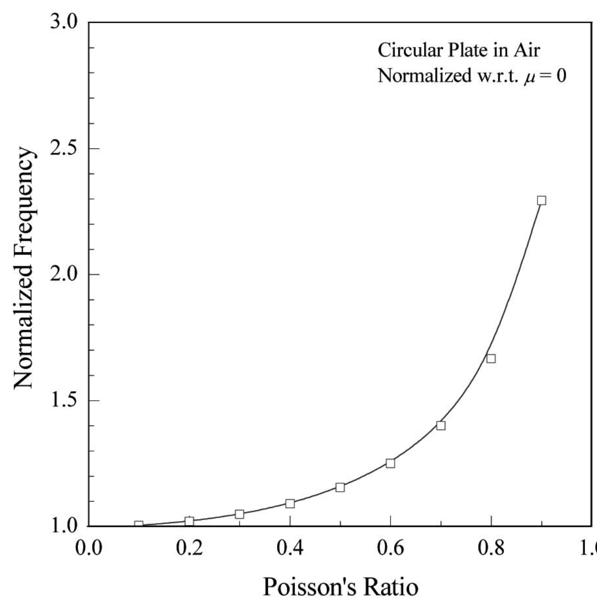


Fig. 4. Relationship Between the Natural Frequency and Poisson's Ratio

where D, ρ, t, E, μ are the flexural rigidity, density, thickness, Young's modulus and Poisson's ratio of the plate, respectively. As mentioned earlier, the natural frequency of the plate in air is proportional to the thickness of the plate. The relationship between the natural frequency and Poisson's ratio is also determined from Eq. (2), and is shown in Fig. 4.

For a plate submerged in fluid, Eq. (1) becomes exceedingly complicated and the relationship between the natural frequency and the plate thickness is not expressed explicitly as in the case of the in-air condition. This greatly complicates the prediction of the modal characteristics of a plate submerged in a fluid with respect to the plate thickness.

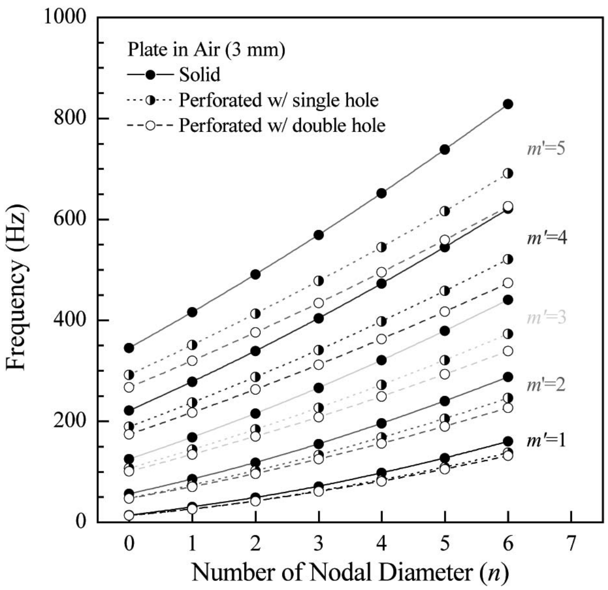


Fig. 5. Frequency Comparisons Between Solid and Perforated Plates in Air

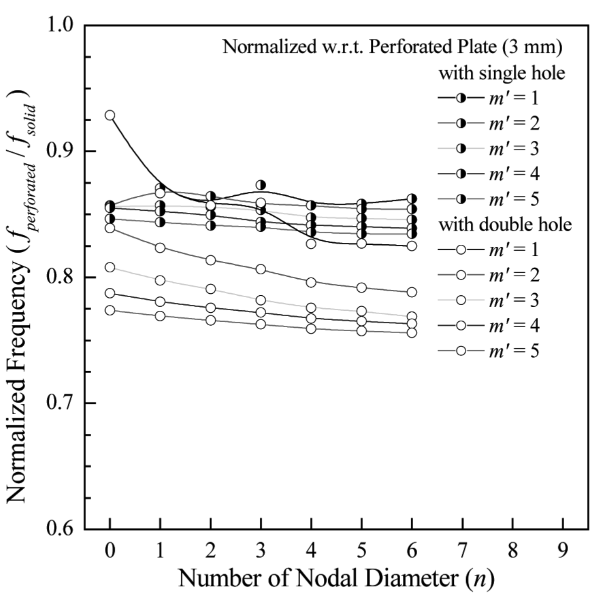


Fig. 6. Normalized Frequencies of Perforated Plate w.r.t. Solid Plates in Air

The frequencies of a solid plate and a perforated plate with single or double hole patterns and their normalized values are shown in Figs. 5 and 6, respectively. In all cases, as the number of nodal circles increases, the frequencies increase, which were not shown with cylindrical shells [8, 9]. The inclusion of holes decreases the frequency significantly, and the effect of holes is more significant for the double hole pattern, becoming more significant as the modal number increases. Typical mode shapes of radial modes are shown in Figs. 7 and 8 for a solid and perforated plate, respectively.

The frequency comparisons between a perforated plate with its original properties and a solid plate with equivalent properties in air are shown in Fig. 9, where the equivalent material properties are calculated from Article A-8000 of Appendix A to the ASME code Section III [3]. Their normalized values as shown in Fig. 10 indicate that using the effective elastic constants of the ASME code generates frequencies that are too low; therefore, it is not reasonable to use the solid plate model with the effective elastic constants provided by the ASME code for a modal analysis of a perforated plate. As the frequency is proportional to the square root of the elastic constant, the effective elastic constants must be increased by a ratio of about six in order to match the frequencies. The effective elastic constants given in the ASME code are restricted to plates with a thickness t of greater than twice the pitch of the hole pattern P; the model studied here has t/P = 30/72 = 0.42, which is not viable for an application to the ASME code values. The effective elastic constants generated by O'Donnell [2] for the thin perforated plate were also found to be too low. This necessitates a redefinition of the equivalent material properties for the modal characteristics of a perforated plate

Considering a circular perforated plate with a triangular or square penetration pattern, as shown in Figs. 11 and 12, where the radius of the plate and pitch of the hole are 1384 mm and 72 mm, respectively, the modal characteristics and the equivalent elastic constants are investigated as a typical case.

As mentioned earlier, the equivalent elastic constants of a perforated plate with a triangular penetration pattern proposed by the ASME code were found to be no more valid for the modal analysis. Therefore, it is necessary to redefine the equivalent elastic constants such that the modal characteristics of the perforated plate with original properties are the same as those of a solid plate with modified equivalent properties. The best means of finding the equivalent constant is as follows:

(a) Develop a finite element model for solid plates and perforated plates with various ligament efficiencies.
(b) Perform the modal analyses for the perforated and solid plates with original properties.

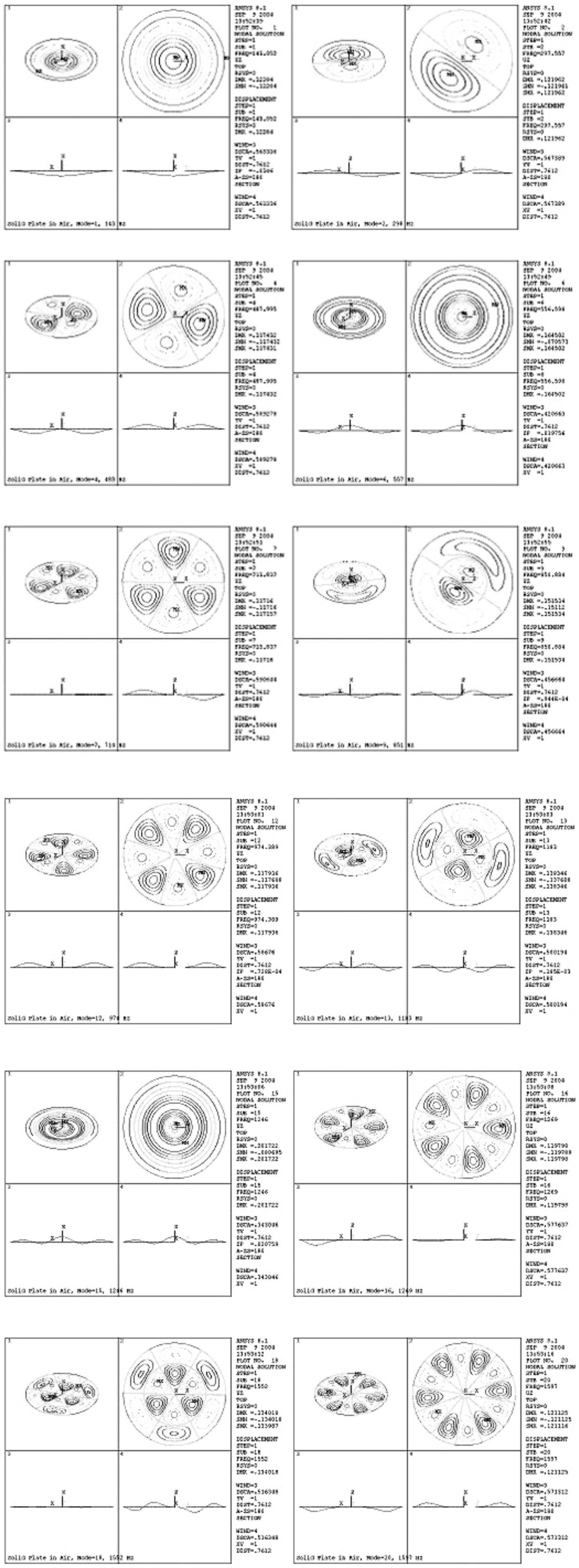


Fig. 7. Typical Mode Shapes of a Solid Plate

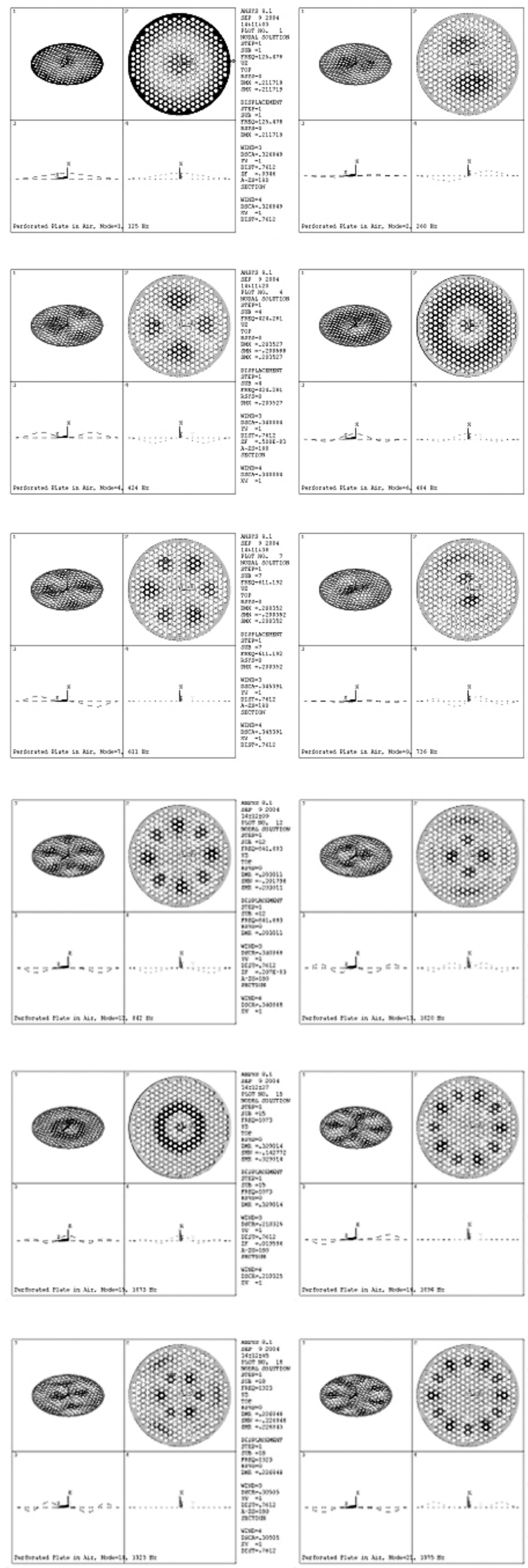


Fig. 8. Typical Mode Shapes of a Perforated Plate

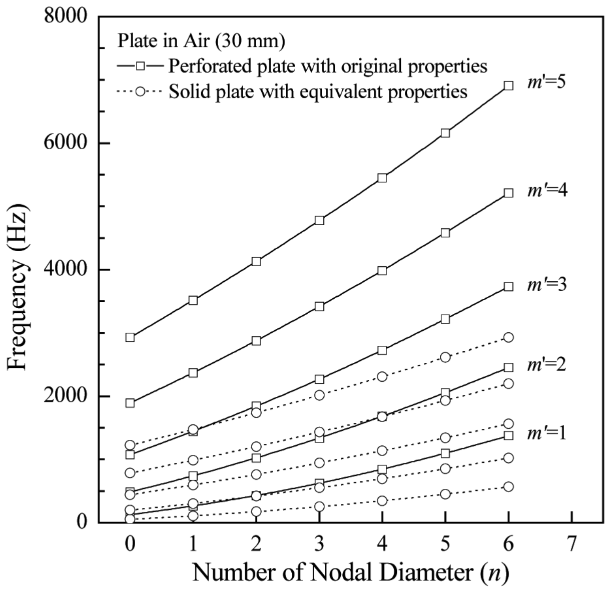


Fig. 9. Frequency Comparisons Between a Perforated Plate with Original Properties and a Solid Plate with Equivalent Properties in Air (thickness = 30 mm)

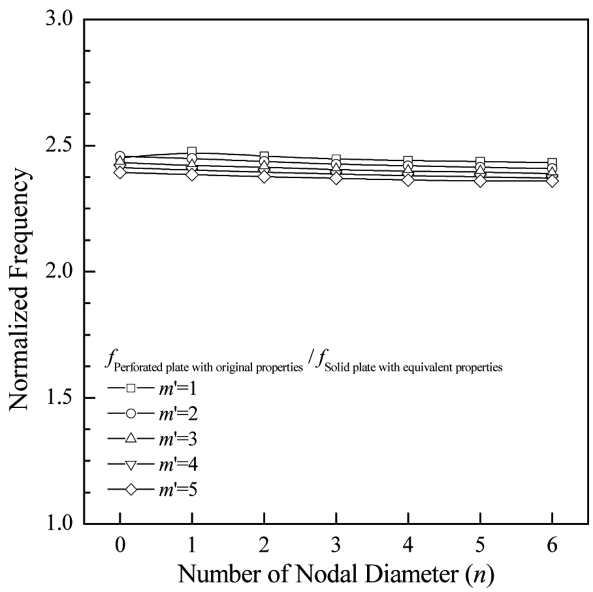


Fig. 10. Normalized Frequencies of a Perforated Plate with Original Properties with Respect to a Solid Plate with Equivalent Properties in Air (thickness = 30 mm)

(c) Compare the frequencies and determine the ratio of frequencies for the perforated plate to those of solid plate.
(d) Find the multipliers of the Young's modulus of solid plate in order to match the frequencies of the perforated plate with the original properties using the relationship between the frequency and Young's modulus.

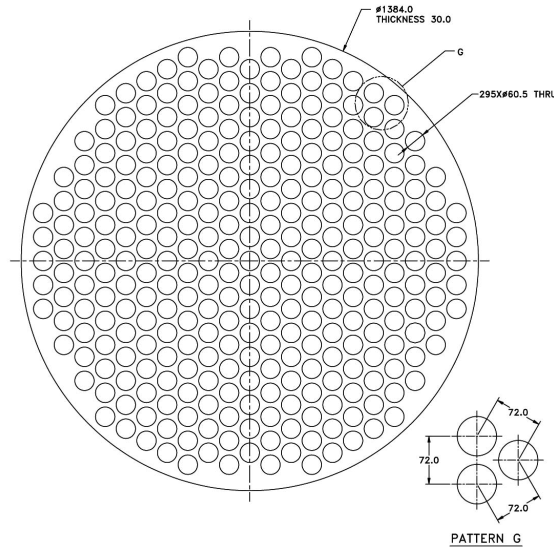


Fig. 11. Perforated Plate with a Triangular Penetration Pattern

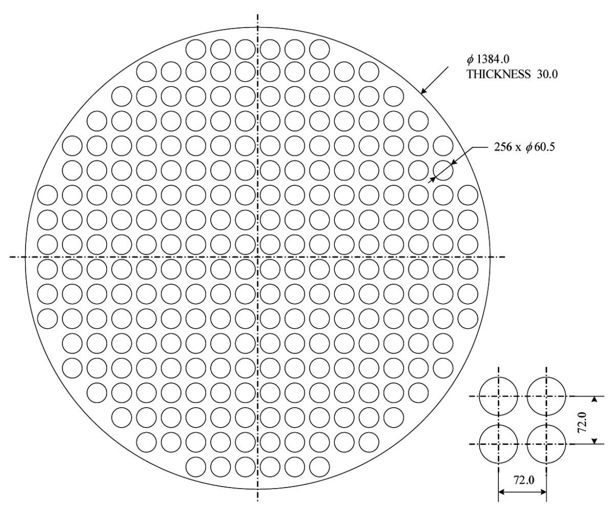


Fig. 12. Perforated Plate with a Square Penetration Pattern

(e) The effective elastic constant for each ligament efficiency is averaged for all modes.

This type of analysis is repeated for several ligament efficiencies $\eta = h/P$. As the frequency is proportional to the thickness of the plate even if it is perforated, it is not necessary to perform several analyses with respect to the thickness. Therefore, in this study a thickness ratio of t/P = 0.05 is considered for a typical case.

The natural frequencies are summarized in Figs. 13 and 14 for the triangular and square penetration patterns, and their normalized values for a perforated plate with respect to a solid plate are obtained. As the elastic constant is proportional to the square of the frequency, the normalized frequencies are squared and these are the effective elastic constants with the same modal frequencies as the perforated plate are obtained for the solid plate. As the effective values

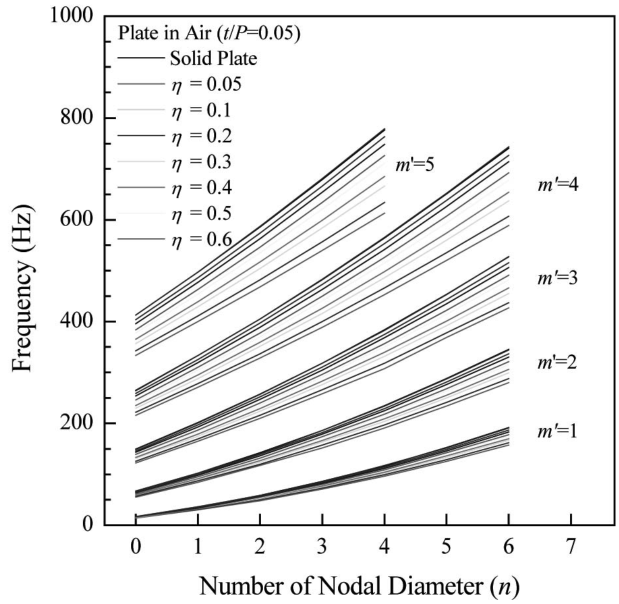


Fig. 13. Natural Frequencies of a Plate for a Solid Plate and Perforated Plates with a Triangular Penetration Patterns for Various Ligament Efficiencies

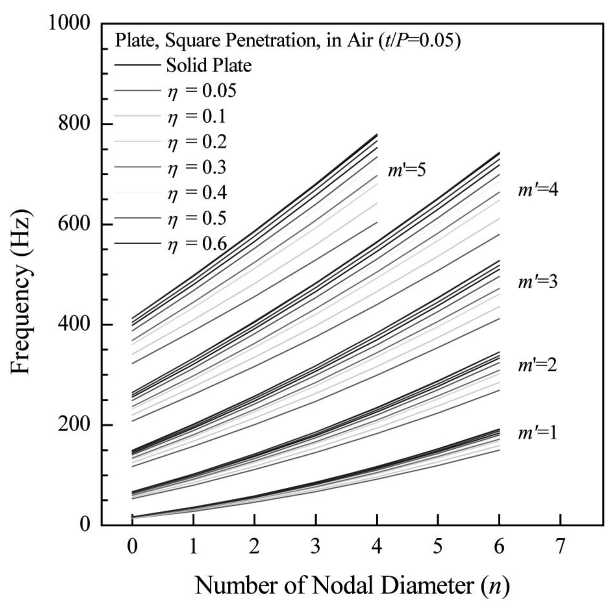


Fig. 14. Natural Frequencies of a Plate for a Solid Plate and Perforated Plates with a Square Penetration Pattern for Various Ligament Efficiencies

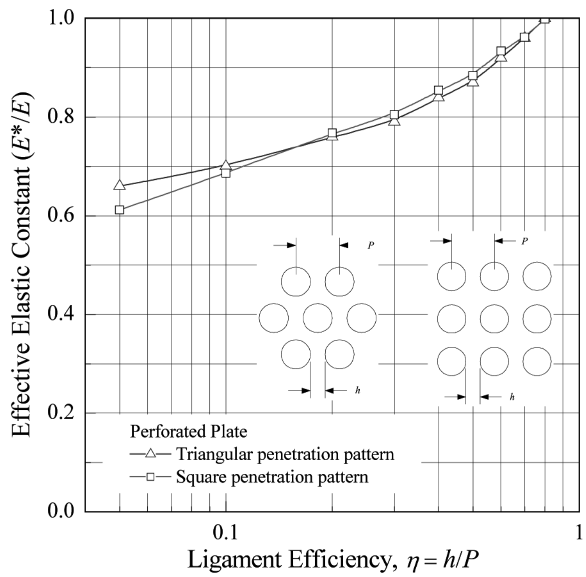


Fig. 15. Effective Elastic Constants of Perforated Plates

depend on the mode numbers, their average values are used. There is a large difference between the effective elastic constants of Slot and O'Donnell [1] and those determined here. Although values from Slot and O'Donnell are for the thick plate $t/P \geq 2.0$, this is not assumed to be valid from the modal characteristic point of view for a perforated plate due to the fact that the frequency is proportional to the thickness of the plate. Figure 15 shows the final effective elastic constants proposed for the modal characteristics of a perforated plate with triangular and square penetration patterns.

The natural frequency of the plate is proportional to the thickness of the plate even for a thick plate, which shows the frequencies of a perforated plate with respect to the thickness with a ligament efficiency of 0.05. Therefore, the effective elastic constants proposed in Fig. 15 can be used for the entire thickness range of the plate.

Figures 16 and 17 show the frequency comparisons between a perforated plate with the original properties and a solid plate with the effective elastic constant determined in this study. Good agreement was found between these data with a difference of less than 3%, verifying the validity of the method developed in this study that calculates the effective elastic constants for a modal analysis of a perforated plate with triangular and square penetration patterns.

4. CONCLUSIONS

It is challenging and can be time consuming to develop a finite element model for a perforated plate. In addition, the effective elastic constants suggested by the ASME

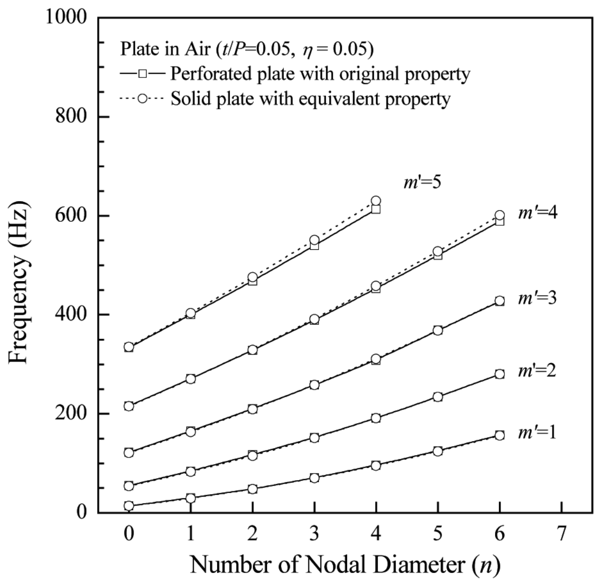


Fig. 16. Comparison of the Frequencies Between a Triangular Perforated Plate with its Original Properties and a Solid Plate with Equivalent Properties

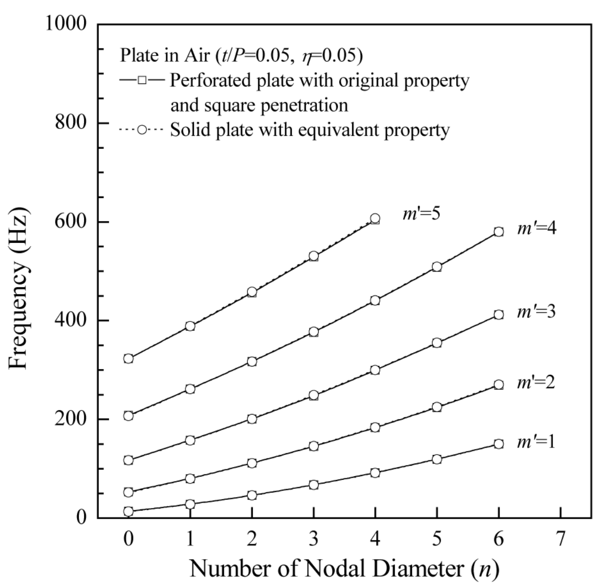


Fig. 17. Comparison of the Frequencies Between a Square Perforated Plate with its Original Properties and a Solid Plate with Equivalent Properties

code are found to be invalid for a modal analysis. Therefore, suggested in this study are the equivalent material properties of a perforated plate that can be used for a dynamic analysis through the performance of several finite element analyses with respect to the ligament efficiencies.

The Young's moduli proposed for the modal analysis of perforated plates with triangular and square penetration patterns are as follows:

$$\frac{E^*}{E} = 0.6106 + 1.1253\eta - 2.7118\eta^2 + 4.0812\eta^3 - 2.1128\eta^4 \tag{3}$$

$$\frac{E^*}{E} = 0.5280 + 2.0035\eta - 5.4758\eta^2 + 7.7474\eta^3 - 3.8968\eta^4 \tag{4}$$

respectively, where E^* is the equivalent Young's modulus.

Using the equivalent Young's modulus defined in this study, a modal analysis of the perforated plate is performed. The frequencies of a perforated plate with its original properties agreed well with those for a solid plate with equivalent properties.

REFERENCES

[1] Slot, T., and O'Donnell, W.J., "Effective Elastic Constants for Thick Perforated Plates with Triangular and Square Penetration Patterns," *Journal of Engineering for Industry*, Vo.93, No.4, pp.935-942 (1971).

[2] O'Donnell, W.J., "Effective Elastic Constants for the Bending of Thin Perforated Plates with Triangular and Square Penetration Patterns," *Journal of Engineering for Industry*, Vol.95, pp.121-128 (1973).

[3] ASME, *ASME Boiler and Pressure Vessel Code*, Section III Rules for Construction of Nuclear Facility Components, Appendix A Stress Analysis Method, Article A-8000 Stresses in Perforated Flat Plates, The American Society of Mechanical Engineers (2004).

[4] Jhung, M.J., Choi, Y.H., and Jeong, K.H., "Fluid Bounding Effect on Natural Frequencies of Fluid-Coupled Circular Plates," *KSME International Journal*, Vol.17, No.9, pp.1297 -1315 (2003).

[5] Jhung, M.J., Choi, Y.H., Kim, H.J., and Jeong, K.H., "Natural Vibration Characteristics of a Clamped Circular Plate in Contact with Fluid," *Structural Engineering and Mechanics*, Vol.21, No.2, pp.169-184 (2005).

[6] ANSYS, *ANSYS Structural Analysis Guide*, ANSYS, Inc., Houston (2004).

[7] Grimes, R.G., Lewis, J.G., and Simon, H.D., "A Shifted Block Lanczos Algorithm for Solving Sparse Symmetric Generalized Eigenproblems," *SIAM Journal on Matrix Analysis and Applications*, Vol.15, No.1, pp.228-272 (1994).

[8] Jhung, M.J., Choi, Y.H., and Jeong, K.H., "Modal Analysis of Coaxial Shells with Fluid-Filled Annulus," *Structural Engineering and Mechanics*, Vol.16, No.6, pp.655-674 (2003).

[9] Jhung, M.J., Jeong, K.H., and Hwang, W.G., "Modal Analysis of Eccentric Shells with Fluid-Filled Annulus," *Structural Engineering and Mechanics*, Vol.14, No.1, pp.1-20 (2002).