REACTIVITY OSCILLATION IN SOURCE-DRIVEN SYSTEMS

S. DULLA*, C. NICOLINO and P. RAVETTO
Politecnico di Torino, Dipartimento di Energetica
Corso Duca degli Abruzzi 24 - 10129 Torino, Italy
*Corresponding author. E-mail : sandra.dulla@polito.it

Received March 18, 2006
Accepted for Publication June 16, 2006

The problem of reactivity oscillations for a point reactor constitutes an interesting aspect of nuclear reactor physics and its solution may give important information for dynamic and safety assessments. The present paper considers the problem of a reactivity oscillation for a source-driven system which involves some specific aspects that introduce significant differences with respect to the source-free situation. Assuming a square-wave shape for the reactivity insertion, the solution is derived by a fully analytical approach. The conditions for stability and instability can be identified in a straightforward way by directly studying the stationarity of the power response. Numerical results presented allow to discuss the role of the system kinetic parameters and of the time-shape of the reactivity wave.

KEYWORDS : Source-driven Systems, Reactivity Oscillations, Point Kinetics

1. INTRODUCTION

The study of the behaviour of a nuclear multiplying system under the insertion of a periodic reactivity is an interesting problem in nuclear reactor kinetics [1]. Oscillated experiments are often conducted in order to obtain information on the physical characteristics of a reactor. An analytic approach was used in the past to show some peculiar features of the point kinetic model which are connected to the presence of the delayed emission phenomenon [2]. However, in that work no source was considered. At present there is a widespread interest for the study of source-injected subcritical nuclear systems. Hence, it is deemed worth-while to apply the same analytical approach also in the present work to obtain the full solution for a source-driven system following a reactivity square wave and to determine the conditions for which a stationary oscillated response may be established. The analytical solution allows to clearly understand the physical role of the source in determining the evolution properties and the stability conditions for the system, under which a steady power oscillation is established.

The investigations on the dynamic response to periodic reactivity insertions may have some interest also for the study and safety assessment of molten salt reactors. In these systems, reactivity oscillations may appear as a consequence of the precipitation of some fissile material to form lumps that are then driven through the core and the primary circuit by the fluid salt motion, thus a positive reactivity is inserted periodically [3].

The present paper considers the problem in the absence of any feed-back phenomenon. Of course the presence of non-linear effects may greatly impact on the stability of the system and on the response that is expected following the same reactivity insertion.

2. THE SOLUTION FOR THE SOURCE-DRIVEN POINT REACTOR EQUATIONS

The analytical solution for a step reactivity insertion for the point reactor equations is well-known. For completeness the main steps for the derivation of the analytical solution of the problem are briefly shown here.

The system of equations defining the point model for one delayed neutron precursor is the following [4]:

$$\begin{cases} \dfrac{dP(t)}{dt} = \dfrac{\rho - \beta}{\Lambda} P(t) + \lambda C(t) + S(t), \\ \dfrac{dC(t)}{dt} = \dfrac{\beta}{\Lambda} P(t) - \lambda C(t), \end{cases} \tag{1}$$

where ρ is the (constant) reactivity value and the other symbols appearing here have their standard meaning. The problem is well-posed once initial conditions for the functions P and C are assigned. For the particular case of a steady-state system driven by a constant source and characterized by subcriticality negative reactivity ρ_0, if an initial condition is assumed to be a steady state, it is given by the following formulae:

$$\begin{cases} P(0) = -\dfrac{\Lambda S}{\rho_0}, \\ C(0) = -\dfrac{S\beta}{\rho_0 \lambda}. \end{cases} \tag{2}$$

The contribution of several families of delayed neutron emitters can be easily included in the model with no further complications.

It is convenient and compact to use a matrix and vector Dirac's notation to carry out the mathematical procedure appearing in the following of the work. The ket symbol $|\cdot\rangle$ indicates a column vector, while the bra symbol $\langle\cdot|$ denotes a row vector. Therefore inner products are simply expressed by the notation $\langle\cdot|\cdot\rangle$. The system (1) can then be written in the following form:

$$\frac{d\,|\boldsymbol{X}(t)\rangle}{dt} = \hat{A}\,|\boldsymbol{X}(t)\rangle + |\boldsymbol{S}\rangle\,, \tag{3}$$

where the unknown state vector $|\boldsymbol{X}\rangle$, the source vector $|\boldsymbol{S}\rangle$ and the characteristic matrix $\hat{A}$ are defined as follows:

$$|\boldsymbol{X}(t)\rangle = \left|\begin{array}{l} P(t) \\ C(t) \end{array}\right\rangle, \quad |\boldsymbol{S}(t)\rangle = \left|\begin{array}{l} S(t) \\ 0 \end{array}\right\rangle, \tag{4}$$

$$\hat{A} = \begin{pmatrix} \dfrac{\rho-\beta}{\Lambda} & \lambda \\ \dfrac{\beta}{\Lambda} & -\lambda \end{pmatrix} \tag{5}$$

The standard technique for solving systems of ordinary differential equations exploits the direct and adjoint eigenvectors of matrix $\hat{A}$ [2], namely:

$$\hat{A}\,|\boldsymbol{\Psi_i}\rangle = \omega_i\,|\boldsymbol{\Psi_i}\rangle\,, \quad \langle\boldsymbol{\Psi_i}|\,\hat{A}^\dagger = \omega_i\,\langle\boldsymbol{\Psi_i}|\,, \tag{6}$$

which can be readily normalized so that:

$$\langle\boldsymbol{\Psi_i}\,|\boldsymbol{\Psi_j}\rangle = \delta_{ij}. \tag{7}$$

The original unknown and the source vectors can be expressed as a linear combination of such eigenvectors, leading to uncoupled first-order ordinary differential equations for the unknown components of vector $|\boldsymbol{X}\rangle$. The full solution can thus be worked out as:

$$\begin{aligned} |\boldsymbol{X}(t)\rangle = \sum_{i=1}^{2} \Big[&\langle\boldsymbol{\Psi_i}\,|\boldsymbol{X}(0)\rangle\, e^{\omega_i t} + \\ &\int_0^t \langle\boldsymbol{\Psi_i}\,|\boldsymbol{S}(t')\rangle\, e^{\omega_i (t-t')} dt' \Big] |\boldsymbol{\Psi_i}\rangle\,. \end{aligned} \tag{8}$$

If a steady source is assumed, the solution can be given a quite useful formulation, as:

$$\begin{aligned} |\boldsymbol{X}(t)\rangle &= \sum_{i=1}^{2} \left[\langle\boldsymbol{\Psi_i}|\ \boldsymbol{X}(0)\rangle\, e^{\omega_i t} + \langle\boldsymbol{\Psi_i}|\ \boldsymbol{S}\rangle\, \frac{e^{\omega_i t}-1}{\omega_i} \right] |\boldsymbol{\Psi_i}\rangle \\ &= \sum_{i=1}^{2} \left[|\boldsymbol{\Psi_i}\rangle\,\langle\boldsymbol{\Psi_i}|\, e^{\omega_i t} \right] |\boldsymbol{X}(0)\rangle + \\ &\quad \left[|\boldsymbol{\Psi_i}\rangle\,\langle\boldsymbol{\Psi_i}|\, \frac{e^{\omega_i t}-1}{\omega_i} \right] |\boldsymbol{S}\rangle \\ &\equiv \hat{\vartheta}(t)\,|\boldsymbol{X}(0)\rangle + \hat{\xi}(t)\,|\boldsymbol{S}\rangle\,, \end{aligned} \tag{9}$$

where the time-dependent *response* matrices $\hat{\vartheta}(t)$ and $\hat{\xi}(t)$ are properly introduced with obvious definitions. These matrices yield the system response in terms of both power and precursor concentrations at time t when applied to the initial state and to the external source.

3. THE SQUARE WAVE REACTIVITY OSCILLATION PROBLEM AND ITS PHYSICAL FEATURES

3.1 The Analytical Solution

A subcritical system driven by a steady source is now considered. It is supposed that an oscillating reactivity is introduced into such a system with a time behaviour described by a square wave as shown in Fig. 1. The analytical solution for a reactivity step written down in the previous section can be used recursively to determine the response of

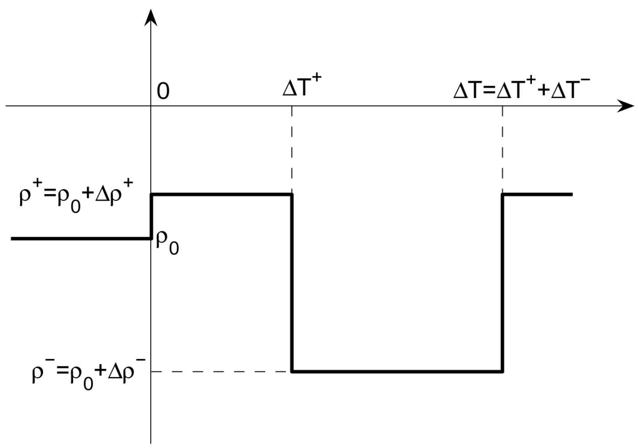


Fig. 1. Square Wave for the Reactivity Oscillation. Upper and Lower Reactivity Limits (ρ^+ and ρ^-) are Indicated, Together with Corresponding Time Intervals ΔT^+ and ΔT^-

the system. In each oscillation period ΔT, the eigenproblem (6) is solved for reactivities ρ^+ (inserted for the time interval ΔT^+) and ρ^- (lasting for ΔT^-) to obtain eigenvalues ω_i^+ and ω_i^-, respectively, together with the corresponding direct and adjoint eigenvectors, $|\Psi_i^\pm\rangle$ and $\langle\Psi_i^\pm|$. Consequently, the response matrices $\hat{\vartheta}(\Delta T^\pm)$ and $\hat{\xi}(\Delta T^\pm)$ can be evaluated using the definitions stemming from Eq. (9). At last, in order to be able to evaluate the effect of each reactivity oscillation, the following matrices are introduced:

$$\hat{\Theta}^{\pm} \equiv \hat{\vartheta}(\Delta T^{\pm}), \quad \hat{\Xi}^{\pm} \equiv \hat{\xi}(\Delta T^{\pm}). \tag{10}$$

For each interval, either ΔT^+ or ΔT^- within one period, it is possible to write an expression having the same structure as Eq. (9), since the reactivity is kept constant. Starting from the initial state, one gets:

$$\left|\boldsymbol{X}(\Delta T^+)\right\rangle = \hat{\Theta}^+ \left|\boldsymbol{X}(0)\right\rangle + \hat{\Xi}^+ \left|\boldsymbol{S}\right\rangle \tag{11}$$

and:

$$\left|\boldsymbol{X}(\Delta T)\right\rangle = \hat{\Theta}^- \left|\boldsymbol{X}(\Delta T^+)\right\rangle + \hat{\Xi}^- \left|\boldsymbol{S}\right\rangle. \tag{12}$$

Equations (11) and (12) can be combined and generalized for any, say the *n*-th, oscillation, included in the interval between $t^{(n)}$ and $t^{(n+1)} = t^{(n)} + \Delta T$:

$$\begin{aligned} \left|\boldsymbol{X}(t^{(n+1)})\right\rangle &= \hat{\Theta}^- \left(\hat{\Theta}^+ \left|\boldsymbol{X}(t^{(n)})\right\rangle + \hat{\Xi}^+ \left|\boldsymbol{S}\right\rangle\right) + \hat{\Xi}^- \left|\boldsymbol{S}\right\rangle \\ &= \hat{\Theta}^- \hat{\Theta}^+ \left|\boldsymbol{X}(t^{(n)})\right\rangle + \hat{\Theta}^- \hat{\Xi}^+ \left|\boldsymbol{S}\right\rangle + \hat{\Xi}^- \left|\boldsymbol{S}\right\rangle \\ &\equiv \hat{\Phi} \left|\boldsymbol{X}(t^{(n)})\right\rangle + \left|\boldsymbol{B}\right\rangle, \end{aligned} \tag{13}$$

which can be used recursively. Matrix $\hat{\Phi}$ fully describes the evolutionary properties of the system (thus it is referred to as *evolution matrix*), while the vector $|B\rangle$ accounts for the contribution of the external source.

The condition for establishing a stationary oscillation is determined through the following obvious requirement:

$$\left|\boldsymbol{X}(t^{(n+1)})\right\rangle = \left|\boldsymbol{X}(t^{(n)})\right\rangle. \tag{14}$$

Using Eq. (14) in Eq. (13), the following non-homogeneous linear system of algebraic equations is obtained:

$$\left(\hat{\Phi} - \hat{I}\right) \left|\boldsymbol{X}(t^n)\right\rangle + \left(\hat{\Theta}^- \hat{\Xi}^+ + \hat{\Xi}^-\right) \left|\boldsymbol{S}\right\rangle = 0, \tag{15}$$

where $\hat{I}$ is the unit matrix. A solution exists for a source-driven system only if the following condition is satisfied [5]:

$$\det\left(\hat{\Phi} - \hat{I}\right) \neq 0. \tag{16}$$

The mathematical existence of a solution does not automatically imply, however, that it is physically meaningful, since also a positivity requirement for both the power and the delayed precursor concentration must be fulfilled. Therefore, the realization of the stationary conditions is connected through matrix $\hat{\Phi}$ to the values of the physical parameters of the system, e.g. effective prompt neutron lifetime and effective delayed neutron fraction, as well as to the parameters of the oscillating reactivity wave, $\Delta T^\pm$ and $\rho^\pm$. The value of the determinant appearing in Eq. (16) determines the possibility to establish a stationary oscillating response in the system. It is worth-while to recall the result also for a source-free system [2]. In this case, the source term is dropped from Eq. (15), hence a homogeneous algebraic system is obtained. Consequently, the stationarity condition for a non-vanishing solution requires that the condition (16) is modified into the following [2,5]:

$$\det\left(\hat{\Phi}-\hat{I}\right)=0. \tag{17}$$

Due to the role played in characterizing the evolution of the system, it is justified to denote $\det\left(\hat{\Phi}-\hat{I}\right)$ as the *critical determinant*.

3.2 The Role of the Eigenvalues of the Evolution Matrix

To characterize the evolution of the system following a given initial state, it is useful to represent the unknown and the source vectors using as a base in the $\Re^2$ space the eigenvectors of $\hat{\Phi}$, which however do not constitute an orthogonal base, hence also the adjoint vectors are needed. For that purpose, the following preliminary problems must be solved:

$$\hat{\Phi}\left|\mathbf{\Gamma_i}\right\rangle=\epsilon_i\left|\mathbf{\Gamma_i}\right\rangle, \quad \left\langle\mathbf{\Gamma_i}\right|\hat{\Phi}^{\dagger}=\epsilon_i\left\langle\mathbf{\Gamma_i}\right|. \tag{18}$$

The (real) eigenvalues ϵ_i are found by solving the secular equation:

$$\det\left(\hat{\Phi}-\epsilon\hat{I}\right)=0, \tag{19}$$

and the eigenvectors can be normalized so that $\langle\boldsymbol{\Gamma}_i|\boldsymbol{\Gamma}_j\rangle=\boldsymbol{\delta}_{ij}$. Starting from the initial state, one can construct the solution after the first oscillation period as:

$$\begin{aligned}\left|\boldsymbol{X}(\Delta T)\right\rangle=&\,\hat{\Phi}\left[x_1\left|\mathbf{\Gamma_1}\right\rangle+x_2\left|\mathbf{\Gamma_2}\right\rangle\right]+\\ &\left[b_1\left|\mathbf{\Gamma_1}\right\rangle+b_2\left|\mathbf{\Gamma_2}\right\rangle\right]\\ =&\,x_1\epsilon_1\left|\mathbf{\Gamma_1}\right\rangle+x_2\epsilon_2\left|\mathbf{\Gamma_2}\right\rangle+\\ &\left[b_1\left|\mathbf{\Gamma_1}\right\rangle+b_2\left|\mathbf{\Gamma_2}\right\rangle\right],\end{aligned} \tag{20}$$

where x_1 and x_2 (b_1 and b_2) are the components of the initial state (external source) vectors with respect to the base constituted by the eigenvectors $|\boldsymbol{\Gamma}_1\rangle$ and $|\boldsymbol{\Gamma}_2\rangle$, respectively. These components can be easily found by projection, as

$$\begin{aligned}x_i&=\left\langle\mathbf{\Gamma_i}\right|\mathbf{X}(0)\rangle,\\ b_i&=\left\langle\mathbf{\Gamma_i}\right|\mathbf{B}\rangle.\end{aligned} \tag{21}$$

The procedure can be applied recursively for all following periods, getting for the n-th oscillation the following expression:

$$\begin{aligned}\left|\boldsymbol{X}(n\Delta T)\right\rangle=&\,x_1\epsilon_1^n\left|\mathbf{\Gamma_1}\right\rangle+x_2\epsilon_2^n\left|\mathbf{\Gamma_2}\right\rangle+\left(1+\sum_{i=1}^{n-1}\epsilon_1^i\right)b_1\left|\mathbf{\Gamma_1}\right\rangle\\ &+\left(1+\sum_{i=1}^{n-1}\epsilon_2^i\right)b_2\left|\mathbf{\Gamma_2}\right\rangle.\end{aligned} \tag{22}$$

The first two terms in the right hand side represent the evolution of the initial state, while the other terms convolve the source contributions. For usual values of the kinetic parameters, one of the two eigenvalues, say ϵ_2, is very close to zero, hence the evolution of the system is linked almost exclusively to ϵ_1. By direct investigation of the sign of the coefficients of the second-degree algebraic equation (19), it is possible to conclude that such eigenvalue is always positive and can be smaller or larger than 1, depending on the oscillation parameters, once the kinetic parameters of the system are fixed. In particular, it turns out to be smaller than 1 if the determinant appearing in Eq. (16) is positive.

The observation of Fig. 2 shows that for a fixed subcriticality there exists a reactivity oscillation amplitude for which this determinant changes sign, and thus for larger amplitudes ϵ_1 becomes larger than 1. At last, one notices that the value of the eigenvalue ϵ_2 is related to $\boldsymbol{\omega}_2^-$, which, being connected to the effective lifetime and dominating in the prompt response, is normally very large in absolute value and negative; it is readily verified that:

$$\lim_{\omega_2^-\to-\infty}\epsilon_2=0. \tag{23}$$

Similar conclusions can be drawn also using the eigenvalues and eigenvectors of the matrix $\hat{\Theta}^+\hat{\Theta}^-$, if one is interested in the solution at times equal to $n\,\Delta T+\Delta T^+$.

3.3 The Asymptotic Behaviour

The properties of the evolution matrix completely define the asymptotic state of the system. From Eq. (22) the asymptotic state can be easily identified, under the condition that both eigenvalues are smaller than 1. On taking the limit for $n\rightarrow\infty$, one obtains the stationary oscillation as:

$$\begin{aligned}\lim_{n\to\infty}\left|\boldsymbol{X}(n\Delta T)\right\rangle&=\lim_{n\to\infty}\sum_{j=1}^{2}\left[x_j\epsilon_j^n+\left(1+\sum_{i=1}^{n-1}\epsilon_j^i\right)b_1\right]\left|\mathbf{\Gamma_j}\right\rangle\\ &=\sum_{j=1}^{2}\left[\left(1+\sum_{i=1}^{\infty}\epsilon_j^i\right)b_j\left|\mathbf{\Gamma_j}\right\rangle\right]\\ &=\sum_{j=1}^{2}\left[\frac{1}{1-\epsilon_j}b_j\left|\mathbf{\Gamma_j}\right\rangle\right],\end{aligned} \tag{24}$$

where the properties of the geometric series are made use of. One can also observe that the initial state does not contribute to the asymptotic behaviour, while both eigenvectors are present, as they are excited by the source term, which somehow reminds the typical physical features of multiplying systems driven by an external source [6]. In the case ϵ_1 is larger than 1, the response will result in a diverging oscillation of exponential type ($\sim \epsilon_1^n$).

An interesting comment can be made concerning the source-free system. In this case the source terms are dropped in Eq. (22). As a consequence, the stationary condition requires that there exists a unitary eigenvalue for the evolution matrix $\hat{\Phi}$, which is also stated by Eq. (17). Furthermore, it can be proved in a straightforward way that the other eigenvalue turns out to be exactly zero. This condition retains strong similarities with the well-known classic criticality condition of reactor physics. For this critical case in which ϵ_1 is exactly 1, if a source is present, the system behaves as a perfect integrator, and the oscillation would be driven to diverge linearly.

Figure 2 reports the behaviour of the critical determinant and of the fundamental eigenvalue ϵ_1. It is clearly seen that for reactivity amplitudes larger than a limiting value the determinant becomes negative (corresponding to an eigenvalue larger than 1) and hence the system is unstable. Such a limiting value for the amplitude decreases as the system becomes closer to criticality, as expected.

To summarize, the possible physical situations are the following:

- source-free system:
  - det $(\hat{\Phi} - \hat{I}) < 0 \cdots \Rightarrow \cdots$ no stationary oscillation can be established, the reactivity oscillation drives the system to exponential divergence;
  - det $(\hat{\Phi} - \hat{I}) = 0 \cdots \Rightarrow \cdots$ a stationary oscillation is asymptotically established;

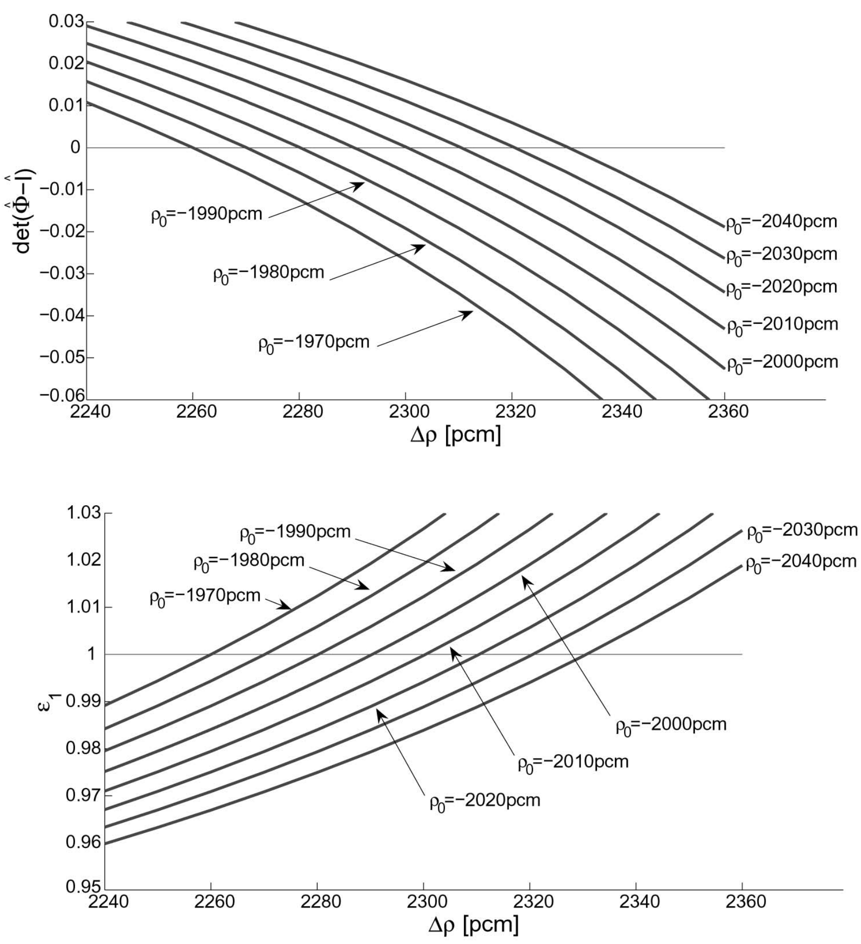


Fig. 2. Behaviour of the Critical Determinant and of the Fundamental Eigenvalue as a Function of the Reactivity Oscillation Amplitude, for Different Values of Subcriticality. The Kinetic Parameters are the Following: $\lambda = 0.1\ s^{-1}$, $\beta = 600$ pcm, $\Lambda = 10^{-4}$ s, $\Delta T^+ = \Delta T^- = 1$ s and $S = 1$ p.u./s (p.u. stands for arbitrary power units)

- det $(\hat{\Phi} - \hat{I}) > 0 \cdots \Rightarrow \cdots$ the system is asymptotically shutting down;
- source-driven system:
  - det $(\hat{\Phi} - \hat{I}) < 0 \cdots \Rightarrow \cdots$ the system is showing an asymptotically exponentially diverging oscillation;
  - det $(\hat{\Phi} - \hat{I}) = 0 \cdots \Rightarrow \cdots$ the system oscillates with a linearly diverging trend;
  - det $(\hat{\Phi} - \hat{I}) > 0 \cdots \Rightarrow \cdots$ the stationary oscillation can be established.

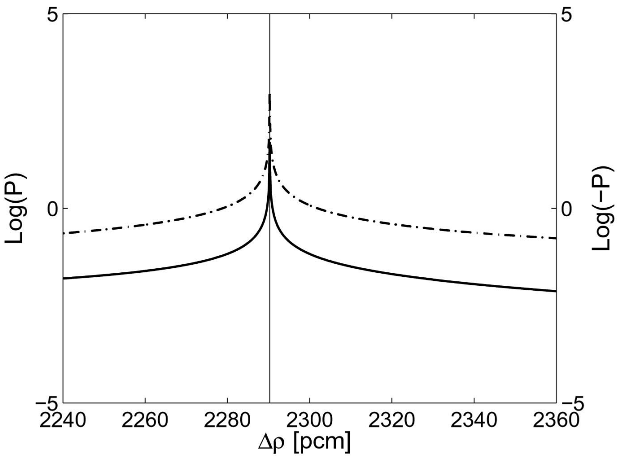


Fig. 3. Asymptotic Values of the Maximum Power (dash-dotted line) and Minimum Power in Each Oscillation (solid line) vs. the Reactivity Amplitude of the Oscillation. Unphysical Results are Obtained for Amplitudes Larger than the Limiting Value. The Kinetic Parameters are the Following: $\lambda = 0.1\ s^{-1}$, $\beta = 600$ pcm, $\Lambda = 10^{-4}$ s, $\Delta T^{+} = \Delta T^{-} = 1$ s, $\rho_0 = -2000$ pcm and $S = 1$ p.u./s

4. RESULTS AND DISCUSSION

Some significant results are now discussed, in order to enlighten the physical features of the problem that is analysed in the previous sections. For all cases the absolute values of the power given in suitable power units (p.u.) are plotted following a unitary steady-state source.

Figure 3 illustrates the behaviour of the asymptotic maximum and minimum values of the power during one reactivity oscillation, as a function of the reactivity amplitude. It can be clearly seen that approaching the limiting value from the left both diverge. For amplitudes larger than the limiting value the critical determinant is still non-vanishing, hence condition (16) is satisfied: therefore mathematically a solution for an asymptotic stationary oscillation still exists, however it retains no physical meaning, giving negative values for power and precursor concentrations.

The following Fig. 4 illustrates the evolution of the power and of the delayed neutron concentration starting from a fixed initial condition for a system allowing an asymptotic stationary oscillation. Also the curves enveloping the maximum and minimum values in each oscillations are drawn.

At last, Fig. 5 describes the effect of different physical system parameters and characteristics of the reactivity oscillation on the evolution of the solution. Various parameters in the reference system are let to vary, one at a time, in order to evidence the effect on the response. All physical effects shown are a direct consequence of the delayed neutron production and of the unbalance obtained during each reactivity oscillation. The reduction of the subcriticality level (Fig. 5(a)) induces an increase of both minimum and maximum power together with an increase of the amplitude

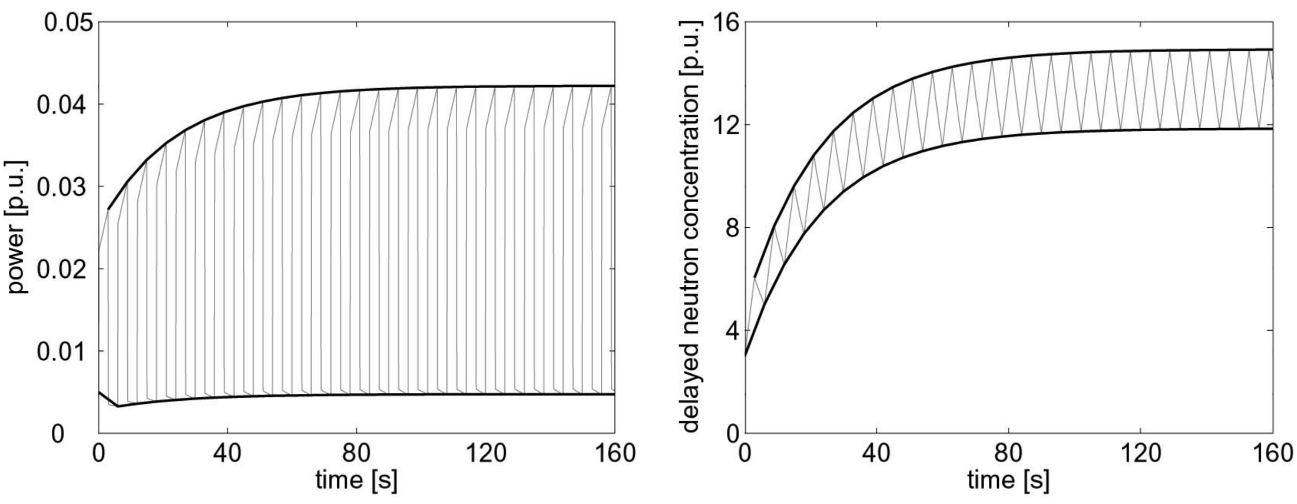
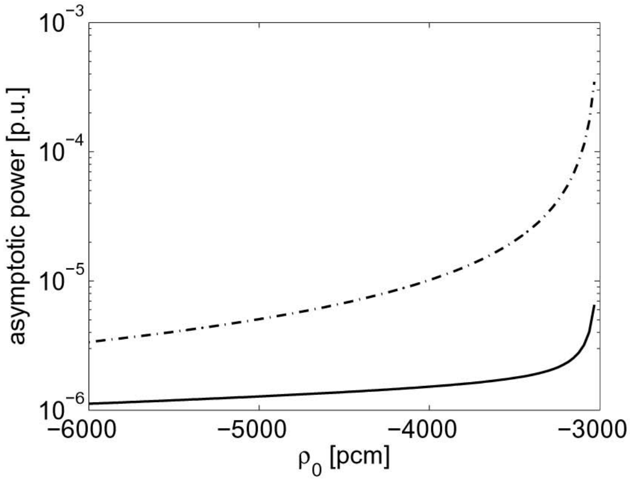
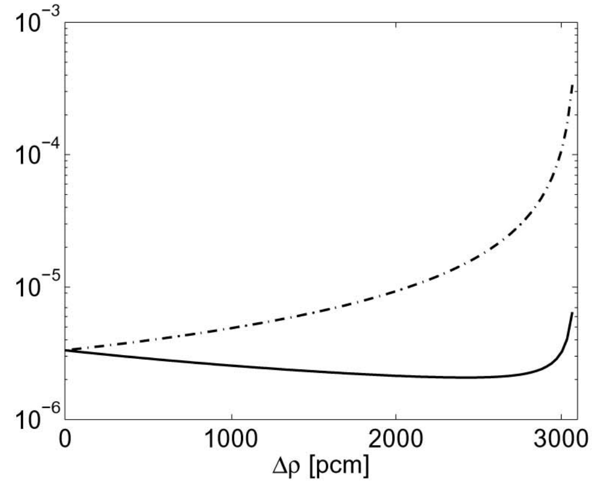


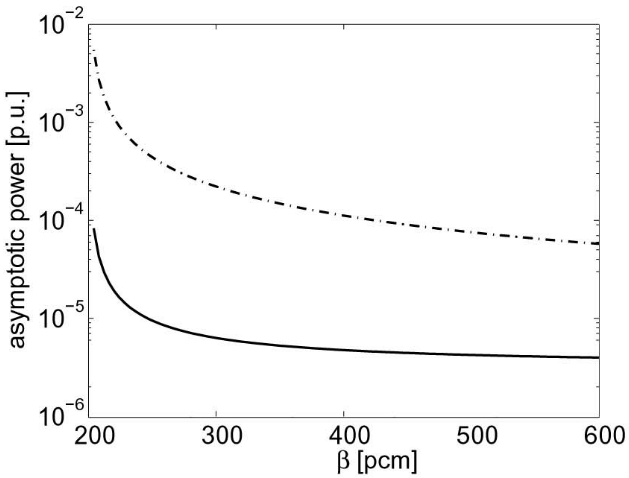
Fig. 4. Evolution of the Power and of the Delayed Neutron Concentration Starting from an Equilibrium Condition. ρ_0, $\Delta\rho$ and Kinetic Parameters are: $\rho_0 = -2000$ pcm, $\Delta\rho^{+} = \Delta\rho^{-} = 2010$ pcm, $\lambda = 0.1\ s^{-1}$, $\beta = 600$ pcm, $\Lambda = 10^{-4}$ s, $\Delta T^{+} = \Delta T^{-} = 3$ s and $S = 1$ p.u./s. The Lines Enveloping the Evolution are also Indicated



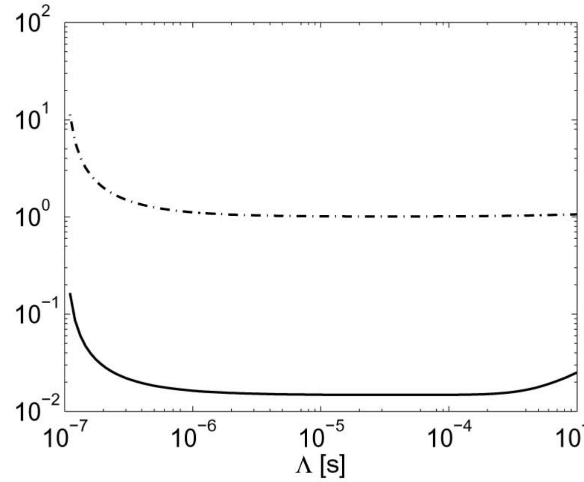
(a)



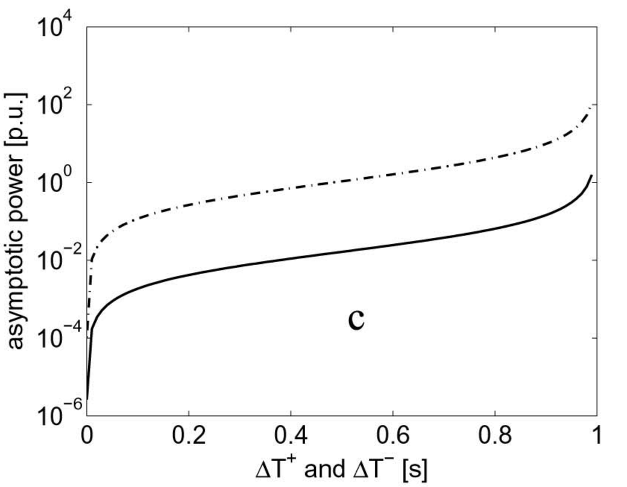
(b)



(c)



(d)



(e)

Fig. 5. Influence of Various Parameters on the Asymptotic Oscillation. Maximal and Minimal Power are Drawn. The Data for the Reference Case are: ρ_0 = - 3000 pcm, $\Delta\rho^+ = \Delta\rho^-$ = 3098 pcm, λ = 0.1 s^{-1}, β = 600 pcm, $\Lambda = 10^{-7}$ s, $\Delta T^+ = \Delta T^-$ = 1 s and S = 1 p.u./s

of the response. On the other hand, an increase of the amplitude of the reactivity oscillation (Fig. 5(b)) at first causes an increase of the maximum power, together with a decrease of the minimum one, due to the increase of the negative portion of the wave; however, when the positive introduction of reactivity becomes large enough, the negative portion does no longer allow to compensate and also the minimum power takes on an increasing trend. A monotonic reduction of the values and of the amplitude of the response is experienced when allowing an increase on the effective delayed neutron fraction (Fig. 5(c)). A contradictory behaviour is associated to the change of the effective prompt neutron generation time (Fig. 5(d)). For low values of this parameter, the asymptotic power increases rapidly while reducing Λ, because of the more prompt system response. However, for systems with a long lifetime (e.g., highly thermalized), the effect of the negative reactivity insertion on the asymptotic power is compensated by the excess production of delayed neutron precursors occurring during the preceding positive reactivity insertion. A change in the dynamic equilibrium between power and precursors is established. As a consequence, the amplitude of the response is reduced. The effect of the frequency of the reactivity oscillation is shown in the last graph (Fig. 5(e)), evidencing an increase of the effect as the frequency is reduced.

5. CONCLUSIONS

The paper solves the problem of reactivity oscillations for a point reactor driven by an external source. A simple square-wave shape for the reactivity insertion is assumed, which allows a fully analytical approach to the solution of the first-order differential system of equations. The problem is of interest for practical applications to the study of the stability of a source-driven systems and to determine the physical situations that may lead the system to drift to a diverging situation.

The possibility for establishing a stationary oscillating power response is studied, determining the condition under which it can be realized, and the differences with respect to a source-free reactor are also pointed out and discussed. Such a condition is connected to the eigenvalues of the system response matrix, which fully controls the type of asymptotic evolution of the system. The unstable situations which may arise for certain levels of subcriticality and under some characteristics of the oscillating reactivity wave are investigated. The numerical results presented show the effects of the kinetic parameters of the system and of the inserted reactivity on its behaviour. In particular, the role of the effective lifetime and of the delayed neutron fraction is analysed, showing the effect of these parameters on the asymptotic power.

The paper shows that oscillating reactivity insertions need to be carefully considered for the stability and safety analysis of multiplying systems, even when performing in the subcriticality regime.

REFERENCES

[1] D.L. Hetrick. *Dynamics of nuclear reactors*. American Nuclear Society, La Grange Park, IL, 1993.
[2] P. Ravetto. Reactivity oscillations in a point reactor. *Annals of Nuclear Energy*, 24:303- 314, 1997.
[3] S. Dulla. *Models and methods in the neutronics of fluid fuel reactors*. PhD Thesis, Politecnico di Torino, Torino, 2005.
[4] Z. Ackasu, G.S. Lellouche and L.M. Shotkin. *Mathematical methods in nuclear reactor dynamics*. Academic Press, New York, 1971.
[5] I. M. Gel'fand. *Lectures on linear algebra*. Dover, New York, 1961.
[6] S. Dulla, P. Ravetto, M.M. Rostagno, G. Bianchini, M. Carta and A. D'Angelo. Some features of spatial neutron kinetics for multiplying systems. *Nuclear Science and Engineering*, 149:88-100, 2005.