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On the Fundamental and Higher-Mode Eigenpairs of Fission Matrix Method and p-CMFD Method in Continuous-Energy Monte Carlo Calculation

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1. Introduction

Recently, Carney et al. [1] implemented fission matrix capability in the MCNP code and contemplated to use it for acceleration of Monte Carlo (MC) calculation. It turned out that the concept of fission matrix was introduced quite early in reactor physics [2] and already used to accelerate MC convergence by Carter and McCormick [3] and by Kitada and Takeda [4].

On the other hand, following the high-order/low-order paradigm structure for acceleration in deterministic transport calculation, Yun and Cho [5] proposed a continuous-energy MC acceleration method using fission source distribution (FSD) obtained by partial current-based coarse-mesh finite difference (p-CMFD) deterministic method [6]. In a similar work, coarse-mesh finite difference (CMFD) acceleration of multi-group MC calculation was proposed by Lee et al. [7].

In this exploratory paper, any relationship between fission matrix and p-CMFD matrices is investigated by applying them to a 1-D continuous-energy thermal reactor test problem. The fundamental and higher-mode eigenpairs are calculated and compared.

2. Relation Between Fission Matrix and p-CMFD Matrix

2.1 Fission Matrix Method

By integrating the neutron transport equation over energy and angle, and segmenting the problem domain into coarse-mesh cells [1], the fission matrix equation is obtained as

$$\hat{S} = \frac{1}{k}\hat{H}\hat{S}\,, \qquad (1)$$

where $\hat{H}$ is fission matrix and $\hat{S}$ is FSD vector. In this derivation, fission matrix ($\hat{H}_{IJ}$) means the number of fission neutrons produced in cell *I* due to fission neutrons born in cell *J* normalized by the number of fission neutrons born in cell *J*. The fission matrix can be directly tallied during the MC calculation. Using the QR algorithm in MATLAB, one can then obtain fundamental and higher-mode eigenpairs for the fission matrix.

2.2 Relation between Fission Matrix Method and p-CMFD Method

Following the p-CMFD methodology, the one-group p-CMFD equation can be expressed in matrix form as:

$$T\Phi = \frac{1}{k}F\Phi\,, \qquad (2)$$

where *T* and *F* are neutron transport and fission matrices, respectively, and Φ is coarse-mesh cell-averaged flux distribution vector. From these p-CMFD matrices, a corresponding form of Eq. (1) is obtained by using the following operations:

$$H = FT^{-1}, \qquad (3a)$$

and

$$S = F\Phi\,. \qquad (3b)$$

Thus, a "fission matrix" can be expressed in terms of p-CMFD matrices using Eq. (3a). Then fundamental and higher-mode eigenpairs can be calculated from this "fission matrix" in the same way. However, Eq. (3a) requires explicit inverse of *T*, which is not trivial. To avoid this problem, the QZ algorithm in MATLAB can be applied directly to Eq. (2) to obtain fundamental and higher-mode eigenpairs.

3. Numerical Results

A 1-D thermal reactor test problem shown in Fig. 1 is solved by the continuous-energy MC code McSLAB [8], which has implemented the fission matrix tally procedure. Calculation conditions are shown in Table I. During the 10 active cycles, fission matrix is directly evaluated. In the meantime, one-group p-CMFD parameters are also tallied to construct p-CMFD matrices.

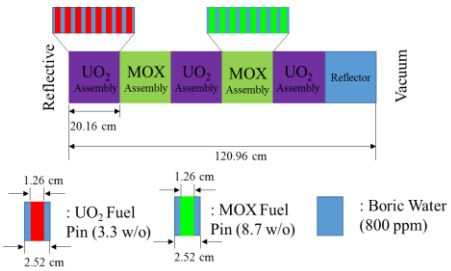

Fig. 1. 1-D thermal reactor test problem

Figs. 2 and 3 show fission matrices obtained by direct sampling of $\hat{H}$ and by Eq. (3a) with inversion of T in p-CMFD matrices, respectively. Since the last two cells in the reflector of the test problem does not contain fissile material, the rank of "fission matrix" *H* decreases from 12 to 10, and the number of non-zero eigenvalues is 10. In Fig. 3, it is observed that column 11 and 12 in the "fission matrix" obtained from p-CMFD matrices are non-zeros, while they are zeros in directly sampled fission matrix. And the bandwidth of the "fission matrix" obtained from the p-CMFD matrices is wider.

Table I. MC calculation conditions

Histories per Cycle	50,000
Inactive/Active Cycles	90/10
Cell Size	10.08 cm (Half-Assembly)

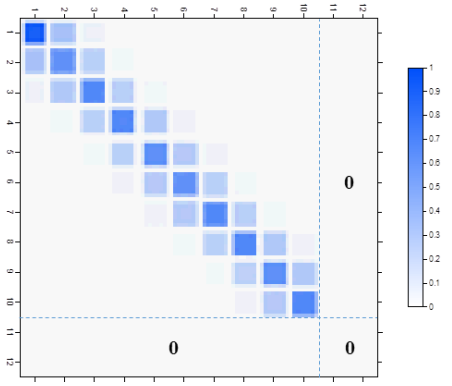

Fig. 2. Fission matrix obtained from direct sampling

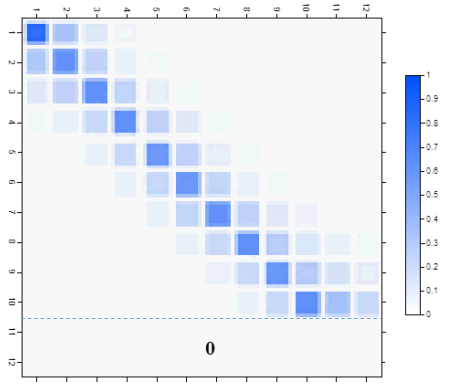

Fig. 3. Fission matrix obtained from p-CMFD matrices

Fig. 4 compares the eigenvalue spectra obtained from directly sampled fission matrix and from p-CMFD matrices, while Figs. 5 and 6 compare fundamental and first-mode FSDs.

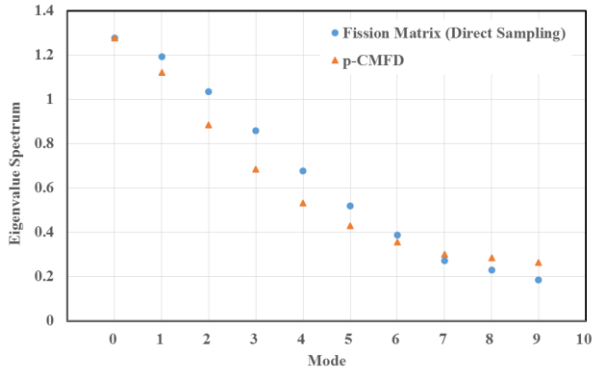

Fig. 4. Eigenvalue spectra obtained from directly sampled fission matrix and p-CMFD matrices

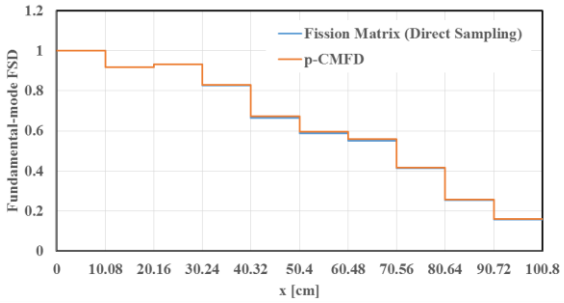

Fig. 5. Comparison of fundamental-mode fission source distributions

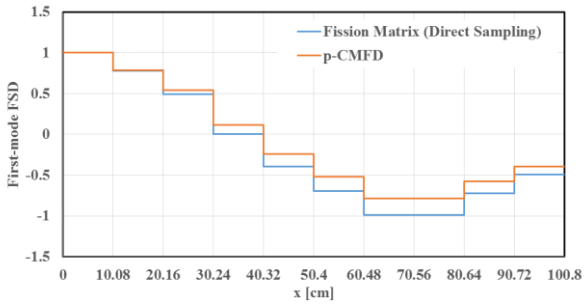

Fig. 6. Comparison of first-mode fission source distributions

4. Summary and Conclusions

The structure, fundamental and higher-mode eigenpairs of the fission matrix obtained from the fission matrix method and the p-CMFD method, respectively, are compared using the numerical results on a test problem reactor with reflector region.

The fission matrix method gives eigenmodes in fission source distributions, but not in flux distributions. Thus, flux distribution in the non-fissile region of the reactor is not available. However, the p-CMFD method gives eigenmodes in both variables.

The numerical results indicate that the fundamental-mode eigenpairs from the two methods are practically identical, but the higher-mode eigenpairs exhibit some differences. The dominance ratio in the p-CMFD method is smaller than that in the fission matrix method, if the coarse-mesh is not too fine. This implies that the p-CMFD method converges faster than the fission matrix method if it is used as low-order equation in acceleration of Monte Carlo calculation. It is also noted that the storage required is linear in N (=number of cells) in the p-CMFD method, while it is quadratic in the fission matrix method.

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