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A Preliminary Study on Sensitivity and Uncertainty Analysis with Statistic Method: Uncertainty Analysis with Cross Section Sampling from Lognormal Distribution

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1. Introduction

Uncertainties of the reactor responses (e.g., k_{eff}, neutron flux, reaction rates) are caused by the code methodology, assumptions in computational simulation, manufacturing uncertainty, and nuclear data. Especially, it is noted that the uncertainty of the nuclear data mainly affects the uncertainties of the response [1]. For the uncertainty analysis, deterministic and statistical methods were known [2, 3]. The uncertainty evaluation with statistical method is performed by repetition of transport calculation with sampling the directly perturbed nuclear data. Hence, the reliable uncertainty result can be obtained by analyzing the results of the numerous transport calculations. One of the problems in the uncertainty analysis with the statistical approach is known as that the cross section sampling from the normal (Gaussian) distribution with relatively large standard deviation leads to the sampling error of the cross sections such as the sampling of the negative cross section. Some collection methods are noted [3, 4]; however, the methods can distort the distribution of the sampled cross sections. In this study, a sampling method of the nuclear data is proposed by using lognormal distribution. After that, the criticality calculations with sampled nuclear data are performed and the results are compared with that from the normal distribution which is conventionally used in the previous studies.

2. Methods

In the nuclear data library, the cross section and the covariance data are included. Conventionally, the distributions of the cross sections were assumed to have Gaussian. However, the cross sections, which have relatively large standard deviation compared with the average cross section, can be negatively sampled with Gaussian. Because the cross sections are inherently positive physical quantities, two methods, which are truncation of the negatively sampled cross sections and direct uses of the negative cross sections without modification, were used in the previous studies [3, 4]. In this study, a sampling method of the cross sections using the lognormal distribution was proposed.

2.1. Distributions and Stochastic Sampling

The normal distribution is given as shown in Eq. (1).

$$N(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), \quad -\infty < x < \infty \tag{1}$$

where, μ and σ are the average and the standard deviation of the normal distribution, respectively. It is noted that the lognormal distribution is defined for the positive real number variables as shown in Figure 1 [5]. The lognormal distribution approximately becomes normal distribution with small standard deviation while the distribution is lognormal distribution with larger standard deviation because the minimum value of the lognormal variable is 0. The cross section is the positive variable; thus, the lognormal distribution as shown in Eq. (2) is tried to use in this study to solve the negative sampling problem of the cross sections.

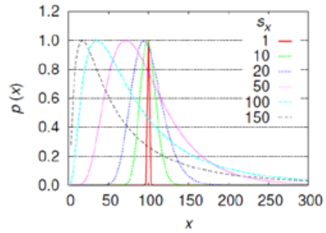


Fig. 1. Property of Lognormal Distribution with Different Standard Deviations, S_x

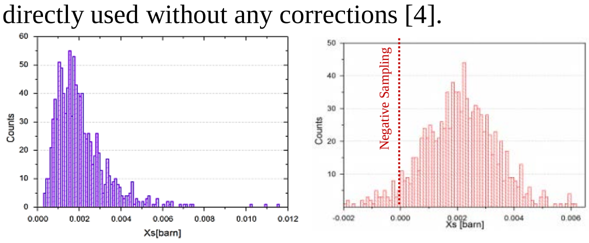
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}x}\exp\left(-\frac{(\log x-\mu)^2}{2\sigma^2}\right), 0 \le x < \infty \tag{2}$$

The normal and lognormal distributions have infinite boundary. In order to sample the cross section with reasonable boundary, it is assumed that the sampling boundary is μ ± 3σ (99.7%). The sampling range of each energy group is given at Eq. (3).

$$m_i - 3\times\sigma_i \le XS \le m_i + 3\times\sigma_i \tag{3}$$

where, m_i= i^{th} energy group cross section
σ_i = standard deviation of i^{th} energy group cross section

By using Eqs. (1) and (2), the cross sections are sampled as shown in Figure 2. In case of the cross section sampled from the normal distribution, the negative cross sections in some energy groups were directly used without any corrections [4].



(a) Lognormal Distribution (b) Normal Distribution
Fig. 2. Results of (n,ɣ) Cross Sections Sampled with Normal and Lognormal Distributions at 44th Energy Group

2.2. Overall Algorithm for Statistic S/U Analysis

The overall algorithm of the sensitivity and uncertainty analysis in this study is shown in Figure 3. At first, the

44 group material cross section library (MATXS format) and covariance data are generated by NJOY code [6] from ENDF-VII.1 cross section library. The cross sections are sampled by using the covariance data with the methods described at Section 2.1. In this study, an automatic cross section sampling and writing program was developed for the sampling the cross sections. After the sampling of the multi-group cross section, the cross sections are arranged by BBC module, and then, the effective multi-group cross section is produced by TRANSX. Finally, the criticality calculation is performed by THREEDANT [7].

For the analysis of the criticality effect caused by the distribution type, S/U analysis with sampling (n,ɣ) cross sections were only pursued. N numbers of the cross section sets were generated and criticality calculations were performed as the same number of the sampled cross section sets for GODIVA benchmark problem [8].

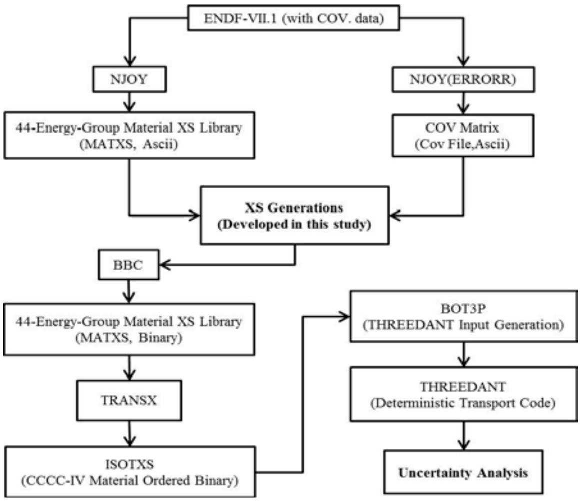


Fig. 3. Flowchart for the Statistic Uncertainty Evaluation

3. Results and discussion

After the criticality calculations of the GODIVA benchmark problem, the effective multiplication factors were obtained. In order to analyze the uncertainty, $\sigma_{k_{eff}}/k_{eff}$ values were calculated for each case. The results are given as shown in Tables I and II. The averages of the k_{eff} were no significant differences with the cross sections sampled by the normal and lognormal distributions. However, $\sigma_{k_{eff}}/k_{eff}$ with the lognormal distribution was increased to 0.031% than that of the normal distribution for N=1000 case (outside of the 95 % confidence interval boundary). Analysis shows that the negative cross sections sampled from the normal distribution can give a small biased value in S&U analysis.

Table I. Uncertainty Analysis Results of k_{eff} Using Sampling from Normal Distribution for N Cross Sections

N	Average k_{eff}	σ_{keff}/k_{eff} %	95% Confidence Interval (σ_{keff}/k_{eff})	
			Lower	Upper
100	1.00129	0.559	0.491	0.650
200	1.00145	0.537	0.489	0.596
500	1.00089	0.517	0.486	0.551
1000	1.00121	0.505	0.484	0.529

Table II. Uncertainty Analysis Results of k_{eff} Using Sampling from Lognormal Distribution for N Cross Sections

N	Average k_{eff}	σ_{keff}/k_{eff} %	95% Confidence Interval (σ_{keff}/k_{eff})	
			Lower	Upper
100	1.00171	0.562	0.493	0.652
200	1.00154	0.534	0.486	0.591
500	1.00127	0.540	0.508	0.575
1000	1.00141	0.536	0.513	0.559

4. Conclusions

In this study, the statistical sampling method of the cross section with the lognormal distribution was proposed to increase the sampling accuracy without negative sampling error. Also, a stochastic cross section sampling and writing program was developed. For the sensitivity and uncertainty analysis, the cross section sampling was pursued with the normal and lognormal distribution. The uncertainties, which are caused by covariance of (n,ɣ) cross sections, were evaluated by solving GODIVA problem. The results show that the sampling method with lognormal distribution can efficiently solve the negative sampling problem referred in the previous studies. It is expected that this study will contribute to increase the accuracy of the sampling-based uncertainty analysis.

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