# **Quantification of J-integral for the circumferentially cracked pipes under mechanical and thermal loads between R6 and A16**

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# **1. Introduction**

As most part of pressurized plant components are subject to various stresses. Stresses applied to plant components can be classified into primary (mechanical loading) and secondary stress (such as thermal and residual stress). When a crack is found in a pressurized plant component under combined primary and secondary stress, it is important to estimate relevant fracture mechanics parameters<sup>[1~3]</sup>. *J*-integral is one of the fracture mechanics parameters used for elasticplastic analysis. Detail procedure for *J*-integral estimation under combined stresses are tabulated in R6 and A16. This paper compares the estimated *J*-integral between R6 and A16 for combined stresses through FE analysis results.[4,5]

# **2.** *J* **estimates under combined mechanical and thermal load in R6, RCC-MR A16**

# *2.1 J estimates for Combined Load in R6 Code[4]*

Reference stress based *J* estimates was developed by Ainsworth. In the reference stress method, the elasticplastic *J*-integral for a cracked pipe under the primary stress only,  $J^p$ , is estimated as

$$
\frac{J^p}{J_{el}^p} = \frac{E\varepsilon_{ref}}{\sigma_{ref}} + \frac{1}{2}L_r^2 \frac{\sigma_{ref}}{E\varepsilon_{ref}}
$$
(1)

Where the reference stress,  $\sigma_{ref}$ , and the proximity parameter for plastic collapse, *Lr*, are defined by the reference load, *Pref*, as

$$
L_r = \frac{\sigma_{ref}}{\sigma_y} = \frac{P}{P_{ref}} \tag{2}
$$

In Eq. (2), *P* denotes the generalized mechanical load. The reference strain is defined as the true strain at the reference stress, determined from true stress-strain data of the material.  $J_{el}$ <sup>*p*</sup> represents the elastically calculated value of J due to mechanical load.  $J_{el}^P$  is defined by the stress intensity factor due to the primary stress,  $K^P$ , as

$$
J_{el}^p = \frac{(K^p)^2}{E}
$$
 (3)

The elastic-plastic *J* under combined primary and

secondary stress, 
$$
J^{p+s}
$$
, can be estimated as  
\n
$$
J^{p+s} = \frac{(K^p + V \cdot K^s)^2}{E} \left( \frac{E \varepsilon_{ref}}{\sigma_{ref}} + \frac{1}{2} L^2_r \frac{\sigma_{ref}}{E \varepsilon_{ref}} \right)
$$
\n(4)

Where  $K^S$  is the stress intensity factor due to secondary stress and *V* is multiplying factor, interaction between primary stress and secondary stress is defined to be covered.

R6 provides both simplified and detailed procedure to estimate *V*-factor for calculating the elastic-plastic value of *J* under combined stress. Simplified procedure is based on the elastic analysis. To quantify the magnitude of the secondary stress, a non-dimensional parameter, *β<sup>1</sup>* is introduced. The equation is defined as

$$
\beta_1 = \frac{\kappa_I^S}{\left(\frac{K_I^P}{L_r}\right)}\tag{5}
$$

*V*-factor is assumed to be a function of the coefficient, *β<sup>1</sup>* and *Lr*, given by

$$
V = \begin{cases} 1+0.2L_r + 0.02\beta_1(1+2L_r) & \text{for } L_r < L_r^* \\ 3.1-2L_r & \text{for } L_r < L_r < 1.05 \\ 1 & \text{for } L_r > 1.05 \end{cases}
$$
 (6)

 $L_r^*$  is determined by the intersection of the first and second lines in Eq. (6). In detailed method, a method of calculation to V-factor is given by

$$
V = \zeta \cdot V_O \tag{7}
$$

In R6, a number of procedures to calculate the *V<sup>o</sup>* are suggested.

$$
V_o = \frac{\sqrt{E'J^s}}{K^s} = \sqrt{\frac{J^s}{J_e^s}}
$$
 (8)

Where  $J_s$  is estimated by elastic-plastic FE analysis. And  $\zeta$  is a function of  $\beta_l$  and  $L_r$ , which is tabulated in R6

# *2.2 J estimates for Combined Load in A16 Code[5]*

The elastic-plastic *J* under combined primary and secondary stress,  $J^{P+S}$ , can be estimated from

$$
J^{p+s} = \left[ \sqrt{J^{me}} + k_{th2}^* \sqrt{J_{el}^{th}} \right]^2 \tag{9}
$$

Where  $J^{me}$  is calculated by EPS option,  $k_{th2}$ <sup>\*</sup> is coefficient of the stress redistribution under combined loading and  $J_{el}$ <sup>th</sup> is calculated using FE analysis. In EPS option *J me* can be estimated from,

$$
J^{me} = J_{el}^{me} \cdot \frac{E \cdot \varepsilon_{ref}^{me}}{\sigma_{ref}^{me}}
$$
(10)

The reference stress, *σref* is deduced from elastic and plastic equivalent stresses on the tensile curve, and this process is shown in Fig.2. A estimation method of equivalent stresses is presented in A16 for each crack geometry and stress condition. A16 proposed  $k_{th2}$ <sup>\*</sup> option to estimate the interaction between primary and secondary stress.  $k_{th2}$ <sup>\*</sup> is obtained using following equation,

 $k_{th2}^* = \beta_2 + \kappa_1 \cdot (k_{th2} - 1)$ (11) Where each coefficients are given in,

$$
\kappa_1 = \frac{\sigma_{el}^{th}}{\sigma_{el}^{th} + \sigma_{ref}^{me}}, \quad \kappa_2 = 0.5 \cdot \frac{\sigma_{2m}}{\sigma_{1m} + \sigma_{gb} + \sigma_{12}} \bullet \text{CDAI}: \beta_2 = 1 + 0.3 \cdot \left(1 - \kappa_2^{0.5}\right) \cdot \left(12\right)
$$

And *kth2* is obtained following the equation,

$$
k_{th2}^2 = \left(\frac{\sigma^{th}}{\sigma_{el}^{th}}\right)^2 \cdot \frac{E \cdot \varepsilon_{ref}^{th}}{\sigma_{ref}^{th}}
$$
(13)

Where  $\sigma_{ref}$ <sup>*th*</sup>,  $\varepsilon_{ref}$ <sup>*th*</sup> and  $\sigma^{th}$  is calculated in accordance with Fig. 3. The value of follow up factor, *rth* in Fig. 3 is given for each crack geometry in A16.  $\sigma_{e l}$ <sup>th</sup> is defined by following equation.

### **3. Result**

This paper analyze for fully circumferential surface crack cases with *a/t*= 0.1 and *n*=10(Ramberg-osgood equation,  $E=200$ GPa and  $\sigma_y=500$ MPa). Axial tension was the only considered factor for the mechanical loading. To generate secondary stress, radial gradient temperature type of thermal load was only applied to the pipes. To confirm load order effect under combined load, two different types of load order were applied to the pipes. First type is applying thermal loading before the mechanical loading and second type is applying thermal loading after the mechanical loading. Elasticplastic analysis of circumferentially cracked pipes was performed using ABAQUS. This paper compares R6 and A16 *J* estimates for combined stresses with FE result. Fig 3. compare R6 and A16 *J* estimates with FE results



Fig. 1. Concept of EPS option in RCC-MR A16 code







Fig. 3. Comparison of J-integral with R6, A16 and FE result

#### **4. Conclusion**

*J* value for the fully circumferential surface crack are conservative in contained yielding regime in A16 method. But in R6 method, less conservative result in contained yielding regime. Under large scale yielding (LSY) regime, R6 simplified method and A16 method produced similar result to J estimation. But detailed method are close to the FE results even for large *L<sup>r</sup>* values. And this paper revealed that occurs when thermal loading is applied after mechanical loading the most severe loading sequence.

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