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Code Accuracy Assessment with Time-Domain Bilateral Kernel Regression

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1. Introduction

Accuracy evaluation of computer code calculations in nuclear safety and performance analysis is becoming more and more important due to preserve greater thermal margin and address best-estimate of transient phenomena. The evaluation process is typically based on the effect of discretization, key parameter selection, richness of measurement data, transient pattern recognition, and decision making criteria (rule-of-thumb or statistical treatment). This paper focuses on the kernel based time-domain (KTD) analysis method to analyze the measurement-prediction discrepancies. KTD method has several advantages over the one of recent prevailing approach of the fast Fourier transform based method (FFTBM) by Kunz *et al*. [1] working in the frequency domain. Recently, US NRC published a NUREG report [2] focusing on the quantitative assessment by using FFTBM. The improved FFTBM introduces signal mirroring technique by rearranging the time domain signal into a set of symmetrised signals. Prosek *et al*. [3] suggest the calculation of measurement-prediction discrepancies the experimental signal $F_{\exp}(t)$ as follows.

The error signal in the time domain is defined as $\Delta F(t) = F_{cal}(t) - F_{\exp}(t)$ where $F_{cal}(t)$ is the calculated signal. The average error amplitude A_f in frequency domain is defined:

$$A_f = \sum_{n=0}^{2^m} |\tilde{\Delta}F(f_n)| / \sum_{n=0}^{2^m} |\tilde{F}_{\exp}(f_n)|, \quad (1)$$

where $n = 0, 1, ..., 2^m$ and m is the exponent defining the number of points $N = 2^{m+1}$ for FFT at frequencies f_n. The A_f factor can be considered a sort of average fractional error and the closer the A_f value is to zero, the more accurate is the result.

This paper describes the time-domain method based on the bilateral kernel regression which has several advantages of 1) no limitation on number of data points, 2) no artificial edge, 3) easy to program and 4) intuitive insight into the error signals.

2. Methods and Results

2.1 Kernel Regression in Time-Domain

The kernel regression filter is a kind of static filter. In this paper we adhere the derivation by Park *et al*. [4] Measurement produces a set of random variables $\{t_i, y_i; i = 1, 2, ..., N\}$ on the interval $\{0 \le t_i \le T\}$. It is assumed that

$$y_i = y(t_i) + \varepsilon \quad (2)$$

where ε is a random noise variable with the mean equal to zero.

The kernel regression estimate of $y(t)$ at $t = \tau$ from this random data is defined as the estimator $\hat{y}(\tau)$ as

$$\hat{y}(\tau) = \sum_{i=1}^{N} y_i k(\tau - t_i) / \sum_{i=1}^{N} k(\tau - t_i) \cdot \quad (3)$$

The function $k(\tau - t_i)$ is the kernel function which can be chosen from a wide variety of symmetric functions.

2.2 Bilateral Kernel Regression

The bilateral filter is a technique proposed by Tomasi and Manduchi [5]. This technique preserves edges by mixing a moving average technique with a nonlinear system of weights. Each neighboring value is weighted on its proximity in space or time (a domain weight). A second weighting factor gives some measurement of local difference (a range weight). In the time domain, this is expressed as:

$$\begin{aligned} k(t) &= k_D(\text{distance}) \times k_G(\text{feature}) \\ &= \exp(-D(t_i, t_q)^2 / \sigma_t^2) \times \exp(-D(y_i, y_q)^2 / \sigma_x^2) \end{aligned} \quad (4)$$

where σ_t^2, σ_x^2 are the variances of the spatial distances for noise rejection and feature preservation, respectively. D is the Euclidean distance and t_q is the query point where the smoothed signal is to be generated in the interval of time series data $\{0 \le t_i \le T\}$.

2.3 Error Amplitude by using KTD method

The average error amplitude $A_{t,k}$ in time domain for measurement-prediction can be similarly defined with Eq. (1) as :

$$A_{t,k} = \sum_{i=1}^{N} |\Delta F(\sigma_k)| / \sum_{i=1}^{N} |F(\sigma_k)|, \quad (5)$$

where $i = 0, 1, ..., N$ and σ_k is the bandwidth of the kernel. $1/\log(\sigma_k)$

Fig. 1 shows the test measurement and code calculation result without high-frequency component. Fig. 2 shows the result with rich high-frequency component giving the increase in error amplitude with increase in $1/\log(\sigma_k)$ which is equivalent to high frequency component in FFTBM analysis.

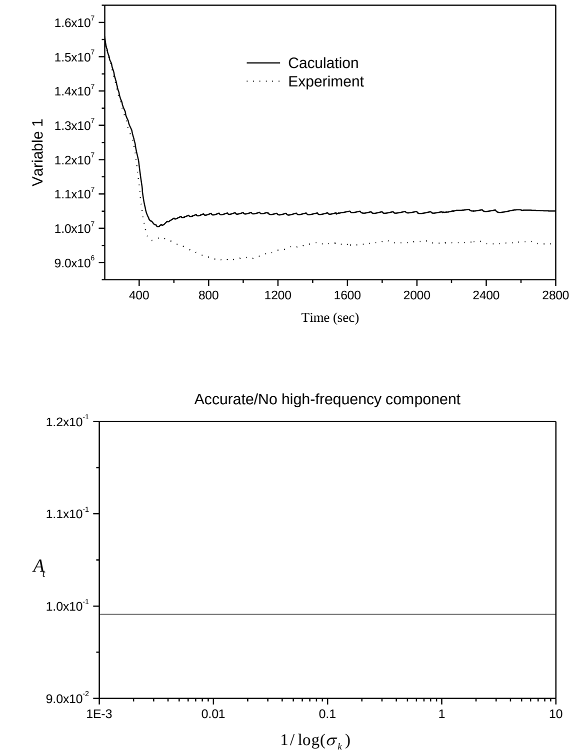

Fig. 1 Accuracy without high-frequency component

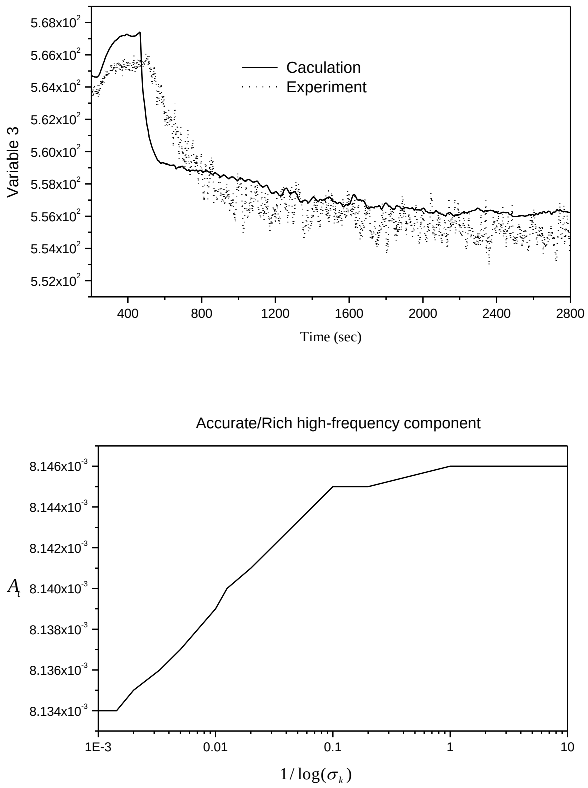

Fig. 2 Accuracy with rich high-frequency component

Fig. 3 shows the relatively large error amplitude in the region of small $1/\log(\sigma_k)$ which means the calculation result show large error in DC component and has no high frequency components.

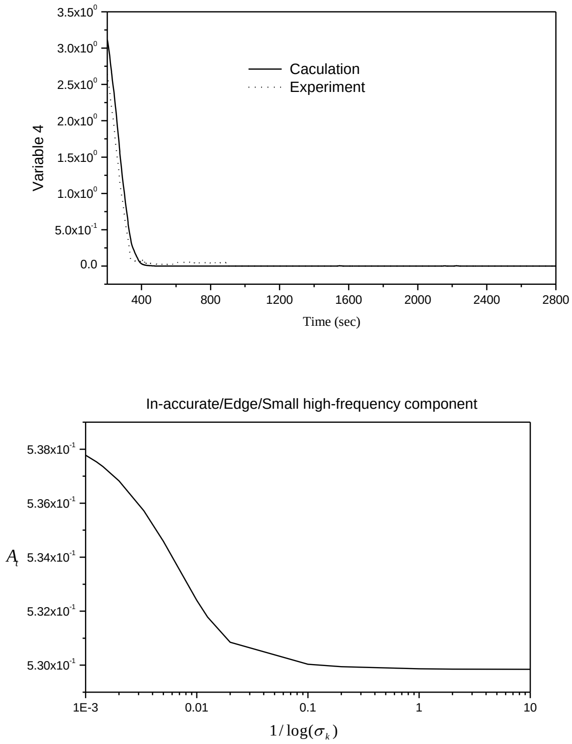

Fig. 3 Accuracy with inaccurate and edge component

3. Conclusions

The code accuracy analysis process is proposed with KTD method of the time-domain bilateral kernel regression which gives several advantages of no limitation on number of data points, no artificial edge, easy to program and intuitive insight into the error signals over typically found difficulties in FFTBM.

KTD method shows the remarkable effectiveness and simplicity over the frequency-domain method for accuracy estimation of computer code calculations.

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