Transactions of the Korean Nuclear Society Autumn Meeting
Gyeongju, Korea, October 24-25, 2013

DVI Film Flow Instability Based on the Normal Mode Analysis

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1. Introduction

As a safety injection method, the direct vessel injection has been intensively developed in Korea and employed in the APR1400. The developing efforts were made from the determination of the number, location and size of DVI nozzles to the experimental demonstrations. Experimental facilities with various scales have shown its complicated phenomena due to highly nonlinear interaction between the steam and injected water flow. The injected DVI water forms a film type flow but very unstable due to the unsteady energetic steam flow which find the exits around the shell of the downcomer of the reactor vessel.

This steam and liquid film interaction leads to the instability on the surface of the film flow and the waves are highly nonlinear to form undercutting, roll over, and finally droplet releasing. The entrained droplets causes a difficulty in the analysis to estimate the water penetrated into the reactor core to cool the nuclear fuels heated up.

Unfortunately, these instabilities on the interface of the DVI liquid film have not been studied appropriately and the conservative estimation of its effect on the coolability has been deduced in the design process and safety analysis of DVI. Actually, the DVI film flow instability should consider the three dimensional wave structure such as horseshoe wave, the solitary like wave formulated in the way of Korteweg-de Vries (KdV) type nonlinear wave formulation, and estimate the entrainment due to such nonlinear waves. Furthermore, DVI film instability even needs a consideration of the curvature of the reactor vessel.

In the present paper, as the first step of nonlinear studies, the appearance of the third order spatial differentiation of the film thickness in the wave propagation equation is to be derived. The two-fluid model in the adiabatic condition is employed and normal mode analysis. Interfacial pressure forces between steam and water need to be modeled for this purpose.

2. Normal Mode Stability of the Two-fluid model

As Lee(1990) Lee and NO(1994) noted, the information propagating structure of the two-fluid model in the abstract phase space often loses its hyperbolic characteristics. Mathematically we call it ill-posed but the loss of hyperbolic information propagation appears often in the real physical world in the form of choking or instability.

Therefore, the hyperbolicity breaking theory has been successfully applied to develop and explain many physical phenomena including CCFL and critical flows.

$$\underline{\underline{A}}\frac{\partial\Psi}{\partial t}+\underline{\underline{B}}\frac{\partial\Psi}{\partial z}=\underline{C} \tag{1}$$

in which $\Psi=\left(\alpha,\bar{p},u_g,u_f\right)^T$.

If we follow the characteristic line of information propagation denoted as $\xi=z+ct$ where c is the wave propagation speed.

$$\left[c\underline{\underline{A}}+\underline{\underline{B}}\right]\frac{\partial\Psi}{\partial\xi}=\underline{C} \tag{2}$$

Therefore, the wave propagation speed needs to be real which has been known as the condition of well-posedness. Therefore, Eq. (1) can be transformed into the following in the phase space of information propagation, in which the wave propagation speed can be interpreted as the eigenvalue in mathematics.

Without proper model of the interfacial momentum transfer terms, the wave speed determined by taking the determination of the matrix multiplied to the derivative of the variables over the characteristic line zero always become complex or imaginary. Only the way to make the eigenvalue real, the transient terms or spatial gradient terms in the interfacial momentum transfer terms such as the virtual mass force or the turbulent dissipation force or the gradient of the interfacial pressure difference.

Such a continuous geometry of the interface in DVI film flow, the virtual mass force and the turbulent dissipation forces are very small but the interfacial pressure difference plays a dominant role in determination of the information propagation

3. Fixed point of the DVI film flow

The phase space analysis needs to determine fixed point which is the bases before having any instability. Therefore, in mathematics it is very simple to be the root of the no gradient in the phase space, in other word

$$\frac{\partial\Psi}{\partial\xi}=\frac{\Delta}{\det\left|c\underline{\underline{A}}+\underline{\underline{B}}\right|}=0 \tag{3}$$

The well-known fixed point was derived by Nusselt based on the velocity profile in the liquid film. Nusselt determine the fixed film thickness as the function of the

Reynolds number. Recently, Lee derived the fixed point for the curved surface which is closer to the DVI film because the inner surface of the reactor vessel is not flat but has the curvature.

$$\delta_{DVI} \approx \frac{\delta_{flat}}{\sqrt[3]{1+\frac{\delta_{DVI}}{R}}} \tag{4}$$

4. Interfacial Pressure Force

It is very hard to formulate the force balance on the interface of the DVI film because of the three dimensional wave structure and nonlinearity which cannot be estimated by the simple harmonic wave functions. In the present study, we set the following interfacial pressure balance:

$$-P_f + 2\mu_f \frac{\partial u_f}{\partial r} = -P_g - \sigma\left(\frac{\partial^2 \eta}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial} r \frac{\partial \eta}{\partial r}\right) - \Delta p_\sigma \tag{5}$$

Even though the curvature of the reactor vessel is very small, the vessel curvature produces the pressure difference as

$$\Delta p_\sigma = \frac{\sigma}{R+\delta_{DVI}} - \frac{\sigma}{R+\delta_{DVI}+\eta} \tag{6}$$

5. Normal Mode analysis for the hyperbolic characteristics of DVI film flow

The determination of the eigen matrix of Eq. (2) gives us the characteristic equation to determine the speed of information propagation which is denoted by c but can be interpreted as $c = \omega / k$ and the characteristic equation become second order equation when we applied the sonic velocity is much greater than the speed of convective information:

$$ac^2 + 2bc + d = 0 \tag{7}$$

where

$$a = \alpha_f \rho_g + \alpha_g \rho_f$$

$$b = \alpha_f \rho_g u_g + \alpha_g \rho_f u_f$$

$$d = \alpha_f \rho_g {u_g}^2 + \alpha_g \rho_f {u_f}^2 - \left\{\alpha_f (1-\beta_g)\left(P_g - P_i\right) - \alpha_g (1-\beta_f)\left(P_f - P_i\right)\right\}$$

The real solution criteria of Eq.(7) give us the limitation of the velocity difference on the interface not to produce dangerous nonlinear wave, in which dangerous means unexpected large amplitude wave. The following inequality is the condition of the onset of nonlinear instability:

$$\left(\frac{\alpha_g \alpha_f \rho_g \rho_f}{\alpha_g \rho_f + \alpha_f \rho_g}\right)(u_g - u_f)^2 \le \left\{\alpha_f (1-\beta_g)\left(P_g - P_i\right) - \alpha_g (1-\beta_f)\left(P_f - P_i\right)\right\} \tag{8}$$

6. Nonlinear waves in the DVI film flow

The solitary wave occurs when we consider the second derivative of the interface of the film in the formulation of the pressure difference force. This second derivative will produce third derivative of the perturbation of the film and it will construct the well-known Korteweg de Vries Equation:

$$-c\frac{\partial \eta}{\partial \xi} + 6\eta \frac{\partial \eta}{\partial \xi} + \frac{\partial^3 \eta}{\partial \xi^3} = 0 \tag{9}$$

Its solution is given

$$\eta = \frac{1}{2} c \sec h^2 \left(\frac{\sqrt{c}}{2}\left(z - ct - \phi\right)\right) \tag{10}$$

This wave is mainly composing all roll waves which is the major source of entrainment as noted by Kataoka and Ishii. Its three dimensional form has been known as horseshoe wave as shown in Fig. 1

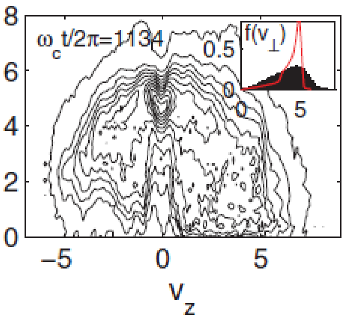

Fig. 1. Horseshoe wave form

7. Conclusions

In the present study, we developed a theoretical basis to study nonlinear wave phenomena on the DVI film flow which highly affect the DVI penetration and liquid droplets entrainment out. We set the hyperbolicity breaking condition by providing the interfacial pressure difference considering the curvature of the reactor vessel. The interfacial pressure difference could generate nonlinear wave such as the horseshoe wave which has been believed as a main source of film break up to produce huge amount of droplets to be entrained out.

Acknowledgement

The present work is partially supported by the middle and long term nuclear research program of ministry of Science, I&CT, and future planning(MSIP) ,

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