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Collocated Scheme on an Unstructured Mesh for Two-phase Flow Analyses

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1. Introduction

The staggered method, in which a control volume for each component of a fluid velocity vector is located on a face normal to its direction, has been popularly used in the area of computational fluid dynamics (CFD). It couples velocity and pressure fields without any artificial ways by a naturally added numerical diffusion. For a Cartesian mesh, the implementation of the staggered method for interior cells of the mesh is straightforward. But it is very tedious to implement boundary conditions on a flow domain for a very complicated geometry. In the meantime, a collocated (or non-staggered) approach in which all the solution variables are located at the centers of the control volumes was developed by Rhie and Chow[1]. They used pressure-weighted interpolation (PWIM) of a cell-face velocity to prevent decoupling of pressure and velocity in the collocated scheme.

For a multi-phase flow with a gravitational body force, the standard form of the Rhie-Chow's scheme makes instability in a flow field, and sometime it fails to get a solution to the flow field. Gu[2] noticed this body-force induced instability, and modified the original Rhie-Chow's scheme. But his scheme is very ad-hoc. Minato et al.[3] used a density-weighted interpolation for a pressure at a cell face to remove a numerical instability in a gas-liquid flow. This density-weighted interpolation method was adopted in the NEPTUNE CFD code by Mechitoua et al.

In this article, the density-weighted interpolation for a pressure gradient was reviewed. And a general formulation for a pressure gradient in a multi-phase flow under gravity was introduced. Also the standard Rhie-Chow's scheme was modified to include the effect of gravity force. Some two-phase flows such as phase separation and free surface flow by a broken dam were solved to evaluate the collocated scheme on unstructured meshes.

2. Numerical Methods

2.1 Solution Algorithm

In this study for the development of a numerical method to analyze gas and water two fluid flows, a three-field model of continuous liquid, dispersed droplets and gas fields was adopted. Each field has its own mass, momentum and energy conservation equations as governing equations. The governing equations are discretized by a finite volume method on an unstructured grid to handle the geometrical complexity of the nuclear reactors. The phasic momentum equations are coupled among the phases and solved with a sparse block Gauss-Seidel matrix solver to increase the numerical stability. A pressure correction equation derived by summing the phasic continuity equations is applied to the unstructured mesh in the context of the cell-centered collocated scheme.

2.2 Pressure Gradient

The phasic momentum equation requires a pressure gradient at a cell center. In an unstructured finite volume method (FVM), the pressure gradient can be obtained by the Gauss theorem as follow:

$$\nabla p_{c0} = 1/\Omega_{c0} \oint p d\vec{A} \approx 1/\Omega_{c0} \sum_f p_f \vec{A}_f \,. \tag{1}$$

p_f is the pressure on a cell face f, and it is usually obtained by simple averaging or distance weighted averaging. If the gravity force is considered in a flow field, the face pressure must be carefully defined because the pressure field has hydrodynamic and hydrostatic effects. Minato et al. introduced the density-weighted interpolation of the pressure. The main idea of the method is depicted in Fig. 1 and Eq. (2)

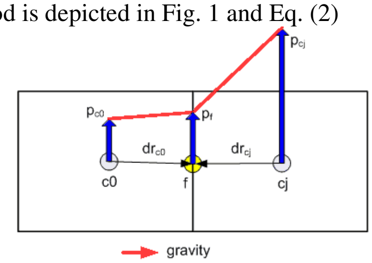

Fig. 1. Pressure profile affected by gravity

$$p_f = p_{c0} + \rho_{c0} g dr_{c0} = p_{cj} + \rho_{cj} g dr_{cj} \tag{2}$$

Here, ρ is a density of two-phase mixture. If dr_{c0} is canceled, then the pressure on the cell face is obtained as follow:

$$p_f = \frac{(1-w_f) p_{c0} / \rho_{c0} + w_f p_{c0} / \rho_{c0}}{(1-w_f)/\rho_{c0} + w_f / \rho_{c0}} \,. \tag{3}$$

It is uncertain that Eq. (3) is applicable to a cell face which is normal to the gravity. In that case, face pressure must recover to a simple averaged value.

Here, a new formulation for a face pressure is introduced.

$$p_f^- = p_{c0} + \rho_{c0} \vec{g} \cdot d\vec{r}_{c0}, \;\; p_f^+ = p_{cj} + \rho_{cj} \vec{g} \cdot d\vec{r}_{cj} \,, \tag{4}$$

$$p_f = 0.5 \times (p_f^- + p_f^+) \,.$$

This formulation guarantees that a face pressure becomes a simple averaged value if the face is normal to the gravity.

2.3 Volume flow on a cell face

A volume flow on a cell face which is a face velocity multiplied by the face area has the key role to the pressure-velocity decoupling.

$$U_{f,k} = \bar{U}_k + \bar{H}_k\left[\overline{(\nabla p - \rho g)} - (\nabla p - \rho g)_f\right]\cdot \vec{A}_f \qquad (5)$$

Eq. (5) is the modified Rhie-Chow's PWIM by Gu. But he failed to use the equation. The bar over the variables in Eq. (5) means averaging of two values at cells c0 and cj. Eq. (5) can be changed into the following equation on the unstructured FVM.

$$U_{f,k} = \bar{U}_k + \bar{H}_k\left[(pp_{cj} - pp_{c0}) - \overline{(\nabla p - \rho g)}\cdot d\vec{r}\right]\frac{A_f}{d\vec{r}\cdot\hat{n}}, \qquad (6)$$

where

$$\overline{(\nabla p - \rho g)} = (1 - w_f)(\nabla p - \rho g)_{c0} + w_f(\nabla p - \rho g)_{cj},$$

$$pp_{cj} - pp_{c0} = (p_{cj} + \rho_{cj}\vec{g}\cdot d\vec{r}_{cj}) - (p_{c0} + \rho_{c0}\vec{g}\cdot d\vec{r}_{c0}).$$

3. Numerical Results

The numerical method proposed in this study was validated by solving a phase separation in a square cavity and dam breaking problems. For the phase separation problem, air and water are equally mixed and distributed in the cavity initially. To test a numerical stability of the modified PWIM, an unstructured triangular mesh was used for the problem. Fig. 2 shows the mesh used which has 800 triangular cells. Fig. 3 is the velocities and iso volume fractions of the air. At t=5.6s, the air and water are almost separated. So the air velocity must be reduced gradually. When Eq. (4) was applied, the air velocity was weakened. But as can be seen at the right figure, Eq. (3) made the velocity field unphysical.

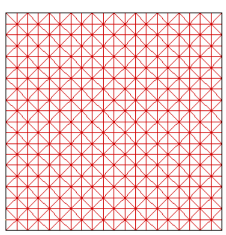

Fig. 2. Unstructured triangular mesh for a phase separation.

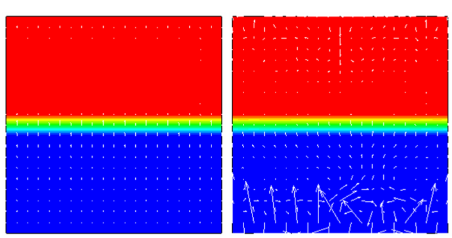

Fig. 3. Iso volume fraction and velocities of air at t= 5.6 s, left: using Eq. (4), right: using Eq. (3).

For the dam breaking problem, a water column is maintained by an artificial membrane until it disappears at t=0s. When it disappears, the water column collapses by gravity. The water front moves and hits the obstacle on the bottom wall. Fig. 4 shows the movement of the water front along time. The proposed method for the face pressure and the modified Rhie-Chow scheme produced the wavy front of the collapsed water very stably.

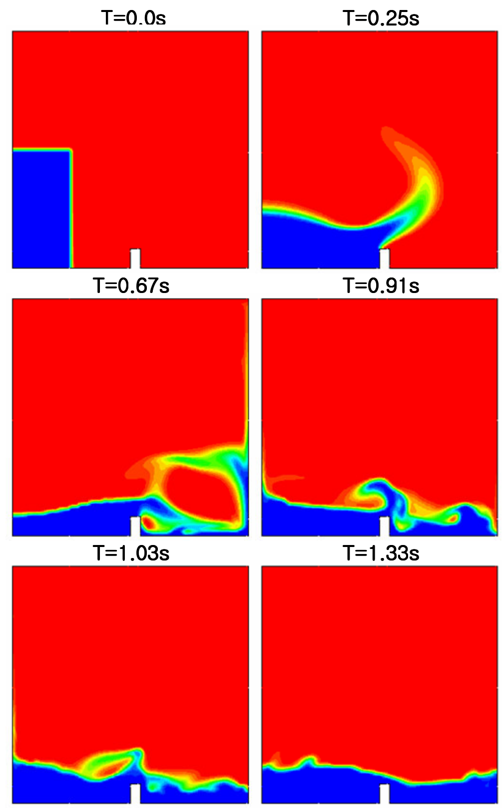

Fig. 4. Dam-break problem, progress of water volume fractions.

4. Conclusion

A modified Rhie-Chow scheme and a new method to define a face pressure for a two-phase flow with a body force were introduced. It was found that the methods increase the numerical stability when the gravity is acting on a flow field.

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