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CFD Analysis of Natural Convective Non-Darcy Flow in Porous Medium

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1. Introduction

The phenomenon of natural convection in the porous medium has been a key issue in many areas, and especially in the core catcher of LMFBR. One of the main features is the loss of momentum described by the famous Darcy's law. However it is only valid for slow flow with low permeability. The flow becomes non-Darcy flow when the Reynolds number is over unity. Therefore, the need of analysis on non-Darcy flow effect in porous medium has been increased. In this paper, we developed and validated the physical model and numerical method with CFX-11.0, and we suggest a way to describe the heat transfer in heat-generating non-Darcy porous medium through scale analysis.

2. CFD Analysis on Porous Media

In this study, the fluid dynamics in porous media is treated as a non-Darcy flow condition that the inertial resistance and the viscous resistance must be considered.

2.1 Physical Model

According to Niven [1], the porosity and particle size determine the flow condition, whether it is Darcy or non-Darcy flow. The momentum loss in non-Darcy flow is expressed as,

$$\frac{\Delta P}{L} = aU + bU^2 \qquad (1)$$

This is also known as Forchheimer's equation. From this, Ergun [2] developed the well-known equation of the relation between momentum loss and the geometrical parameters.

$$\frac{\Delta P}{L} = \frac{150\mu}{\varphi_p^2 d_p^2}\frac{(1-\varepsilon)^2}{\varepsilon^3}U + \frac{1.75\rho_f}{\varphi_p d_p}\frac{(1-\varepsilon)}{\varepsilon^3}U^2 \qquad (2)$$

When Darcy equation is introduced, the permeability, Forchheimer's coefficient *b*, and momentum loss can be expressed as follows.

$$\kappa = \frac{\varepsilon^3 d_p^2}{150(1-\varepsilon)^2} \qquad (3)$$

$$b = \frac{1.75}{\sqrt{150}}\frac{\rho_f}{\sqrt{\kappa}}\frac{1}{\varepsilon^{3/2}} \qquad (4)$$

$$\frac{\Delta P}{L} = \frac{\mu}{\kappa}U + \frac{1.75}{\sqrt{150}}\frac{\rho_f}{\sqrt{\kappa}}\frac{1}{\varepsilon^{3/2}}U^2 \qquad (5)$$

Based on these, the governing equations of mass, momentum, and energy were implemented in ANSYS CFX-11.0.

$$\frac{\partial}{\partial t}\varepsilon\rho + \nabla \bullet \left(\rho\vec{K}\cdot\vec{U}\right) = 0$$

$$\frac{\partial}{\partial t}\left(\varepsilon\rho\vec{U}\right) + \nabla \bullet \left(\rho\left(\vec{K}\cdot\vec{U}\right)\otimes\vec{U}\right) - \nabla \bullet \left(\mu\vec{K}\cdot\left(\nabla\vec{U} + \left(\nabla\vec{U}\right)^T\right)\right) = -\varepsilon\nabla p - \varepsilon\vec{R}\cdot\vec{U}$$

$$\frac{\partial}{\partial t}\left(\varepsilon\rho H\right) + \nabla \bullet \left(\rho\vec{K}\cdot\vec{U}H\right) - \nabla \bullet \left(\mu\vec{K}\cdot\nabla H\right) = \varepsilon S$$

Where $\vec{K}$ is the area porosity tensor, R is momentum loss involving the loss by gravity, and S is a heat source within the porous media.

The porous media is assumed to be homogeneous and the density variation of the fluid follows the Boussinesq's assumption. The fluid is treated as an incompressible Newtonian fluid and laminar.

2.2 Numerical Method

The physical domain is a square rectangular enclosure. The mesh was generated with hexagonal mesh and included 50 X 50 nodes, uniformly. In transient analysis, zero velocity and the reference temperature was applied as an initial value. The convergence criterion was below 10^{-5} for all averaged residual RMS value.

2.3 Validation

The natural convection occurs with the heat transfer on side wall due to the density difference. For validation, the porous media with constant temperature on side wall and with the heat generating inside was modeled.

The results of numerical analysis for validation of the case of only constant side wall temperature were compared with the previous results of Nithiarasu et al. [3]. Most of the data agreed with the data of Nithiarasu et al. [3] and reasonable trends were found. Furthermore, the natural convection was strengthened as the difference of wall temperature increased. Porosity effect was also considered, but we found the influence of porosity was negligible.

For the case of the porous media with heat generating inside as well as the constant temperature on side wall, the results showed a good agreement with the previous works (Table 1). The influence of Darcy number was examined and we found that the natural circulation can be depressed more by the denser porous media (Fig. 1).

For deeper level of validation, transient analysis was also conducted. The steady state was attained at about 1 or 2 minutes as the natural circulation became stable.

3. Scale Analysis

To describe the flow behavior, previous works applied the Darcy flow assumption. However, non-Darcy flow is also important and must be considered.

Table 1 Internal heating and constant temperature of side wall

Ra	Ra_I	Da	Porosity	Nu (present)	Nu [1]
10^5	10^3	10^{-2}	0.4	3.61	2.84
10^5	10^3	10^{-4}	0.4	1.06	1.02
10^5	10^3	10^{-6}	0.4	1.02	0.954
10^5	10^5	10^{-2}	0.4	3.28	2.4
10^5	10^5	10^{-4}	0.4	0.6	0.57
10^5	10^5	10^{-6}	0.4	0.55	0.51
10^5	10^7	10^{-2}	0.4	-42.42	-44.1
10^5	10^7	10^{-4}	0.4	-46.10	-46.4
10^5	10^7	10^{-6}	0.4	-47.89	-48.4

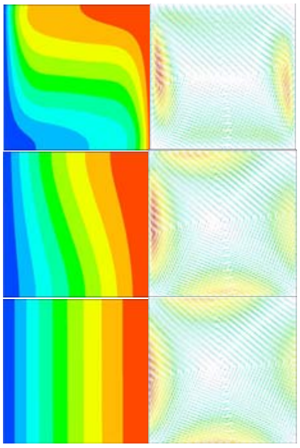

Fig.1 Internal heating and constant side wall temperature (Darcy number)

The non-dimensional parameters are as follows.

$$Ra = \frac{g\beta SKd^3}{2k\alpha\nu} \qquad Nu = \frac{Sd^2}{2k\Delta T} \tag{6}$$

The governing equations are

$$\nabla \cdot \vec{u} = 0 \tag{7}$$

$$\frac{\mu}{K}\vec{u} = -\nabla P + \rho\vec{g} \tag{8}$$

$$\vec{u} \cdot \nabla T = \alpha\nabla^2 T + \frac{S}{\rho c} \tag{9}$$

$$\rho = \rho_r[1 - \beta(T - T_r)] \tag{10}$$

Let's consider the momentum balance in the sublayer, where the magnitude of the flow resistance due to the porous medium is comparable to the buoyancy force due to the temperature.

$$\frac{\mu}{K}u + \frac{\rho}{b}u^2 \sim \frac{g\beta Sd\delta}{k} \tag{11}$$

where the second balance is from

$$q \sim k\frac{\Theta}{\delta} \sim Sd \qquad \Theta \sim \frac{Sd\delta}{k} \tag{12}$$

Θ is temperature at the edge of the sublayer with thickness of δ. Because the molecular motion is dominant in the sublayer, the molecular velocity can be estimated and when applied to eq.(11) the scale relation is derived as

$$\frac{d}{\delta} \sim (Ra - A)^{1/2} \tag{13}$$

where $Ra = \frac{g\beta SKd^3}{k\alpha\nu}$ $\quad A = \frac{\rho\alpha Kd^2}{b\delta^3\mu}$ (14)

For the expression of the turbulent Peclet number, similar way of what Kim [4] did was applied and the following scale relation can be obtained.

$$\frac{u_c d}{\alpha} \sim (Ra - A)^{2/3} \tag{15}$$

Finally, substituting eq.(13) and (15) into eq.(16), Nusselt number correlation in terms of Rayleigh number can be derived.

$$Nu \sim \left(\frac{\delta}{d} - \frac{\alpha}{u_c d}\right)^{-1} \sim \left(\left(\frac{d}{\delta}\right)\left(1 - \left(\frac{u_c d}{\alpha}\right)^{-1}\left(\frac{d}{\delta}\right)\right)\right)^{-1} \tag{16}$$

$$Nu = \frac{C_1(Ra - A)^{1/2}}{1 - C_2(Ra - A)^{-1/6}} \tag{17}$$

where $Nu = \frac{Sd^2}{2k\Delta T}$ $\quad A = \frac{\rho\alpha K^{1/2}d^2 C_F}{\mu\delta^3}$ and $\delta_H = \alpha\left(\frac{C_F K^{1/2}}{g\beta K\Delta T}\right)^{1/2}$. C_1 and C_2 are constants of order 1 and could be determined empirically or analytically.

4. Conclusion

Based on Ergun [2] equation, CFD methodology was developed to understand the natural convection within the porous media for, both Darcy flow and non-Darcy flow condition. Furthermore, the physical model and numerical method was validated showing good agreements with the previous works. To suggest a theoretical base for non-Darcy flow case, Nusselt number correlation was derived by the scale analysis.

As an extension of this study, the further analysis on applications, such as core catcher in LMFBR will be needed.

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