A Preliminary Study on Calculation of Inter-Pebble Dancoff Factor in a Pebble Type Core

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1. Introduction

The Dancoff factor is an entering probability of the neutron escaped from specific fuel kernel to another one without the interaction with moderators. Currently, Dancoff factors are mainly evaluated from stochastic methods, hence a research on analytical method is considerably insufficient in this field. In order to analytically evaluate Dancoff factor considering doubleheterogeneous effect, inter-pebble and intra-pebble Dancoff factors should be calculated, respectively. Intra-pebble Dancoff factor related with the fuel kernels in one pebble was analyzed in past study [1]. For the evaluation of inter-pebble Dancoff factor, fuel region to region Dancoff factor (FRDF) was defined and the method to calculate the FRDF is developed in this study. The result is compared with the calculation result of the MCNP5 code.

2. Methodology

The process for analytical solution of inter-pebble Dancoff factor consists of three steps, as shown in Figure 1. The escape probability from initial fuel region is firstly calculated. FRDF is defined as an entering probability of the neutron escaped from a specific fuel region to another one without any collision with moderator. In the second step, this FRDF is estimated. Finally, entering probability to fuel kernels in the other fuel regions is analyzed.



Fig. 1. Process for Calculating Inter-Pebble Dancoff Factor

From the definition of inter-pebble Dancoff factor, it can be given by Eq. (1).

$$\mathbf{D}_{\text{inter}} = \mathbf{P}_{\text{esc}} \cdot \mathbf{P}_{\text{frr}} \cdot \mathbf{P}_{\text{fent}} \tag{1}$$

where, P_{esc} = neutron escape probability from the initial fuel region

$$P_{frr} = FRDF$$

 P_{fent} = neutron entering probability into the fuel kernel in the other fuel pebbles

D_{inter}=inter-pebble Dancoff factor

The calculation of P_{esc} is pursued by using SSM method in past study [1]. In this study, a method to calculate FRDF is developed.

2.1 Analysis Method for Calculation of FRDF

It is assumed that the pebbles are piled up with BCC structure in pebble type reactor. The fuel pebbles are numbered cubically from the source pebble as shown in Figure 2. The fuel pebbles with the same array number include all fuel pebbles on the surface and vertex in a cube. Hence, the number of fuel pebbles with the array number 1 is 26 and array number 2 includes 98 fuel pebbles. The positive axes, that are the half quarter part of whole space, are only considered because the pebbles are symmetrically piled up from the center pebble.



Fig. 2. Fuel Pebble Arrangement with Array Number in BCC Structure

It is assumed that the source is isotropic. For the generation of isotropic source, the Gauss-Legendre quadrature sets are used. The azimuthal angle is divided into the same number of orthogonal angle. Finally, weights in Gauss-Legendre quadrature sets are considered.

2.2 Calculation of Fuel Region to Region Dancoff Factor (FRDF)

Neutrons have a collision probability with moderator before entering to another fuel region. For the calculation of the collision probability, the penetration distance of neutrons passing the moderator region between the fuel regions should be properly estimated.

The line equation with the specific direction from the source can be given by Eq. (2).

$$\mathbf{x} = \mathbf{t} \cdot \sin\phi \cdot \cos\theta = At \tag{2a}$$

$$\mathbf{y} = \mathbf{t} \cdot \sin \phi \cdot \sin \theta = Bt \tag{2b}$$

$$z = t \cdot \cos \phi = Ct$$
(2c)
where, $A = \sin \phi \cos \theta$

$$B = \sin\phi\sin\theta$$
$$C = \cos\phi$$

The distance from pebble center point to the line is calculated to decide that a neutron penetrates some pebbles or not. The distance equation is given as the followings:

$$t_1 = (aA + bB + cC)/(A^2 + B^2 + C^2)$$
(3a)

$$d = \sqrt{(At_1 - a)^2 + (Bt_1 - b)^2 + (Ct_1 - c)^2}$$
(3b)
where,(a,b,c) = center point of a pebble,(x,y,z)

 t_1 =t value of the distance from (a,b,c)point d=distance from line to center point of pebble

The neutron passes through the pebble where the distance, d, is shorter than the pebble radius. When the neutron passes the moderator pebble or graphite shell of fuel pebble, the attenuation distance with moderator is given as the following:

$$t'_{1} = \frac{(aA+bB+cC) + \sqrt{(aA+bB+cC)^{2} - (A^{2}+B^{2}+C^{2})(a^{2}+a^{2}+a^{2}-R_{p}^{2})}}{A^{2}+B^{2}+C^{2}}$$
(4a)

$$t'_{2} = \frac{(aA+bB+cC) - \sqrt{(aA+bB+cC)^{2} - (A^{2}+B^{2}+C^{2})(a^{2}+a^{2}+a^{2}-R_{p}^{-2})}}{A^{2}+B^{2}+C^{2}}$$
(4b)

$$l = \sqrt{A^{2}(t'_{1} - t'_{2}) + B^{2}(t'_{1} - t'_{2}) + C^{2}(t'_{1} - t'_{2})}$$
where, R_{p} = fuel pebble radius (4c)

 $t'_1, t'_2 = t$ values that the line meets the pebble surface

l = distance to pass through the pebble

Before entering the fuel region, the neutron is attenuated with moderator enclosing fuel region. The distance between the surface of fuel region and fuel pebble on the line is given by Eq. (5).

$$t'' = \frac{(aA+bB+cC) - \sqrt{(aA+bB+cC)^2 - (A^2+B^2+C^2)(a^2+a^2+a^2-R_{fr}^2)}}{A^2+B^2+C^2}$$
(5a)

$$l' = \sqrt{A^2(t''-t'_2) + B^2(t''-t'_2) + C^2(t''-t'_2)}$$
(5b)

where, $R_{fr} = fuel region radius$

- t" = t value that the line meets the fuel region *surface*
- *l'* = distance between fuel region and fuel pebble

Using the attenuation distance and the weight values of Gauss-Legendre quadrature sets, the FRDF can be represented as the following:

$$\mathbf{P}_{\rm frr} = \sum w(\phi) \cdot e^{-\Sigma l_t} \tag{6a}$$

$$l_t = \sum_i l(i) + l' \tag{6b}$$

where, $w(\phi)$ = weight from Gauss-Legendre quadrature sets

 l_t = total distance to pass moderator from the source point to another fuel region

3. Calculation Results and Discussion

FRDF was calculated with Matlab program by the method above. The calculations were pursued for array number 1 to 10, respectively. The radius of fuel region is 2.5cm and the pebble radius is 3cm with 0.61 packing fractions. Gauss-Legendre quadrature sets are chosen to 128 and 512 order in this study. These calculations were also carried out by MCNP5 [1] code for the BCC structure. The calculation results are shown in table 1.

Table I. Result of Fuel Region to Region Dancoff Factor

Array Number	128 Order	512 Order	MCNP5	Rate of Entering (512 Order)
1	2.0425e-01	2.0493e-01	2.0538e-01	83.44%
2	3.8339e-02	3.8170e-02	3.8200e-02	15.54%
3	2.2156e-03	2.1789e-03	2.2000e-03	0.89%
4	2.6177e-04	2.6832e-04	2.2740e-04	0.11%
5	4.3444e-05	4.1001e-05	-	0.02%
6	1.2117e-07	7.7520e-08	-	0.00%
7	00	3.9776e-11	-	0.00%
8	2.7169e-10	6.3673e-11	-	0.00%
9	0000	0000	-	0.00%
10	0000	0000	-	0.00%
Total	2.4511e-01	2.4559e-01	2.4610e-01	100%

The FRDF reaches 0.2451 and 0.2456 for 128 and 512 orders, respectively. These results agree well within 0.5% difference compared with the MCNP5 results. From the analysis result of FRDF, it is clear that array number 3 or below can be neglected in the calculation of inter-pebble Dancoff factor.

4. Conclusions

As this study is preliminary work to calculate interpebble Dancoff factor, the analytic method to calculate FRDF is developed. The result from the method is also compared with MCNP5 result. The results in this study give a good agreement with MCNP5. The method in this study can be easily utilized to analyze the tendency of inter-pebble Dancoff factor. In the future study, the inter-pebble Dancoff factor will be evaluated with the calculation of neutron entering probability into the fuel kernel in the other fuel pebbles. Consequently, a novel methodology, for which analytical evaluation of Dancoff factor including the spatial dependency developed from this study, can be installed to deterministic computer code for pebble-type core which will be developed in future.

Acknowledgment

This work was supported in part by the Ministry of Education, Science and Technology [MEST] of Korea through the Nuclear Hydrogen Development and Demonstration [NHDD] Project coordinated by Korea Atomic Energy Research Institute (M20406010002-05J0101-00212) and the SRC/ERC (R11-2000-067-01001-0), and the Ministry of Knowledge Economy (2008-P-EP-HM-E-06-0000).

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