An Assessment of a LOFT L2-5 LBLOCA Uncertainty Based on ACE-RSM: Complementary Work for the OECD BEMUSE Phase-III Program

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1. Introduction

All participants of the OECD BEMUSE Phase-III program [1] utilize the increased number of samples in their uncertainty quantification on the LOFT L2-5 LBLOCA than one required in the first order of Wilks' formula (i.e., 59). This was mainly due to one of the observations at the end of BEMUSE-II that the number of code runs should be increased instead of 59 runs that were utilized as a reference case to reduce the dispersion of the uncertainty. Main objective of this paper is to provide a complementary method (based on the ACE algorithm) to the method based on the Wilks' formula, which can be assessed with a limited number of code runs. The ACE method [2] offers distinctive advantages over the traditional nonlinear regression techniques, which require a priori selection of the functional forms and often modifications of the functional relationships, even with a much better data fitting than the traditional RSM (response surface model).

2. Methods and Results

In this section, key feature of the ACE (alternating conditional expectation) algorithm is briefly introduced first and then it is applied to the BEMUSE Phase III LBLOCA uncertainty analysis. Finally, the ACE-RSM-based uncertainty analysis results on the blowdown PCT are compared with those of the corresponding MARS 2.3 [3].

2.1 Brief Introduction of the ACE Algorithm

For a dependent variable y and multiple independent variables $(x_i, i = 1,..., p)$, the objective of the ACE algorithm is to find optimal transformations $\theta(y)$ and $\phi_i(x_i)$ that maximize the statistical correlation between $\theta(y)$ and $\sum_i^p \phi_i(x_i)$, by treating each value of $\theta(y)$ as the expectation of several realizations of the sum of $\sum_i^p \phi_i(x_i)$. For a set of N data points (x_{1j}, y_j), the algorithm finds a transformation $\theta(y)$ of y and a functional fit $\phi(x)$ such that the square error in the regression of $\theta(y_j)$ and $\phi(x_{1j})$ [2]

$$e^2 = \frac{1}{N}\sum_1^N \left[\theta(y_j) - \phi(x_{1j})\right]^2 \quad (1)$$

is minimized. With a judicious choice of $\theta(y)$, the error in the above equation could vanish, if $\theta(y_j)$ equals $\phi(x_{1j})$ for every point. In practice, however, this idealized situation does not occur because the data contain randomness and so do $\theta(y_j)$ and $\phi(x_{1j})$. Thus, $\theta(y_j)$ is considered, in the ACE algorithm, the expectation of several realizations of $\phi(x)$ for the j'th point, rather than a single unique realization $\phi(x_{1j})$ as in conventional regression analysis. When a convergence is attained with an iterative scheme [2], the data in each transformed variable are usually smooth and slowly varying. Then, selecting simple functional forms for the transformations and performing standard regression analysis for each transformation, we can obtain the final functional form of y versus $x_1,...,x_p$ if $\theta(y)$ has an inverse function: $y = \theta^{-1}[\sum_{i=1}^p \phi_i(x_i)]$.

2.2 MARS Simulation for the PCT Uncertainty Analysis

In order to assess the PCT uncertainty for a LBLOCA blowdown phase, MARS simulation [3] has been made for 14 uncertainty parameters (see Table I), covering (a) physical models employed in the code and (b) initial and boundary conditions for the LBLOCA simulation. The random variance of each uncertain parameter was determined by a crude Monte Carlo sampling method within the specified range of the corresponding probability distribution (see Table I). Any dependency between parameters was not considered in the sampling process. Multiple sets of 100 samples were implemented to identify the effect of different sets of random samples on the PCT value, consequently resulting in several thousands of PCT values for its statistical analysis.

Table I: Input parameters for PCT uncertainty analysis

x_i	Parameters and ranges: ±2 σ (or min/max)		PDFs
1	Liquid heat transfer	± 20%	Normal
2	Nucleate boiling heat transfer	± 23.2%	Normal
3	AECL lookup CHF table	± 74%	Normal
4	Transition boiling	± 32%	Normal
5	Film boiling heat transfer	± 36%	Normal
6	Vapor heat transfer	± 20%	Normal
7	Peaking factor(Fq)	± 14.96%	Normal
8	Cold gap size	± 20.98mm	Uniform
9	Gap conductance	± 80%	Uniform
10	Fuel conductivity	± 10%	Normal

11	Decay heat	± 6.6%	Normal
12	Break area ratio	0.7~1.15	Uniform
13	Pump two-phase performance	0.0~1.0	Uniform
14	Downcomer lateral loss coeff.	0.0~1.0	Uniform

2.3 Formulation of the ACE-RSM Models and Results

The RSM is a surrogate model to an original model (or code), which is very useful when it takes a long time to obtain the relevant output values and thus the number of evaluations through it is limited to at most several tens or hundreds. Based on the MARS input x_i (p=14) and output values for the PCT (y) uncertainty analysis, the corresponding ACE-RSM models can be formulated through the following procedures:

(Step-1): Derive two types of the ACE-transformed functional forms for independent and dependent variables with the prepared N sample input and output values: $x_i \sim \hat{\phi}_i(x_i)$ and $y \sim \hat{\theta}(y)$.

(Step-2): Perform a (piece-wise) linear regression between the transformed variables, $\hat{\phi}_i(x_i)$ and $\hat{\theta}(y)$:

$$\hat{\theta}^*(y) = \alpha_0 + \sum_{i=1}^{p=14} \alpha_i \hat{\phi}_i(x_i). \qquad (2)$$

(Step-3): Derive the final functional form between the original input and output variables, x_i and y:

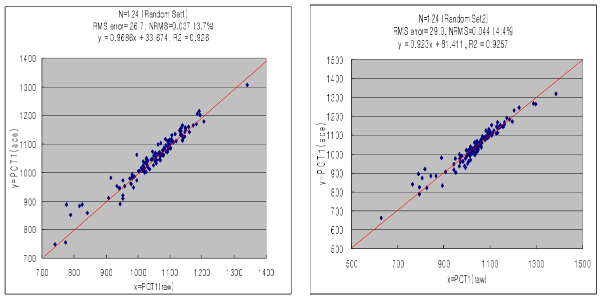
$$\hat{y} = \hat{\theta}^{*-1}\left\{\alpha_0 + \sum_{i=1}^{p=14} \alpha_i \hat{\phi}_i(x_i)\right\}. \qquad (3)$$

Then, 3 typical ACE-RSM models on the PCT have been formulated based on a limited number of sample runs: (a) *one from N=124 (Random Set-1): Model-1*, (b) *one from N=124 (Random Set-2): Model-2*, (c) *one from N=300: Model-3*. Figures 1-2 show one-to-one comparison of the ACE-RSM-based analysis results for the three ACE-RSM models with the corresponding original MARS results. In addition, Table II shows 5% and 95% PCT values for the ACE-RSM models and the corresponding MARS values.

The comparison with the MARS results explains that except for the lower PCT values (subject to highly nonlinear behavior), the ACE-Models trace the original MARS results relatively well. From the qualitative aspect, the ACE-Model-3 (based on a larger number of samples) is quickly converged to the original results, compared with ACE-Model-1 and Model-2. From the quantitative aspect, the accuracy bound for ACE-Model-3 is in between ΔT |MARS-ACE|=5K for 95% PCT and 20K for 5% PCT in the case of N=3000. The above results indicate that the ACE-RSM models could give an appropriate surrogate model to the original MARS code, but they also show a greater or less dependency on the utilized number of samples as in the conventional RSM.

3. Conclusions

The ACE-RSM approach has been applied to assess the blowdown PCT uncertainty for the LOFT L2-5 Experiment as a complementary work to the OECD BEMUSE Phase-III program. The present result has shown that the ACE-RSM approach would be effective in the uncertainty quantification with a limited number of code runs. A greater or lesser dependency on the utilized number of samples can be reduced with more efficient sampling schemes like the LHS approach.



(a) ACE-Model-1 (b) ACE-Model-2

Figure 1 Comparison with MARS result (ACE-Models)

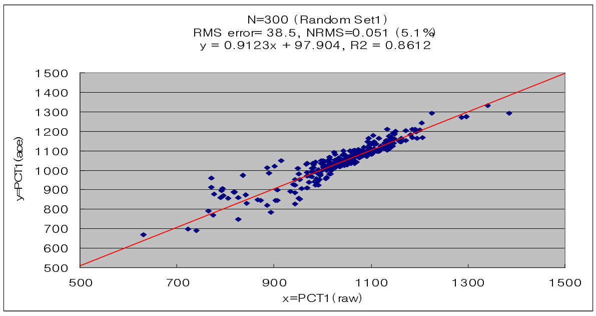


Figure 2 Comparison with MARS result (ACE-Model-3)

Table II: 5/95% PCT (K) values for three ACE-RSM models

N	MARS		ACE-RSM-1		ACE-RSM-2		ACE-RSM-3	
	5%	95%	5%	95%	5%	95%	5%	95%
124	**790.0**	**1286.5**	**824.3**	**1200.8**	**846.1**	**1212.0**	794.0	1204.8
300	**817.7**	**1171.3**	884.8	1189.5	886.9	1178.5	**852.6**	**1182.2**
500	805.8	1172.1	881.7	1182.6	883.0	1176.6	846.7	1173.4
1000	810.7	1173.5	864.8	1181.4	867.7	1177.4	835.0	1173.7
3000	815.5	1174.4	861.2	1179.0	867.8	1177.7	834.4	1176.0
6000	-	-	859.3	1179.3	867.5	1178.6	834.8	1177.1
9000	-	-	857.5	1179.0	864.8	1178.6	N/A	-
10000	-	-	857.4	1178.6	864.5	1178.6	N/A	-

REFERENCES

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