Collapse Moment Prediction for the Wall-Thinned Pipe Bends Using Fuzzy Neural Networks and Uncertainty Analysis

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1. Introduction

The pipe bends and elbows are regarded as critical components in piping systems of nuclear power plants because they are incorporated into piping systems to allow modification of the isometric routing and more importantly pipe bends are usually incorporated to reduce anchor reaction forces. Also, the pipe bends and elbows are capable of absorbing considerably large thermal expansion and seismic movement through the energy dissipation as a result of local plastic deformation so that they maintain the integrity of piping system under transiently loading conditions [1]. However, significant care must be taken to avoid their collapse moment. Therefore, it is important to accurately assess the safety margin for a collapse of pipe bends and elbows under various operating conditions.

2. Fuzzy Neural Networks

2.1 Fuzzy Neural Network (FNN)

The fuzzy model is constructed from a collection of fuzzy if-then rules. The inputs and outputs of the fuzzy model are real-valued variables. Therefore, instead of considering the Mamdani type fuzzy if-then rules that requires time-consuming defuzzification calculation, a Takagi-Sugeno type fuzzy inference system is used where the i-th fuzzy rule for k-th time instant data is described as follows:

$$\begin{aligned}&\text{If } x_1(k) \text{ is } A_1^i(k) \;\; AND \cdots AND \;\; x_m(k) \text{ is } A_m^i(k),\\ &\text{then } \hat{y}^i(k) \text{ is } f^i\left(x_1(k), \cdots, x_m(k)\right)\end{aligned} \tag{1}$$

The fuzzy model identification can be accomplished through clustering of numerical data. A subtractive clustering (SC) method is used as the basis of a fast and robust algorithm for identifying a fuzzy model and assumes the availability of N input/output training data $\left(\mathbf{x}^T(k), y(k)\right)$ where $\mathbf{x}^T(k) = \left(x_1(k), x_2(k), \cdots, x_m(k)\right)$, $k = 1, 2, ..., N$. It is assumed that the data points have been normalized in each dimension. The method starts by generating a number of clusters in the $m \times N$ dimensional input space. The SC method considers each data point as a potential cluster center and uses a measure of the potential of each data point, which is defined as a function of the Euclidean distances to all other input data points:

$$P(k) = \sum_{j=1}^{N} e^{-4\left\|\mathbf{x}(k)-\mathbf{x}(j)\right\|^2 / r_\alpha^2}, \;\; k = 1, 2, ..., N \tag{2}$$

When the cluster estimation method is applied to a collection of input/output data, each cluster center is in essence a prototypical data point that exemplifies a characteristic behavior of the system and each cluster center can be used as the basis of a fuzzy rule that describes the system behavior. Therefore, a complete fuzzy system identification algorithm can be developed based on the results of the SC technique. A number of n Takagi-Sugeno type fuzzy rules can be generated, where the premise parts are fuzzy sets, defined by the cluster centers that are obtained by the SC algorithm. The membership function $A^i(\mathbf{x}(k))$ of an input data vector $\mathbf{x}(k)$ to a cluster center $\mathbf{x}^*(i)$ can be defined as follows:

$$A^i\left(\mathbf{x}(k)\right) = e^{-4\left\|\mathbf{x}(k)-\mathbf{x}^*(i)\right\|^2 / r_\alpha^2} \tag{3}$$

The fuzzy inference system output $\hat{y}(k)$ is calculated by the weighted average of the consequent parts of the fuzzy rules as follows:

$$\hat{y}(k) = \sum_{i=1}^{n} A^i\left(\mathbf{x}(k)\right) f^i\left(\mathbf{x}(k)\right) \Big/ \sum_{i=1}^{n} A^i\left(\mathbf{x}(k)\right) \tag{4}$$

The function $f^i\left(\mathbf{x}(k)\right)$ is a polynomial in the input variables as follows:

$$f^i\left(\mathbf{x}(k)\right) = \sum_{j=1}^{m} q_{ij} x_j(k) + r_i \tag{5}$$

The output of the fuzzy model given by Eq. (5) can be rewritten as

$$\hat{y}(k) = \sum_{i=1}^{n} \bar{w}^i(k) f^i\left(\mathbf{x}(k)\right) = \mathbf{w}^T(k)\mathbf{q} \tag{6}$$

The fuzzy model should be optimized to accomplish the desired performance. The optimization is accomplished by a genetic algorithm combined with a least-squares method.

2. Uncertainty Analysis

In this paper, we use statistical and analytical uncertainty analysis methods.

2.1 Statistical Method

The statistical uncertainty analysis works by generating many bootstrap samples of the training data set and retraining the data-based model parameters on each bootstrap sample. After repetitive sampling and training, the resulting predictions provide a distribution for the output value. This distribution can be used to calculate prediction intervals. In this study, the bootstrap pairs sampling algorithm which is one of statistical methods is used. The available data is divided into development data and test data. The development data consists of a large pool of data from which training and verification samples can be drawn. The test data is fixed. The pool of development data represents all available data, excluding a fixed test data set. The estimate with a 95% confidence interval for an arbitrary test input $\mathbf{x}_0$ is

$$\hat{y}_0 \pm 2\sqrt{Var(\hat{y}_0)+bias^2} = \hat{y}_0 \pm \delta . \quad (7)$$

2.2 Analytic Method

The variance of the predicted output can be estimated as follows:

$$Var(y_0-\hat{y}_0) \approx \sigma^2 + \mathbf{f}_0^T \mathbf{S} \mathbf{f}_0 \approx s^2 + s^2 \mathbf{f}_0^T \left(\mathbf{F}^T\mathbf{F}\right)^{-1} \mathbf{f}_0 \quad (8)$$

The matrix $\mathbf{F}$ is called the Jacobian matrix of first order partial derivatives with respect to the parameters determined from the least squares. The estimate with a 95% confidence interval is

$$\hat{y}_0 \pm 2s\sqrt{1+\mathbf{f}_0^T \left(\mathbf{F}^T\mathbf{F}\right)^{-1} \mathbf{f}_0} = \hat{y}_0 \pm \delta \quad (9)$$

3. Application to the collapse moment prediction

The three fuzzy models are trained for three kinds of data sets. They consist of the extrados, intrados and crown defect locations, respectively, which has smaller errors compared with results using only one data set. The number of rules for the three fuzzy models was automatically determined by the SC method [2]. To determine the antecedent parameters, such as the membership function parameters, we used the genetic algorithm to optimize the cluster radius and we used the least squares method to optimize the consequent parameters q_{ij} and r_i . To conduct an uncertainty analysis, FNN was trained by training data sets to analyze uncertainty analysis. They are verified through test data sets using 170 intrados cases, 170 extrados cases and 32 crown cases. Table 1 shows the performance of the fuzzy model predicting the collapse moment and Table 2 shows its uncertainty analysis results. Fig. 1 shows the predicted error and normalized predicted interval.

4. Conclusions

In this paper, FNN has been used to predict the collapse moment due to the wall-thinned defects of bends in piping systems. Three fuzzy models were trained for three data sets divided into the three classes of extrados, intrados, and crown defects. The relative RMS errors are about 0.6% for the training data and 0.8% for the test data. The RMS error of the fuzzy models for the test data is only a little greater than the RMS error for the training data. Therefore, if the fuzzy models are trained first by using a number of data including a variety of loading conditions and defect geometry cases, they can accurately estimate the collapse moment for any other defect cases. The FNN was accurate sufficiently for application to the collapse moment prediction of the wall-thinned pipe bends.

REFERENCES

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[2] M. G. Na, J. W. Kim, and I. J. Hwang, "Collapse Moment Estimation by Support Vector Machines for Wall-Thinned Pipe Bends and Elbows," *Nucl. Eng. Des.*, vol. 237, no. 5, pp. 451-459, Mar. 2007.

Table 1. Performance of the fuzzy model

	Training data		Test data	
	Relative maximum error (%)	Relative RMS Error (%)	Relative maximum error (%)	Relative RMS Error (%)
Extrados defects	5.3760	0.5476	4.5134	0.8095
Intrados defects	3.6368	0.5035	6.0006	0.9354
Crown	4.7385	0.6717	2.3344	0.7841

Table 2. Uncertainty analysis results on collapse moment of wall-thinned pipe bends

Thinning Location	Analytic Method		Statistic Method	
	Data numbers out of Prediction interval per total numbers	Un-certainty (%)	Data numbers out of Prediction interval per total numbers	Un-certainty (%)
Extrados	9/170	5.29	7/170	4.12
Intrados	14/170	8.24	3/170	1.76
Crown	3/32	9.38	3/32	9.38

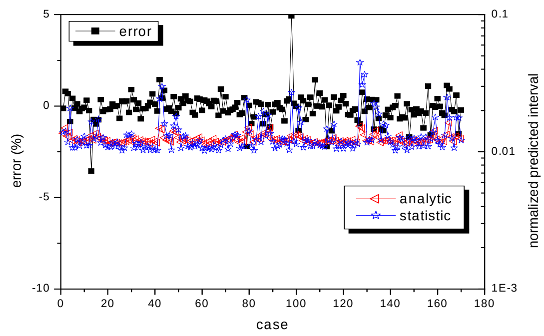

Fig. 1. Predicted error and normalized predicted interval