A Semi-Implicit Numerical Method with a Symmetric Pressure Matrix for multi-dimensional two-phase flows

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1. Introduction

More sophisticated computer codes are needed for a detailed analysis of a two-phase flow in a nuclear reactor coolant system. Although extensive two-phase models were implemented into the system codes such as RELAP5 and TRAC, there computation scale is too large for the analysis of local behavior. Recently, computation fluid dynamics (CFD) codes are applied to this area. However, their two-phase flow applications are somewhat limited due to the lack of two-phase flow models. Hence, a component-scale thermal-hydraulic code CUPID [1,2,3] had been developed at KAERI aiming at providing numerical solutions for issues such as downcomer boiling in the APR1400.

In the CUPID-I code, a numerical method was proposed [1,2] based on the semi-implicit method of the REPLAP5 for the application to multi-dimensional non-staggered unstructured grids. It had been verified against several conceptual problems. The pressure equation was obtained by solving mass, energy and momentum equations simultaneously, where the resulting pressure matrix became asymmetric. Unlike the one-dimensional case where the matrix is easily solved since it is basically tri-diagonal, it is expensive to solve the asymmetric matrix and sometimes it fails to get the solution. In the CUPID-M code [3], the pressure equation was modified so that the pressure matrix became symmetric by solving mass and momentum equations. The phase change and transient density terms of the mass conservation equations were linearized with respect to pressure. The steady state solutions were in good agreement with that of CUPID-I and the pressure solver was more fast and stable. However, the transient solution differed from CUPID-I since the energy variation was not considered in the mass conservation equations.

In this study, the mass conservation equations are linearized with respect to energy as well as pressure to improve the transient solution. The calculations are compared to that of CUPID-I. The developed code is verified against FLUENT code for standard problems.

2. Governing Equations

The two-phase governing equations are employed for the transient two-phase analysis. The continuity, momentum, and energy equations are;

$$\frac{\partial}{\partial t}(\alpha_k \rho_k) + \nabla \cdot (\alpha_k \rho_k \underline{u}_k) = \Gamma_k \tag{1}$$

$$\frac{\partial}{\partial t}(\alpha_k \rho_k \underline{u}_k) + \nabla \cdot (\alpha_k \rho_k \underline{u}_k \underline{u}_k) = -\alpha_k \nabla P + \nabla \cdot [\alpha_k \tau_k] + \alpha_k \rho_k \underline{g} + P \nabla \alpha_k + M_k^{mass} + M_k^{drag} + M_k^{VM} \tag{2}$$

$$\frac{\partial}{\partial t}[\alpha_k \rho_k e_k] + \nabla \cdot (\alpha_k \rho_k e_k \underline{u}_k) = -\nabla \cdot (\alpha_k q_k) + \nabla \alpha_k \tau_k : \nabla \underline{u}_k - P \frac{\partial}{\partial t}\alpha_k - P \nabla \cdot (\alpha_k \underline{u}_k) + I_k + Q'''_k \tag{3}$$

where α_k, ρ_k, $\underline{u}_k$, P_k, and Γ_k are the k-phase volume fraction, density, velocity, pressure, and interface mass transfer rate, respectively. M_k represents the interfacial momentum transfer due to the mass exchange, the drag, and the virtual mass.

3. Numerical Methods

The overall numerical algorithm is the same as that of the previous codes versions [1,2,3]. Only the linearization method is improved for the mass conservation equations. The mass conservation equations for liquid and gas are added as the following equation.

$$\sum_k \frac{1}{\rho_k}\left(\nabla \cdot \alpha_k \rho_k \underline{u}_k^{n+1}\right) = \sum_k \left(\frac{\Gamma_k^{n+1}}{\rho_k} - \frac{\alpha_k}{\rho_k}\frac{\Delta \rho_k}{\Delta t}\right) \tag{4}$$

For a steam the volumetric mass transfer rate is defined as

$$\Gamma_v = -\frac{H_{iv}(T^s - T_v) + H_{il}(T^s - T_l)}{h_v^* - h_l^*}, \tag{5}$$

where $(h_v^*, h_l^*) = (h_v^s, h_l); \Gamma_v \geq 0,\ (h_v^*, h_l^*) = (h_v, h_l^s); \Gamma_v < 0$.

Eq. (5) is linearized with respect to the pressure and energy changes.

$$\begin{aligned}\Gamma_v^{n+1} = &-\frac{H_{iv}}{h_{iv} - h_{il}}\left(\frac{\partial T_s}{\partial p} - \frac{\partial T_v}{\partial p}\right)p' - \frac{H_{il}}{h_{iv} - h_{il}}\left(\frac{\partial T_s}{\partial p} - \frac{\partial T_l}{\partial p}\right)p' \\ &- \frac{H_{iv}}{h_{iv} - h_{il}}\left[\left(\frac{\partial T_s}{\partial e_g} - \frac{\partial T_v}{\partial e_g}\right)e_g' + \left(\frac{\partial T_s}{\partial X_n} - \frac{\partial T_v}{\partial X_n}\right)X_n'\right] \\ &- \frac{H_{il}}{h_{iv} - h_{il}}\left[\left(\frac{\partial T_s}{\partial e_g}\right)e_g' + \left(\frac{\partial T_s}{\partial X_n}\right)X_n' - \left(\frac{\partial T_l}{\partial e_l}\right)e_l'\right] \\ &- \frac{H_{iv}}{h_{iv} - h_{il}}(T_{sat} - T_v) - \frac{H_{il}}{h_{iv} - h_{il}}(T_{sat} - T_l)\end{aligned} \tag{6}$$

The transient density term is linearized as

$$\frac{\Delta \rho_k}{\Delta t} = \left(\frac{\partial \rho_k}{\partial p}\right)\frac{p'}{\Delta t} + \left(\frac{\partial \rho_k}{\partial e_k}\right)\frac{e_k'}{\Delta t}. \tag{7}$$

Since the energy equation is not involved in the pressure equation, the old time values are used for the energy and non-condensable mass changes.

$$p' = p^{n+1} - p^{n}, e_k' = e_k^{n} - e_k^{n-1}, X_n' = X_n^{n} - X_n^{n-1} \quad (8)$$

4. Numerical Results

At first the improved model was applied to the boiling in a vertical pipe where the transient solutions were different between CUPID-I and CUPID-M. Figure 1 shows that the present method predicts well the CUPID-I result. Next, the new CUPID code has been compared to the FLUENT code for three cases of U-tube Manometric oscillation, cavitation and rising gas plume. Figures 2,3, and 4 compares the results of CUPID and FLUENT. All the calculations showed a good agreement between the two codes.

5. Conclusions

The pressure equation in the CUPID code was improved such that the pressure matrix become symmetric. The calculations were fast and stable and showed a good agreement with the previous results. It also showed a good agreement with the FLUENT results.

REFERENCES

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[3] H. Y. Yoon et al., “An Unstructured SMAC Algorithm for Thermal Non-equilibrium Two-Phase Flows,” *International Communications in Heat and Mass Transfer,* Vol. 36, 16–24 (2009).

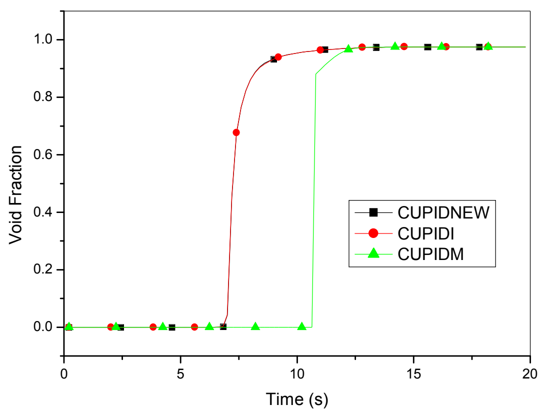


Figure 1. Transient void fractions at y = 1.8 m

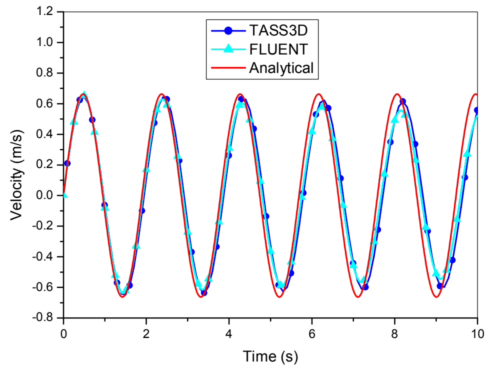


Figure 2. U-tube Manometric oscillation

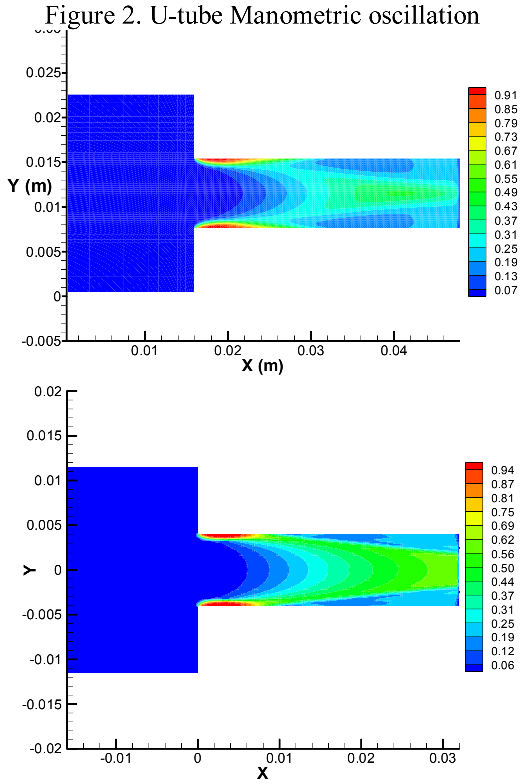
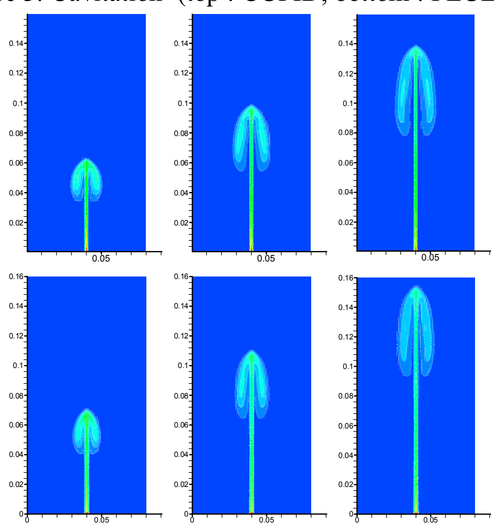


Figure 3. Cavitation (top : CUPID, bottom : FLUENT)



0.4s 0.8s 1.2s

Figure 4. Rising gas plume
(top : CUPID, bottom : FLUENT)