# A Monte Carlo Depletion Method with Leakage Corrected Critical Spectrum

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# 1. Introduction

In conventional deterministic depletion codes, e.g., CASMO or HELIOS [1], a critical spectrum (or leakage corrected spectrum) is used for the depletion of burnable nuclides and the buildup of fission products. The critical spectrum is obtained by a critical buckling search such as the  $B_1$  method. However, the deterministic depletion codes have limitations in treating complex geometry and continuous energy.

The Monte Carlo method overcomes these limitations, but the Monte Carlo depletion codes such as the MONTEBURNS code [2] do not use the critical spectrum [3].

Thus, in this paper, we introduce an approach to Monte Carlo depletion with leakage corrected spectrum, for the first time to our knowledge, in which the streaming (leakage) term of the transport equation is treated as an eigenvalue term and transformed into an extended albedo boundary condition problem. This approach is implemented in the MONTEBURNS code. In our new approach, the MCNP code [4] is modified to solve the albedo boundary condition problem, while the MONTEBURNS code is modified to obtain an appropriate albedo value at current burnup time step by the secant method. Numerical tests of our new implementation are performed on a representative PWR assembly problem.

# 2. Leakage Correction Using Extended Albedo Boundary Condition

In our approach, the MCNP code is wrapped around by the secant method. Thus, the MCNP code solves the following albedo boundary balance equation for eigenvalue  $k_{eff,i}$ , with given albedo  $\alpha_i$  by adjusting the reflected Monte Carlo particle weight:

$$H\psi = \frac{1}{k_{eff,i}} F\psi, \qquad (1)$$

$$\psi(\vec{r},\hat{\Omega},E) = \alpha_i \psi(\vec{r},\hat{\Omega}',E), \text{ for } \hat{n} \cdot \hat{\Omega} < 0, \vec{r} \in \Gamma, (2)$$

where *i* is the iteration index of the secant method. The definition of parameters and operator notations are standard [5]. Then,  $\alpha_{i+1}$  is updated untill  $k_{eff}$  becomes

1.0 as follows:

$$\alpha_{i+1} = \alpha_i - \frac{k_{eff,i} \left(\alpha_{i-1} - \alpha_i\right)}{k_{eff,i-1} - k_{eff,i}},$$
(3)

where  $\alpha_1 = 1.0$  and  $\alpha_2 = 0.5$  are used for the initial

values.  $3\sigma$  (the standard deviation of  $k_{eff,i}$ ) is chosen as an iteration stop criterion. A flow chart of the leakage correction in the modified MONTEBURNS code is shown in Fig 1.



Fig. 1. Flow chart of leakage corrected spectrum depletion in the modified MONTEBURNS code

# 3. Numerical Results

A representative PWR assembly problem composed of 18x18 fuel rods is considered for the test of the leakage corrected MONTEBURNS code. A detailed description of the test problem is shown in Fig.2 and Table I.



Fig. 2. Configuration of a PWR assembly problem

| Region    | Material  | Radius [cm] |
|-----------|---|-------------|
| Fuel      | $4 \text{ w/o UO}_2$<br>( $\rho$ =10.89 [g/cm <sup>3</sup> ]) | 0.4095      |
| Gap       | Water<br>(p=1.0 [g/cm <sup>3</sup> ])                         | 0.4180      |
| Clad      | Zircalory-4<br>(ρ=6.489 [g/cm <sup>3</sup> ])                 | 0.4750      |
| Moderator | Water<br>(p=1.0 [g/cm <sup>3</sup> ])                         | -           |

Table I: Material properties of a PWR assembly problem

A comparison of the  $k_{inf}$  along burnup and the albedo values is shown in Fig. 3.



Fig. 3. Trend of  $k_{inf}$  and albedo vs. burnup

As shown in Fig. 4, the leakage corrected spectra are harder than the conventional case at initial state, while the spectra in two cases tend to be closer at 50.9 [GWD/MTU] burnup.



Fig. 4. Normalized neutron flux spectra in two burnup stages

Due to the difference in spectrum, the fission to capture ratio and one group cross-sections of fuel material in the leakage corrected case show discrepancies as in Fig. 5. The discrepancies in depleted or produced nuclides, e.g., Pu<sup>239</sup>, are also shown in Fig. 6.



Fig. 5. Fission to capture ratio and one group cross-sections in fuel material vs.burnup



Fig. 6. Comparison of Pu<sup>239</sup> vs. burnup

#### 4. Conclusions

The leakage corrected critical spectrum depletion was introduced in this paper and implemented in the MONTEBURNS code using the concept of extended albedo boundary condition.

Numerical results on a representative PWR assembly problem showed considerable discrepancies in buildup and depletion of nuclides between the leakage corrected case and the conventional case.

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