An SP_3 Nodal Method Applied to Fast Reactor Core Analysis

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1. Introduction

The goal of the simplified P_N (SP_N) method is to obtain a relatively inexpensive approximation to the transport equation that contains most of the transport physics characteristics lacking in the diffusion theory. Numerous studies have been performed to investigate the accuracy of the SP_N equations and it has been confirmed that the SP_N equations are significantly more accurate than the diffusion equations but with almost the same computational efforts. [1-3]

In this paper, SP_3 equations are solved by using the nodal expansion method based on a conformal mapping for fast reactor core analysis of a hexagonal geometry. The partial current response matrices are constructed for the coupled nodal SP_3 equations by applying the relationship between the partial currents and the surface-averaged fluxes. The response matrices are solved non-linearly. Then to accelerate the convergence, a coarse mesh rebalancing scheme is adopted.

2. Methods and Results

SP_3 equations in the x-y geometry are given as

$$-\frac{\partial^2}{\partial x^2}\frac{1}{3\Sigma_1^g}\phi_0^g(x,y)-\frac{\partial^2}{\partial y^2}\frac{1}{3\Sigma_1^g}\phi_0^g(x,y)+\Sigma_0^g\phi_0^g(x,y) = Q^g(x,y)+\frac{\partial^2}{\partial x^2}\frac{2}{3\Sigma_1^g}\phi_2^g(x,y)+\frac{\partial^2}{\partial y^2}\frac{2}{3\Sigma_1^g}\phi_2^g(x,y) \quad (1)$$

and

$$-\frac{\partial^2}{\partial x^2}\left(\frac{1}{3\Sigma_3^g}+\frac{28}{81\Sigma_1^g}\right)\phi_2^g(x,y)-\frac{\partial^2}{\partial y^2}\left(\frac{1}{3\Sigma_3^g}+\frac{28}{81\Sigma_1^g}\right)\phi_2^g(x,y)+\frac{35}{27}\Sigma_2^g\phi_2^g(x,y) = \frac{\partial^2}{\partial x^2}\frac{14}{81\Sigma_1^g}\phi_0^g(x,y)+\frac{\partial^2}{\partial y^2}\frac{14}{81\Sigma_1^g}\phi_0^g(x,y) \quad , \quad (2)$$

where $\Sigma_n^g=\Sigma_t^g-\Sigma_{sn}^g, \quad n=0,1,2,3$ and the cross sections are assumed to be position-independent.

Chao and Tsoulfanidis showed that the diffusion equation in a hexagon in the x-y geometry can be easily transformed to the one in a rectangle as shown in Fig. 1 by introducing a scale function for a conformal mapping. [4] The Laplacian diffusion operator in Eqs. (1) and (2) is invariant under a conformal mapping, in the sense of only a scale change, without generating crossed terms:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)=\left[\frac{1}{g^2(u,v)}\right]\left(\frac{\partial^2}{\partial u^2}+\frac{\partial^2}{\partial v^2}\right),$$

where $g(u,v)=|dz/dw|$ is the nonnegative mapping linear scale function and its square is the mapping area scale function.

The SP_N equations are much like a diffusion equation in their forms and thus the SP_3 equations in a hexagon can also be transformed to those of a rectangle in the same way.

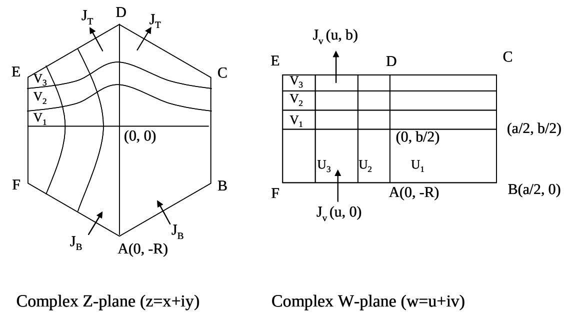

Fig. 1. Conformal mapping of a hexagon to a rectangle

By integrating the conformal mapped SP_3 equations transversely along v, we have

$$-D_0\frac{d^2}{du^2}\phi_0(u)+\Sigma_0\overline{g^2}(u)\phi_0(u)=\overline{g^2}(u)Q(u)-L_0(u)+2D_0\frac{d^2}{du^2}\phi_2(u) \quad (3)$$

and

$$-\left(\frac{7}{9}D_2+\frac{28}{27}D_0\right)\frac{d^2}{du^2}\phi_2(u)+\frac{35}{27}\Sigma_2\overline{g^2}(u)\phi_2(u) = -\frac{14}{27}L_0(u)+\frac{14}{27}D_0\frac{d^2}{du^2}\phi_0(u)-\frac{7}{9}L_2(u) \quad , \quad (4)$$

where $L_i(u)$ = transverse leakage from the i-th moment of the current at the top and bottom surfaces,
$Q(u)$ = effective neutron source.

For a given Q_i and L_i, the one-dimensional flux moments of Eqs. (3) and (4) are approximated with the following expansions:

$$\phi_0(u)=A_0\cosh\kappa_0 u+B_0\sinh\kappa_0 u+a_{00}+a_{01}w_1(u)+a_{02}w_2(u) \quad (5)$$

and

$$\phi_2(u)=A_2\cosh\kappa_2 u+B_2\sinh\kappa_2 u+a_{20}+a_{21}w_1(u)+a_{22}w_2(u) \quad , \quad (6)$$

where

$$\kappa_0^2=\frac{\overline{g^2}(a/2)\Sigma_0}{D_0}, \quad \kappa_2^2=\frac{\overline{g^2}(a/2)\frac{35}{27}\Sigma_2}{\frac{7}{9}D_2+\frac{28}{27}D_0},$$

and w_n are orthogonal polynomials with respect to $\overline{g^2}(u)$ over an interval of (-a/2, a/2). The five coefficients in Eqs. (5) and (6) can be determined with the following five conditions for $\phi(u)$: two boundary conditions for $\phi(u)$ in terms of the net currents on the surfaces of $u=\pm a/2$, matching the first moment (w_1) and second moment (w_2) projections on the two sides of Eqs. (3) and (4), and the overall neutron balance in the node.

The accuracy and efficiency of the conformal mapped nodal SP_3 method have been tested for three fast reactor problems. The first fast reactor problem is the two-dimensional SNR benchmark problem [5] and the second one is the two-dimensional KNK benchmark problem [6] of the NEACRP benchmark problems. The last one is the KALIMER-150 problem, which is a simplified two-dimensional model of the KALIMER-150 core [7].

The numerical results of the conformal mapped nodal diffusion and the SP_3 methods with a flat transverse leakage approximation are compared with those of TWOHEX [8] using a number of triangular meshes per hexagon. TWOHEX is a two-dimensiona1 discrete ordinates code.

The reference solution was obtained by TWOHEX with the eigenvalue convergence criterion of $1x10^{-6}$ and normalization for the 6 neutrons in the whole core. In this paper, the outermost assembly region was excluded for the comparison with the reference solutions since the fluxes are re1atively small and somewhat large errors are introduced in the outermost assemblies. These large errors are associated with using the flat transverse leakage approximation for the conformal mapped nodal SP_3 method in this paper.

Eigenvalues, power errors, and calculation times are summarized in Table I. As seen from this table, the eigenvalue error by the SP_3 method is decreased by about 60% when compared to that by the diffusion method. The RMS and maximum power errors are improved by about 70% and 45%, respectively by the SP_3 method. However, the calculation time by the SP_3 method is increased by about 1.3 to 3.5 times when compared to that of the diffusion method, about 2.3 times on average.

Table I. Comparison of the eigenvalues, powers and CPU times

		k_{eff} Error (pcm)		Power Error[a] (%) RMS (Maximum)		CPU Time[b] (sec)	
		DT	SP_3	DT	SP_3	DT	SP_3
SNR-300	Rods-out	-993	-786	1.21(2.23)	0.93(1.88)	0.094	0.125
	Rods-in	-936	-424	1.52(3.11)	0.99(2.23)	0.078	0.125
KNK	Rods-out	-1,066	-255	1.81(3.06)	0.71(1.34)	0.031	0.109
	Rods-in	-2,130	296	2.30(4.13)	0.73(1.21)	0.031	0.094
KALIMER-150	Rods-out	-1,083	-223	3.18(4.72)	0.70(1.55)	0.359	0.703
	Rods-in	-1,345	-24	3.52(4.96)	0.96(2.04)	0.328	0.688

[a]Outermost assembly region excluded. For KALIMER-150, IVS region also excluded.
[b]CPU time on Mobile Intel Pentium M 740J, 1.733 GHz

3. Conclusions

Hexagonal nodal SP_3 equations were formulated and implemented to have more accurate results than those by diffusion calculations from the neutronics analysis of fast reactors without large increase in computation time. To overcome singularity problem, which occurs when transverse integration is applied to a hexagon in a conventional manner, a hexagon was conformally mapped to a rectangle in this study.

Numerical results show that the conformal mapped nodal SP_3 method can be effectively used for the neutronics analysis of fast reactors. The conformal mapped nodal SP_3 method improves the eigenvalue and power prediction accuracy considerably when compared to those by the conformal mapped nodal diffusion method while its calculation time is not increased too much.

The transverse leakage profile at each surface of a hexagon is assumed to be flat in this study. This causes large flux errors in the outermost assemblies. Although the flux level is relatively small in those assemblies and the accuracy of the current conformal mapped nodal SP_3 method is good enough to be applicable for the neutronics analysis of fast reactors, it would be worthwhile improving the transverse leakage approximation as well as having the capability of three-dimensional calculation first of all.

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