An Analytical Evaluation for Spatial-Dependent Intra-Pebble Dancoff Factor

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1. Introduction

Intra-pebble Dancoff factor (IPDF) is the probability that a neutron leaving a fuel kernel will enter another kernel without any collision in the moderator within the same pebble. The evaluation of Dancoff factor for the pebble type reactor is important in order to solve the slowing down equation in resonance region. Even though many studies to evaluate IPDF have been done, an analytical evaluation for the spatial-dependent intra-pebble Dancoff factor (SIPDF) has not been tried yet. The analytical evaluation of SIPDF is useful to understand the effect of SIPDF for the various geometries. In this study, a model is developed to get the analytical solution for SIPDF.

2. Model and Method

The fuel kernels which are distributed in the fuel region of one pebble have IPDF depending on the position from the center of a fuel pebble. The SIPDFs are analytically calculated by the Sphere Shell Model (SSM) developed in this study.

2.1 Sphere Shell Model

Firstly, to calculate SIPDF, any source kernel position in the fuel region of the pebble is chosen. The neutron emits uniformly from the center point of source kernel. The neutron will enter another fuel kernel with a probability. To calculate the probability, SSM developed in this study is a model which has the concentric shells with the same thickness from the source kernel. The fuel kernels are stochastically located at the midpoint of each shell. This model is shown in Figure 1.

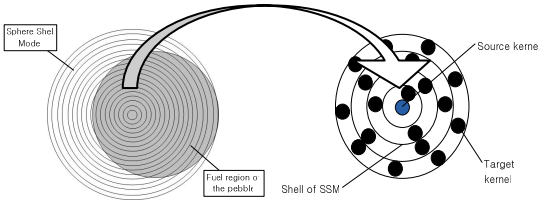

Fig. 1. The SSM at Any Location of Fuel Kernel and Fuel Kernel Distribution in Each Shell

2.2 Determination of the Fuel Kernel Number in Each Shell of SSM

In SSM, IPDF depends on the number of fuel kernels in each shell because the number is applied to the final equation of IPDF deduced.

It is assumed that the lattice in this study is defined as a region including one fuel kernel within the shell and all lattices have the same volume. The number of fuel kernels in each shell is given by Eqs. (1a) and (1b).

$$n_{\text{nth shell}} = V_{\text{nth\_shell}} / V_{\text{lattice}} \quad (1a)$$

$$V_{\text{lattice}} = V_{\text{fuel region}} / n_{\text{a pebble}} \quad (1b)$$

where, n=number of fuel kernels

To calculate the volume of each shell bounded by pebble, the intersection volume of two spheres is used as shown in Figure 2.

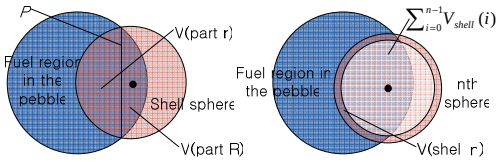

Fig. 2. n-th Intersection Volumes and Shell Volume

The equations of two spheres are as the followings:

$$x^2 + y^2 + z^2 = R^2 \quad (2a)$$

$$(x - u(i))^2 + y^2 + z^2 = r^2 \quad (2b)$$

where,
R = radius of fuel region of a pebble
r= n-th shell radius of SSM
u(i)= center location of SSM from center of pebble

The surface of intersection of both spheres is as the followings:

$$p = \{(R^2 - r^2) + (u(i))^2\}/(2u(i)) \quad (2c)$$

$$p' = p - u(i) \quad (2d)$$

where, p = intersection surface for the pebble sphere
p' = coordinates movement of p
(intersection surface for SSM sphere)

The partial intersection volumes of the spheres (as shown in Figure 2) are as the followings:

$$V_{part,R} = \int_{|p|}^{R} \pi(\sqrt{R^2 + x^2})^2 dx \quad (2e)$$

$$V_{part,r} = \int_{|p'|}^{r} \pi(\sqrt{r^2 + x^2})^2 dx \quad (2f)$$

The intersection volume equations rely on the conditions as shown Figure 3. And the equations are in Table 1.

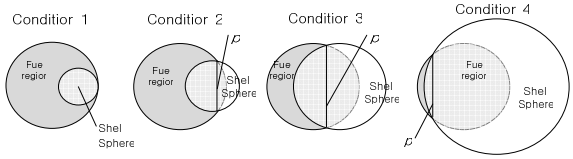

Fig. 3. Conditions in Each Calculation

Table I. Intersection Volume in Each Condition

Condition	Volume
{R-(u(i)+r)}<0	$V_{its} = \frac{4\pi}{3} r^3$
R≥*p*>u(i)	$V_{its} = V(r) - V_{part,r} + V_{part,R}$
0≤*p*≤u(i)	$V_{its} = V_{part,r} + V_{part,R}$
-R≤*p*<0	$V_{its} = V(R) - V_{part,R} + V_{part,r}$

Where, V_{its}= intersection volume

Finally, the n-th shell volume is given by Eq. (2g).

$$V_{nth\_shell} = V_{it\,\sec} - \sum_{i=0}^{n-1} V_{shell}(i) \quad (2g)$$

2.3 Determination of the SIPDF and IPDF

$\mathbf{p_{f,n}}$ is the probability that a neutron emitted from the fuel source kernel will enter a fuel kernel of the n-th shell of SSM without interaction with the moderator. $\mathbf{p_{m,n}}$ is the probability that a neutron emitted from the fuel source kernel will have the interaction with the moderator between the source and target kernel.

It is assumed that the source kernel is an isotropic point source and the neutron emitted from the source kernel has a collision with moderator on the probability as a function of mean distance (the average value between a maximum and minimum distance). These probabilities are given by

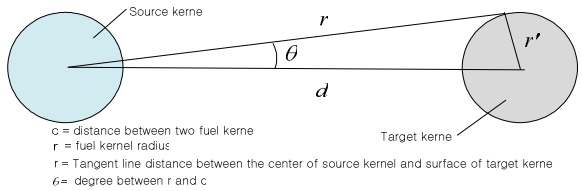

Fig. 4. Probability Calculation for Source and Target Kernel in SSM

$$p_{f,n} = \int_0^{\theta} r \cdot d\theta \int_0^{2\pi} d\phi \cdot r\sin\theta/(4\pi r^2) \cdot e^{-\Sigma_t a} = \frac{1}{2}(1-\cos\theta)\cdot e^{-\Sigma_t a} \quad (3a)$$

$$p_{m,n} = \frac{1}{2}(1-\cos\theta)\cdot(1-e^{-\Sigma_t a}) \quad (3b)$$

where,

$a = \{(d^2 - kr^2)^{1/2} + d - 3\cdot kr\}/2$, *the mean distance*

$\sum_t$= *total macroscopic cross-section of the graphite*

$\mathbf{p_{esc,n}}$ is the probability that a neutron will not enter a fuel kernel of the n-th shell of SSL as well as not interact with the moderator between the fuel kernels.

$$p_{esc,n} = 1 - p_{f,n} - p_{m,n} \quad (4a)$$

Finally, the SIPDFs as the radial positions are determined as the following:

$$D(u(n)) = n(1)\cdot p_{f,1} + n(2)\cdot(p_{esc,1})\cdot p_{f,2} + n(3)\cdot(p_{esc,1}\cdot p_{esc,2})\cdot p_{f,3} + \ldots + n(n)(\prod_{i=1}^{n-1} p_{esc,i})p_{f,n} = \sum_{i=1}^{n} n(i)\cdot p_{f,i}\cdot(\prod_{j=0}^{i-1} p_{esc,j}) \quad (5a)$$

where, u(n) = n'th radial position

n(i) = number of fuel kernel in i'th shell

$p_{esc,0}$= *1*

Also, the IPDF can be calculated as the following:

$$D = \sum_{i=1}^{n} D(u(i))n(i) / \sum_{i=1}^{n} n(i) \quad (5b)$$

3. Results and Discussion

IPDFs as a function of the position using SSM were calculated with Matlab program. The calculations were pursued for the number of 15,000 and 30,000 fuel kernels, respectively. The radius of fuel region is 2.5cm and the shell thickness is equal to the diameter of fuel kernel.

These calculations were also carried out by MCNPX code with the cubic lattice. The calculation results are shown in Figure 5.

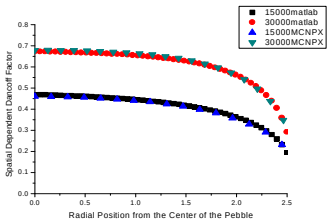

Fig. 5. SIPDFs Calculated by the Developed Method and MCNPX

The average values of IPDF reach 0.359 and 0.548 for 15,000 and 30,000 fuel kernels, respectively. These results agree well within 2% difference comparing with the previous study [1] by using MCNP code.

4. Conclusions

In this study, the sphere shell model is developed to evaluate SIPDF analytically. The deduced SIPDFs using this model were calculated for the some cases and compared with MCNPX calculation and the other study. The result from the model developed in this study gives a good agreement with MCNPX and the other study [1]. The method in this study can be easily utilized to analyze the tendency of SIPDF for the various geometries because it is faster than the MCNP method as well as good accuracy. This result is helpful to analyze the resonance absorption effect when the pebble type reactors are designed. And it can be utilized to the multi-group calculation in resonance region after the calculation of inter-pebble Dancoff factor.

Acknowledgment

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REFERENCES

[1] J.L. Kloosterman, A.M. Ougouag, "Spatial Effects in Dancoff Factor Calculations for Pebble-Bed HTRs," American Nuclear Society Topical Meeting in Mathematics & Computations, Avignon, France, Sept.12-15, 2005.