Development of Advanced SAM Using Constrained Simulated Annealing

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1. Introduction

SAM (Shape Annealing Matrix) is a mathematical synthesis matrix that offsets the difference of the in-core detector signal and the ex-core detector signal of a CPC (Core Protection Calculator). OPR1000 has a digitalized CPC that calculates the DNBR and LPD values continuously to generate reactor trip signals using an ex-core detector. The CPC calculates four channels (A, B, C, D) of reactor power independently, which one consist of three sub-channel (upper, middle, lower) ex-core detector signals to synthesize the axial power distribution.

Currently, the SAM is determined by the "Least Squares Method". Generally, 40~60 items of snapshot file data that contain the axial power distribution data of in-core detector signals at various power level in a FPA (Fast Power Ascension) test are used in the SAM generation.

But current "Least Squares Method" is very sensitive to measurement noise and has a tendency to amplify the noise.

Recently, EOC (End of Cycle) CPC RMS (Root Mean Square) errors exceeded 8% (the limit for penalty) due to the extended life cycle and the changed EOC axial power distribution compared to the BOC when the is SAM determined. If the RMS error limit (8%) is violated, the snapshot data should be recollected by Xenon-Oscillation at 80% of the reactor power and the SAM recalculated. Otherwise, a penalty will occur. This implies a decrease in the capacity factor as well as the safety margin.

To resolve this issue, various studies have investigated these problems in the US. One of these attempts is known as the CI-SAM (Cycle Independent SAM) calculation method for CE-type PWR plants. However, the CI-SAM is also associated with the problem of measurement noise amplification. Thus, the "Constrained Simulated Annealing method" was developed by KHNP. It is a more constructive and accurate new SAM determination method that mitigates the measurement noise amplification problem.

2. SAM using Least Square Method

The ex-core signal mainly depends on its most peripheral fuel assembly power. Therefore, the ex-core signal has a close relationship with its peripheral power rather than its average reactor power. SAM could be set as a 3x3 matrix that offsets the linear vectorial relationship between the core peripheral power and its three sub-channel (upper, middle, lower) ex-core power.

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = [\mathrm{SAM}] \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{pmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{pmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \tag{1}$$

In Eqs. (1) and (2), the peripheral power (p_i) and its ex-core power (d_i) are acquired through the process of a FPA test. The SAM (S_{ij}) matrix is unknown. The determination of SAM is a mathematically over-determined problem as one SAM set should be generated from 40~60 cases. Hence, determination of the SAM is the process of a finding solution that minimizes the degree of deviation error among the various power levels and various instances of axial power distribution data. Usually, the Gaussian least squares method is used to solve this type of problem.

If N instances of axial power distribution data are collected, Eq. (1) can be modified as follows:

$$\begin{pmatrix} d_{i1}^1 & d_{i2}^1 & d_{i3}^1 \\ d_{i1}^2 & d_{i2}^2 & d_{i3}^2 \\ \dots & & \\ d_{i1}^N & d_{i2}^N & d_{i3}^N \end{pmatrix} \begin{bmatrix} s_{i1} \\ s_{i2} \\ s_{i3} \end{bmatrix} = \begin{bmatrix} p_i^1 \\ p_i^2 \\ \dots \\ p_i^N \end{bmatrix} \tag{2}$$

To solve the Eq. (2), the equation is formulated in a more suitable form for the least squares fitting, as follows:

$$\begin{bmatrix} s_{i1} \\ s_{i2} \\ s_{i3} \end{bmatrix} = (D^T D)^{-1} D^T p \tag{3}$$

Here, $D = \begin{pmatrix} d_{i1}^1 & d_{i2}^1 & d_{i3}^1 \\ d_{i1}^2 & d_{i2}^2 & d_{i3}^2 \\ \dots & & \\ d_{i1}^N & d_{i2}^N & d_{i3}^N \end{pmatrix}, \quad p = \begin{bmatrix} p_i^1 \\ p_i^2 \\ \dots \\ p_i^N \end{bmatrix}$

The solution of Eq. (3) is the SAM.

3. Ill-Posedness [1] of Least Square Method

The sensitivity to noise data can be estimated using a condition number (= Δsolution/Δdata) of the squares matrix (D^TD). The higher condition numbers result in a greater degree of ill-posedness. The value of the condition number of (D^TD) used to calculate the SAM is of the level of several thousands. To resolve this ill-posedness in the least square method, KHNP developed the Constrained Simulated Annealing method.

[1] A small error results in a larger error. In this case the matrix is called "ill-posed"

4. Constrained Simulated Annealing

4.1 Constraint

SAM is a unique matrix which has its physical and mathematical meaning between the peripheral core power and the ex-core detector signals. Using the inverse SAM, two constraints [1] are suggested to solve the equation.

$$\begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix}^{-1} = \begin{pmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{pmatrix} \qquad (4)$$

$$\begin{aligned} & w_{11} > w_{12} > w_{13} \\ & w_{21} < w_{22} > w_{23} \quad (5) \\ & w_{31} < w_{32} < w_{33} \end{aligned}$$

To give the SAM a physically convincing value, all of the inverse SAM elements (w_{ij}) of Eq. (4) should be positive because all elements represent the neutron detect probability and thus should have positive value regardless of where they are located. Additionally, the signs of inequality between the w_{ij} values in (5) indicate that the closer detector to each part of the core (upper, middle, lower core vs. upper, middle, lower detector) has a larger value corresponding to its position. Consequently, two constraints for the advanced SAM determination algorithm are established. The constraints of the SAM from Eq. (1) are set as follows:

① S_{11}, S_{22}, S_{33} must have a positive value

② S_{12}, S_{21}, S_{23}, S_{32} must have a negative value

4.2 Simulated Annealing

To calculate the SAM, KHNP used the Simulated Annealing method for statistical and numerical optimization. A constraint can be applied to this method easily. To develop the advanced SAM for solving algorithms, a new optimization algorithm (Simulated Annealing) was tested with constraints. Consequently, the simulated annealing method was confirmed to determine not only the global optimum value; it also has less possibility to fail in the event of difficult functions due to the robustness of the algorithm.

To convert the SAM problem to a Simulated Annealing (CSA) applicable optimization model, Eq. (3) becomes Eq. (6):

$$\min\{\| (D^T D)\, s_{CSA} - D^T p \|_2\}, \quad s.t.\ G s_{CSA} > d \qquad (6)$$

Here, the cost functions are as follows:

$$E(s_i) = [(D^T D) s_i - D^T p]^2 \qquad (7)$$

The global optimization value can then be found by the Metropolis criteria upon acceptance [2], as in Eq. (8) below.

$$P = \exp\left(-\frac{\Delta E}{k_b T}\right) \qquad (8)$$

The simulated annealing method applied as the SAM determination algorithm is that of Goffe's [2], as referenced below. Its algorithm source [3] program is opened to the public.

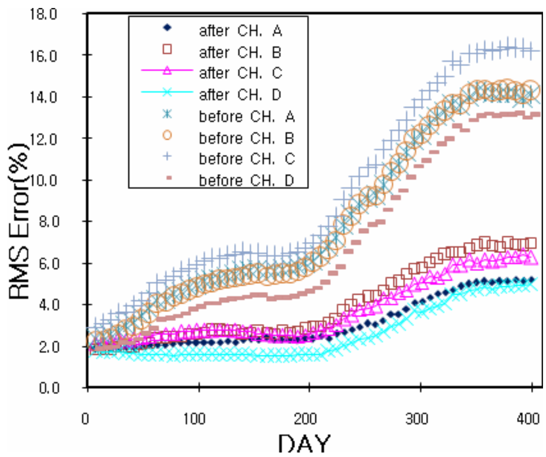
4.3 Constrained Simulated Annealing

The user cost function is converted to a SAM appropriate cost function, as exemplified by Eq. (7). The constrained condition with Goffe method was supplemented to eliminate the data noise interference. The advanced SAM algorithm was embodied as the computerized code CEFAST. The outcome is as follows:

Table 1. SAM Value between the least square and Constrained Simulated Annealing (U3C8)

	Least Square			CSA	
Channel C SAM					
4.6883	-0.7878	-1.0442	4.8062	-0.9539	-0.9222
0.8789	2.0013	0.4546	-0.4981	3.9370	-0.9660
-2.5675	1.7867	3.5894	-1.1511	-0.2031	5.0482
Inverse SAM					
0.2425	0.0366	0.0659	0.2266	0.0576	0.0524
-0.1645	0.5385	-0.1161	0.0418	0.2671	0.0587
0.2554	-0.2419	0.3835	0.0533	0.0239	0.2124
Test Value =	**5.4390**				**4.0110**

Figure 1. RMS Error Trend between the least square and Constrained Simulated Annealing (U3C8)



5. Conclusion and Acknowledgments

A CSA method embodied as CEFAST was highly satisfactory in terms of its results within the technical requirements throughout all core follow calculations. This study was carried out under the KHNP R&D program over a period of four months.

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