Multigroup Transient Calculation Capability in RENUS Nodal Kinetics Code

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1. Introduction

Various research and code development have been carried out to solve the time-dependent neutron diffusion problems by means of the nodal method from 30 years ago[1,2]. But there is still the need for faster and yet more accurate *multi-group* nodal kinetics codes to achieve more readily high-fidelity coupled simulation of neutronic and thermal/hydraulic behaviors in advanced cores. In this regard, a multigroup nodal kinetics code named RENUS is being developed at Seoul National University.

Last year, the flux solver of RENUS employing the source expansion nodal method (SENM) within a two-level coarse mesh finite difference (CMFD) formulation[3] was developed and verified for steady-state applications. It was shown that SENM provided accurate and efficient solutions for multigroup problems. This work here is to add a transient calculation capability to the RENUS code retaining the basic flux solution framework. The newly developed capability is tested for a MOX core transient benchmark problem involving multigroup group cross sections and thermal-hydraulic feedback.

2. Methods

2.1 CMFD Formulation for transient problem

A multigroup transient problem consists of a time-dependent neutron balance equation and a precursor balance equation as follows:

$$\frac{1}{v_g}\frac{\partial \phi_g(\mathbf{r},t)}{\partial t} = \chi_g\left(1-\beta(\mathbf{r},t)\right)\psi(\mathbf{r},t) + \sum_{k=1}^{6}\chi_{dgk}\lambda_k C_k(\mathbf{r},t) + \sum_{g'=1}^{G}\Sigma_{g'g}(\mathbf{r},t)\phi_g(\mathbf{r},t) - \left(\nabla\ J_g(\mathbf{r},t) + \Sigma_{rg}(\mathbf{r},t)\phi_g(\mathbf{r},t)\right) \quad (1)$$

$$\frac{\partial C_k(\mathbf{r},t)}{\partial t} = \beta_k(\mathbf{r},t)\psi(\mathbf{r},t) - \lambda_k C_k(\mathbf{r},t) \quad (k = 1\ 6) \quad (2)$$

where g and k denote the indices of energy group and precursor's group. λ and β are the decay constant and the delayed neutron fraction.

Integration of the balance equations over a node m and division by the node volume are performed to obtain spatially discretized equations. In this process, the CMFD relation can be introduced. Spatially discretized matrix forms of the two equations are expressed simply as follows.

$$\frac{1}{v_g}\frac{d\boldsymbol{\varphi}_\mathbf{g}(t)}{dt} = \mathbf{s}_\mathbf{g}(t) - \mathbf{L}_\mathbf{g}(t)\boldsymbol{\varphi}_\mathbf{g}(t) \quad (3)$$

$$\frac{d\mathbf{C}_k(t)}{dt} = \boldsymbol{\beta}_\mathbf{k}\boldsymbol{\psi}(t) - \boldsymbol{\lambda}_\mathbf{k}\mathbf{C}_k(t)\ . \quad (4)$$

The temporal discretization is also performed using the theta method as follows:

$$\left(\mathbf{L}_g^{n+1} + \frac{1}{\theta v_g \Delta t}\mathbf{I}\right)\boldsymbol{\varphi}_g^{n+1} - \mathbf{s}_g^{n+1} = \frac{1}{\theta v_g \Delta t}\boldsymbol{\varphi}_g^{n} - \frac{\bar{\theta}}{\theta}\left(\mathbf{s}_g^n(t) - \mathbf{L}_g^n(t)\boldsymbol{\varphi}_g^n(t)\right). \quad (5)$$

At the new time point *n+1*, all terms placed on the RHS of Eq. (5) are known. In $\mathbf{s}_g^{n+1}$, the delayed neutron source term included and coupled with the precursor balance equation. The precursor equation is to be solved before solving Eq. (5). With integrating factor, the solution is obtained simply as the following.

$$C_k^{m,n+1} = C_k^{m,n}e^{-\lambda_k(t_{n+1}-t_n)} + \beta_k^n\int_{t_n}^{t_{n+1}}\psi_m(t')e^{-\lambda_k(t-t')}dt'. \quad (6)$$

In this work, the unknown function of fission source $\psi_m(t')$ from t_n to t_{n+1} is assumed as a quadratic form as the following.

$$\psi_m(t) = \psi_m^n + \frac{\psi_m^{n+1}-\psi_m^{n-1}}{2\Delta t}(t-t_n) + \frac{\psi_m^{n+1}-2\psi_m^n+\psi_m^{n-1}}{2\Delta t^2}(t-t_n)^2. \quad (7)$$

With the delayed neutron source term expressed in terms of the fission sources, the linear system of Eq. (5) has a similar form to the steady-state one and thus it can be solved with same solution module.

2.2 Transient Nodal Solution

The transient nodal calculation starts with the time-dependent 1-D neutron balance equation and precursor equation which are obtained by spatial discretization and transverse integration as follows for each node:

$$\frac{1}{v_g}\frac{\partial \phi_g(z,t)}{\partial t} = \chi_g\left(1-\beta\right)\psi(z,t) + \sum_{k=1}^{6}\chi_{dgk}\lambda_k C_k(z,t) + \sum_{g'=1}^{G}\Sigma_{g'g}\phi_z(z,t) - \left(L_{xy}(z,t) + \frac{\partial J_{zg}(z,t)}{\partial z} + \Sigma_{rg}\phi_g(z,t)\right) = R_g(z,t) \quad (8)$$

$$\frac{\partial C_k(z,t)}{\partial t} = \beta_k\psi(z,t) - \lambda_k C_k(z,t) \quad (k = 1\ 6) \quad (9)$$

where

$$L_{xy}(z,t) = \frac{J_{xg}^r(z,t) - J_{xg}^l(z,t)}{h_x} + \frac{J_{yg}^r(z,t) - J_{yg}^l(z,t)}{h_y}.$$

With a temporal discretization of the neutron balance equation and the analytic solution of the precursor equation, the final form of the time-dependent neutron balance equation can be obtained as the same form as the form used in steady-state calculation as the following.

$$-D\frac{d^2\phi_g^{n+1,l+1}(z)}{dz^2}+\Sigma_{rg}\phi_g^{n+1,l+1}(z)= \chi_g\psi^{n+1,l+1}(z)+\sum_{\substack{g'=1\\ g'\neq g}}^{G}\Sigma_{gg'}\phi_{g'}^{n+1,l+1}(z)+s_{ts}(z)-L_{xy}^{n+1,l}(z) \quad (10)$$

where l denotes source iteration of SENM and $s_{tz}(z)$ is transient specific source term which is approximated to *quadratic* polynomials and its coefficients are obtained by using the average values of adjacent nodes. Therefore, the transient nodal kernel can be implemented simply by merely adding the transient specific source term into the steady-state coding.

3. Results

In order to examine the accuracy of the transient calculation method introduced above, two benchmark problems were solved by the RENUS code. The first one is the well known two-group benchmark problem, NEACRP A1[4]. The other one is Part 4 of the NEA MOX transient benchmark problems[5] provided with 2G, 4G and 8G cross sections.

As shown in Table 1, the RENUS transient calculation generated essentially the same solution as the reference solution. The difference in the peak time and power is less than 0.01 sec and 4.18%, respectively. The error of the power integral is 0.80%.

Table 2 shows the results for the NEA MOX transient benchmark with various numbers of energy group. Since no reference solution is given for this problem, no error information is given in the table. Only two-group solutions were available for codes other than PARCS and RENUS. It is noted that the multigroup results give higher peak power than two-group. Nonetheless but the difference in the power integral between the two-group and multigroup results is only about 1~2 %-sec. This means that the multigroup effect is not as serious as originally expected.

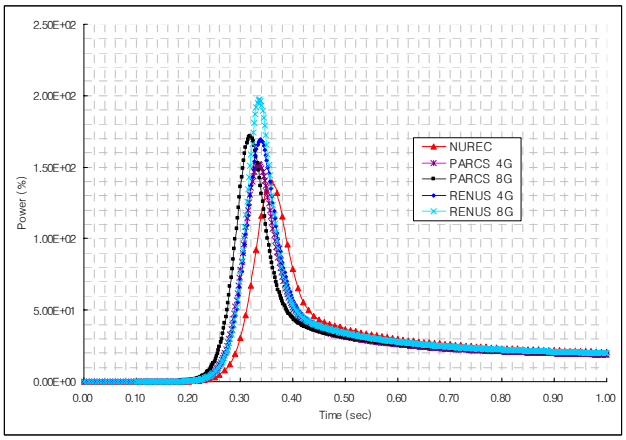
Table 1. NEACRP A1 Results

	Ref.	RENUS	Rel. Error (%)
Boron Conc. (ppm)	561.2	561.3	0.02
Peak Time (sec)	0.538	0.543	-
Peak Power (%)	126.8	121.5	-4.18
Power Integral (%-sec)	111.7	112.6	0.80
Final Fuel Temp. (K)	952.5	956.9	0.47
Final Cool. Temp. (K)	566.4	566.5	0.02

Table 2. NEA MOX Results

CODE	Peak Time (sec)	Peak Power (%)	Power Intg. (%-sec)
CORETRAN	0.33	166	26.4
NUREC	0.36	139	28.4
PARCS 2G	0.34	142	27.2
PARCS 4G	0.33	152	27.8
PARCS 8G	0.32	172	29.1
SKETCH-INS	0.34	144	28.0
RENUS 2G	0.36	146	28.3
RENUS 4G	0.34	169	29.4
RENUS 8G	0.34	197	30.4

Fig 1. Power behavior for NEA MOX Benchmark Prob.



4. Conclusion

A multigroup calculation capability based on the source expansion nodal method and the two-level CMFD formulation was developed and verified. The transient results for the NEACRP A1 and NEA MOX transient benchmark problems indicate that the transient capability of the RENUS code works properly. However, the effect of the multigroup calculation turned out not to be serious than expected.

References

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