An Analytical Study on the Compton Wavelength

Sun-Tae Hwang and Dong-Kwan Shin
TEKCIVIL Corporation
301 Songjeon B/D, 1182 Dunsan-dong, Seogu, Daejeon 302-121, Korea

1. Introduction

Mathematically, the Compton wavelength λ_c of a particle is given by $\lambda_c = h/mc$, where h is the Planck constant, m is the particle's rest mass and c is the speed of light. For the electron as a reference constant, it is about 2.426×10^{-12} m, intermediate between the size of an atomic nucleus and an atom. The λ_c means a critical distance below which certain quantum mechanical effects take place for any particle. The energy of a photon of this wavelength is equal to the rest mass energy $m_e c^2$ of an electron[1]. Now, the λ_c is analytically reviewed.

2. Compton Scattering Effect

Compton scattering effect is the result of a high energy photon colliding with an electron. First, consider the interaction between a photon with momentum vector $\mathbf{p}_\gamma$ and an electron initiallly at rest with mass m_e. Next call $\mathbf{p}'_\gamma$ and $\mathbf{p}'_e$ the momenta of the photon and the electron after the interaction, respectively. By conservation of momentum, the relation between the initial and final momenta is

$$\mathbf{p}_\gamma = \mathbf{p}'_\gamma + \mathbf{p}'_e \tag{1}$$

Reqaranging and squaring both sides, Eq.(2) can be obtained.

$$p_\gamma^2 + p'^2_\gamma - 2\mathbf{p}_\gamma \cdot \mathbf{p}'_\gamma = \mathbf{p}'^2_e \tag{2}$$

The total relativistic energy E and momentum $\mathbf{p}$ of a particle are related to its rest mass m by the invariant relation, $\mathbf{p} \cdot \mathbf{p} c^2 - E^2 = -m^2 c^4$.

Now, by conservation of energy,

$$p_\gamma + m_e c = p'_\gamma + (m_e^2 c^2 + p'^2_e)^{1/2} \tag{3}$$

Rearranging and squaring both sides, Eq.(4) is obtained.

$$p_\gamma^2 + p'^2_\gamma - 2p_\gamma p'_\gamma + 2m_e c(p_\gamma - p'_\gamma) + m_e^2 c^2 = m_e^2 c^2 + p'^2_e \tag{4}$$

Subtraction of Eq.(2) from Eq.(4) yields

$$m_e c(p_\gamma - p'_\gamma) = p_\gamma p'_\gamma - \mathbf{p}_\gamma \cdot \mathbf{p}'_\gamma \tag{5}$$

Finally, dividing both sides by $m_e c p_\gamma p'_\gamma$, Eq.(6) for the relation between the energies of the incident and the scattered photons is obtained.

$$1/E'_\gamma - 1/E_\gamma = (1/m_e c^2) \cdot (1 - \cos\theta) \tag{6}$$

where θ is the angle between their final momentum vectors. In the meantime, considering the wavelike character of photons, their energy $E = h\upsilon = hc/\lambda$ and momentum p are related by the relativistic equation for electron of rest mass of zero, namely $p = h\upsilon/c = E/c$. Accordingly, from Eq.(6) the formula for "wavelength shift" in the wavelength of incoherently scattered photons is obtained as follows:

$$\Delta\lambda = (h/m_e c) \cdot (1 - \cos\theta) \tag{7}$$

where θ is the angle between the trajectories of the incident and scattered photon. When the electron moves in free space with a momentum *p*, its wavelength is given by $\lambda_e = h/p$ which is known as the de Broglie wavelength[2]. Meantime, even though the value $h/m_e c$ is called the Compton wavelength of the electron, this is not an actual wavelength but really a proportionality constant for the wavelength shift $\Delta\lambda$. Looking at Eq.(7), it become clear that the entire shift can be measured purely in terms of the angle at which the photon gets scattered. Everything else on the right side of Eq.(7) is a constant.

3. Compton Wavelength λ_c

Normally, the Compton wavelength λ_c is associated with an incident photon colliding with an electron at rest. The incident photon imparts some momentum to an electron and scatters with less energy. By just using the kinematics in Compton scattering event, the change in the wavelength of photon can be expressed in the form Eq.(7). Generally, the Compton wavelength of a particle is equal to h/mc. In particular, the λ_c of the electron is the characteristic length scale of QED(quantum electrodynamics).

The CODATA 2002 value for λ_c of the electron is $2.4263102175\times10^{-12}$m with a standard uncertainty of $0.0000000033\times10^{-12}$m[3]. Other particles have different values. CODATA(Committee on Data for Science and Technoloy) was established in 1966 as an interdisciplinary committee of the International Council of Science (ICSU), formerly the International Council of Scientific Unions. It seeks to improve the compilation, critical evaluation, storage, and retrieval of data of importance to science and technology. The CODATA recommended values of fundamental physical constants are published at the NIST Reference on Constants, Units, and Uncertainty[4].

4. Conclusions

The Compton wavelength λ_c can be thought of as a fundamental limitation on measuring the position of a particle, taking quantum mechanics and special relativity into accouint. In quantum mechanics, this is a convenient unit of length that is characteristic of an elementary particle and equals to Planck's constant *h* divided by the product of the particle's mass *m* and the speed of light *c*. Hence, λ_c depends on *m*.

The quantum field theory of light postulates that photons behave like particles except for the absence of any rest mass. Using just kinematics, the change in the wavelength of X-ray photon $\Delta\lambda = \lambda' - \lambda$ can be expressed in the form Eq.(7). The Compton wavelength λ_c is actually the de Broglie wavelength for the electron moving at the speed of light.

The Compton scattering shows how such a collision in Section 3 might be represented, with an X-ray photon striking an electron at rest and being scattered away from its original path while the electron receives an impulse and is recoiled.

References

[1] http://www.britannica.com/eb/topic-130389/Compton-wavelength

[2] Clayton, Roderick K., Light and Living Matter Volume 1: The Physical Part, McGraw Hill Book Company, New York, NY, p.14n, 1970 (http://hyperphysics.phy-astr.gsu.edu/hbase/debrog.html)

[3] CODATA 2002 value for Compton wavelength for the electron from NIST.

[4] CODATA Recommended Values of the Fundamental Physical Constants: 2006 (http://physics.nist.gov/cuu/Constants/codata.pdf), NIST.