Transactions of the Korean Nuclear Society autumn Meeting
PyeongChang, Korea, October 30~31, 2008

The validation region of the Boussinesq approximation for nuclear reactor operation and atmospheric pressure conditions

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1. Introduction

In natural convection problem, buoyancy term is important than other terms. An exact solution fully considering the buoyancy term has not existed until now. Some approximation is required to asses the engineering problem. The Boussinesq approximation is the simplest among the other methods. The approximation is derived based on the following:

1. Density is assumed constant except the momentum equation.
2. All other fluid properties are assumed constant.
3. Viscous dissipation is negligible.

These assumptions constricted application region of the Boussinesq approximation. From condition 1 and 2, the approximation is constricted in the incompressible liquid region. Condition 3 constricted that the acoustic phenomena cannot be treated.

The constriction, however, is so abstract as not to asses the validation of the Boussinesq approximation with the problems, directly. The present study allows an explicit region of the validity of the Boussinesq approximation. This means that a solution about natural convection can be meaningful only when the conditions under which the approximation is valid are explicitly known.

Investigation of the explicit region was made by Spiegel and Veronis[1]. They used an order of magnitude method for a perfect gas. Mihaljan[2] derived the Boussinesq equations on the thin layer flow using the small parameter expansion technique. Gray[3] expanded the Mihaljan's method to the liquid and considered all fluid properties to vary with temperature and pressure. Present study expanded the Gray's result to the condition of the nuclear reactor operation region and severe accident condition.

The results will show the validity and limitation region of the Boussinesq approximation applicable to the severe accident analysis.

2. Derivation and Results

2.1 Formulation and Problem description

The derivation of the approximation begins with the general governing equations for a Newtonian fluid of variable properties and zero second viscosity. The tensor forms of these equations are followings[4]:

$$\frac{D\rho}{Dt}+\rho\frac{\partial V_j}{\partial x_j}=0 \tag{1}$$

$$\rho\frac{DV_i}{Dt}=-\frac{\partial P}{\partial x_i}-\rho g k_i+\mu\frac{\partial \Gamma_{ij}}{\partial x_j}+\Gamma_{ij}\frac{\partial \mu}{\partial x_j} \tag{2}$$

$$\rho c_p\frac{DT}{Dt}=K\frac{\partial^2 T}{\partial x_j^2}+\frac{\partial K}{\partial x_j}\frac{\partial T}{\partial x_j}+\alpha T\frac{DP}{Dt}+\mu\Phi \tag{3}$$

, where

$$\Gamma_{ij}=\frac{\partial V_i}{\partial x_j}+\frac{\partial V_j}{\partial x_i}-\frac{2}{3}\frac{\partial V_k}{\partial x_k}\delta_{ij},$$

$$\Phi=\frac{1}{2}\Gamma_{ij}\left(\frac{\partial V_i}{\partial x_j}+\frac{\partial V_j}{\partial x_i}\right).$$

In order to complete these equations, there are required to determine the relationship of the fluid properties. These necessary property equations are approximated by the Taylor linear expansion following as:

$$\rho=\rho_o\left[1-\alpha_o\left(T-T_o\right)+\beta_o\left(P-P_o\right)\right]$$

$$c_p=c_{po}\left[1+a_o\left(T-T_o\right)+b_o\left(P-P_o\right)\right]$$

$$\mu=\mu_o\left[1+c_o\left(T-T_o\right)+d_o\left(P-P_o\right)\right]$$

$$\alpha=\alpha_o\left[1+e_o\left(T-T_o\right)+f_o\left(P-P_o\right)\right]$$

$$K=K_o\left[1+g_o\left(T-T_o\right)+h_o\left(P-P_o\right)\right]$$

, where ρ is density; c_{po}, heat capacity; μ, dynamic viscosity; α, thermal expansion coefficient; K thermal conductivity.

To simplify the problem, one will consider the Benard-convection problem as shown in Fig. 1. Two conditions are investigated on the problem, atmospheric condition and reactor operation condition. Atmospheric condition is related to the severe accident problem.

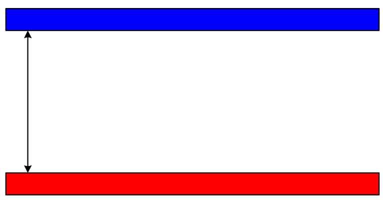

Fig. 1. Control volume of the Rayleigh-Benard problem.

2.2 Boussinesq approximation equations

Nondimensionalized equations are derived using the nondimensionalized parameter as table 1. Velocity is nondimesionalized using the "free-fall" velocity of a thermal.

Table I: Non-dimension parameters

	Dimensional parameter	Non-dimensionalized parameter
Length	x_i	$\frac{x_i}{L}$
Temperature	$T-T_o$	$\frac{T-T_o}{T_s-T_o}$
Velocity	V_i	$\frac{V_i}{\sqrt{\alpha_o g\theta L}}$
time	t	$t\frac{\sqrt{\alpha_o g\theta L}}{L}$
Dynamic pressure	$P-P_s$	$\frac{P-P_s}{\rho_o \square (\alpha_o g\theta L)}$
Pressure difference	$P-P_o$	$\frac{P-P_o}{\rho_o gL}$
Stress tensor	Γ_{ij}	$\Gamma_{ij}\sqrt{\frac{L}{\alpha_o g\theta}}$
Dissipation term	Φ	$\Phi\frac{L}{\alpha_o g\theta}$

Boussinesq approximation equations are derived considering some approximation related to the order of magnitude. Coefficients of equations have the order of magnitude less than 0.1. and other terms are assumed as the order 1. Extended Boussinesq equations are following as[3],

$$\frac{\partial \tilde{V}_j}{\partial \tilde{x}_j}=0, \quad (4)$$

$$\frac{D\tilde{V}_j}{D\tilde{t}}=-\frac{\partial\left(\tilde{P}-\tilde{P}_s\right)}{\partial \tilde{x}_i}+\left(\tilde{T}-\tilde{T}_s\right)k_i+\left(\frac{\Pr}{Ra}\right)^{0.5}\left\{\frac{\partial \tilde{\Gamma}_{ij}}{\partial \tilde{x}_j}\right\}, \quad (5)$$

$$\frac{D\tilde{T}}{D\tilde{t}}=\frac{1}{(\Pr Ra)^{1/2}}\frac{\partial^2 \tilde{T}}{\partial \tilde{x}_j^2}-\frac{\alpha_o gL}{c_{po}}\left(\frac{T_o}{\theta}\right)\tilde{V}_3+\frac{\alpha_o gL}{c_{Po}}\left(\frac{\Pr}{Ra}\right)^{1/2}\tilde{\Phi}. \quad (6)$$

2.3 Valid rage of the Boussinesq approximation

Inequalities to determine the valid region is derived in the used approximation on deriving the extended Boussinesq equations.

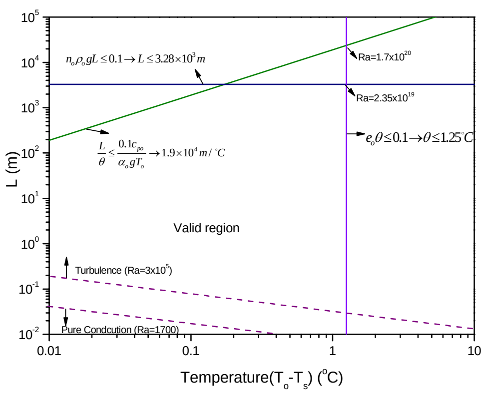

Fig. 2. Valid region map of the Boussinesq approximation of the 1 atm condition.

Two inequalities are derived as neglecting the pressure work and dissipation term. Other two conditions are derived in comparisons with the order of magnitude of coefficients.

Fig. 2 and Fig. 3 are result of the inequalities. In 1 atm condition, Boussinesq equations are valid in the region to a maximum Rayleigh number of 10^{19}. The pressure condition is similar with the severe accident analysis. Rayleigh number of the natural convection for the in-vessel retention(IVR) is order of 10^{16} to 10^{17}. Fig. 2 shows that the Boussinesq approximation is valid to analyze the natural convection for IVR. In case of increasing the pressure upto the 15MPa, the maximum Rayleigh number is decreased to the order of 10^{18} in Fig. 3.

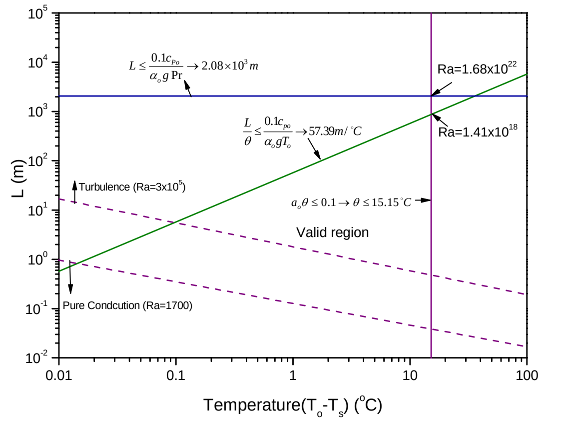

Fig.3. Valid region map of the Boussinesq approximation of the reactor operation condition.

3. Conclusions

Boussinesq approximation equations are derived using the small parameter expansion. Inequalities derived by the approximation determined the valid region of the approximation. The valid regions explicitly are investigated on the two conditions, atmospheric pressure condition and normal operation condition. These results and methods will be expected to introduce the explicit limitation when it is applicable to the Boussinesq approximation to the natural convection problem.

REFERENCES

[1] E. A. Spiegel and G. Veronis, On the Boussinesq approximation for a compressible fluid, Astrophys. Jl., Vol. 131, pp. 442-447, 1960.

[2] J. M. Mihaljan, A rigorous exposition of the Boussinesq approximations applicable to a thin layer of fluid, Astropys. Jl., Vol. 136, pp. 1126-1133, 1962.

[3] D. D. Gray and A. Giorgini, The validity of the Boussinesq approximation for liquids and gases, Int. J. Heat Mass Transfer, Vol. 19, pp. 545-551, 1976.

[4] G. K. Batchelor, An Introduction to Fluid Dynamics, Cambridge University Press, London, 1967.