Analysis of a bubble coalescence in the multiphase lattice Boltzmann method

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1. Introduction

Recently, the lattice Boltzmann method (LBM) has gained much attention for its ability to simulate fluid flows, and for its potential advantages over a conventional CFD method. The key advantages of LBM are, (1) suitability for parallel computations, (2) absence of the need to solve the time-consuming Poisson equation for a pressure, and (3) an ease with multiphase flows, complex geometries and interfacial dynamics may be treated[1].

To study the effect of the mobility coefficient Γ and the width of the interface layer, two stationary bubbles without a collision are considered. The gap of the two bubbles is taken as 4, while the width of the interface (w) and the mobility coefficient Γ are varied. In the present work, the lattice Boltzmann model for multiphase flows proposed by Zheng et al. [4] is used for simulating two stationary bubbles without a collision. By adopting a finite difference gradient operator of a sufficient isotropy, the spurious currents can be made smaller. The main objective of the present work is to establish the lattice Boltzmann method as a viable tool for the simulation of multiphase or multi-component flows.

2. Methods and Results

2.1 Methodology

Here, we consider a flow with two phases which have different densities. The low density and high density are noted as ρ_L and ρ_H respectively. The flow can be described by the Navier-Stokes equations and an interface evolution equation as [4]

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \vec{u}) = \theta_M \nabla^2 \mu_\phi \qquad (1)$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \vec{u}) = 0 \qquad (2)$$

$$\frac{\partial n\vec{u}}{\partial t} + \nabla \cdot (n\vec{u}\vec{u}) = -\nabla \cdot P + \mu \nabla^2 \vec{u} + \vec{F}_b \qquad (3)$$

where θ_M is called mobility, μ_ϕ is the chemical potential, P is the pressure tensor, $\vec{F}_b$ is the body force, and n, ϕ are defined as

$$n = \frac{\rho_A + \rho_B}{2}, \phi = \frac{\rho_A - \rho_B}{2}$$

where ρ_A and ρ_B are the density of fluid A and fluid B respectively.

Under the lattice Boltzmann framework, Eq. (1) can be solved by iterating the evolution equation for a set of distribution functions. These distribution functions evolve with a modified lattice Boltzmann equation and BGK approximation,

$$g_i(x + e_i\delta t, t + \delta t) = g_i(x,t) + \Omega_i + (1-q)\delta g_i$$

with

$$\Omega_i = \frac{g_i^0(x,t) - g_i(x,t)}{\tau_\phi}$$

$$\delta g_i = g_i(x + e_i\delta t, t) - g_i(x,t)$$

where g_i is the distribution function, Ω_i is the collision term, τ_ϕ is the dimensionless single relaxation time, e_i is the lattice velocity, and q is a constant coefficient.

In Eq. (3), the term $\nabla \cdot P$ is related to the surface tension force. This force can be rewritten as a potential term,

$$\vec{F}_s = -\nabla \cdot P = -\phi \nabla \mu_\phi - \nabla p_0$$

where $p_0 = nc_s^2$, c_s is the speed of sound.

The potential form for the surface tension force is adopted to keep the energy conservation. Mathematically, the potential form and stress form are identical. However, numerically, the discretization error is different[2,3,4]. Thus, it is useful to eliminate spurious currents.

The lattice Boltzmann implementation of Eqs. (2) and (3) can be described as

$$f_i(x + e_i\delta t, t + \delta t) = f_i(x,t) + \Omega_i$$

with

$$\Omega_i = \frac{f_i^0(x,t) - f_i(x,t)}{\tau_n} +$$

$$(1 - \frac{1}{2\tau_n})\frac{w_i}{c_s^2}[(\vec{c}_i - \vec{u}) + \frac{(\vec{c}_i \cdot \vec{u})}{c_s^2}\vec{c}_i](-\phi \nabla \mu_\phi + \vec{F}_b)\delta t$$

The equilibrium distributions satisfy the conservation laws as

$$\phi = \sum_i g_i, n = \sum_i f_i$$

$$\vec{u} = [\sum_i f_i \vec{c}_i + \frac{1}{2}(-\phi \nabla \mu_\phi + \vec{F}_b)]/n$$

The details are Ref. [4].

The chemical potential is chosen as

$$\mu_\phi = A(4\phi^3 - 4\phi^{*2}\phi) - \kappa \nabla^2 \phi$$

Following the same procedure as [4], we can obtain the

profile along the normal direction of the interface

$$\phi = \phi^* \tanh(2\zeta / w)$$

Where ζ is the coordinate which is perpendicular to the interface, and w is the thickness of the interface layer,

$$w = \frac{\sqrt{2\kappa / A}}{\phi^*}$$

For a flat interface, the surface tension coefficient can be evaluated by Rowlinson and Widom[5].

$$\sigma = \int \kappa \left(\frac{\partial \phi}{\partial \zeta} \right)^2 d\zeta$$

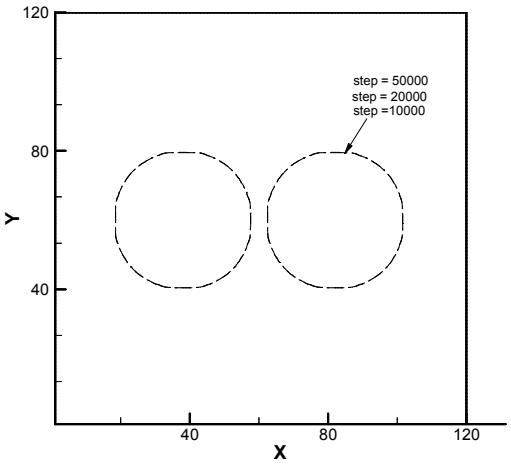
We can obtain the surface tension coefficient as

$$\sigma = \frac{4\sqrt{2\kappa A}}{3} \phi^{*3}$$

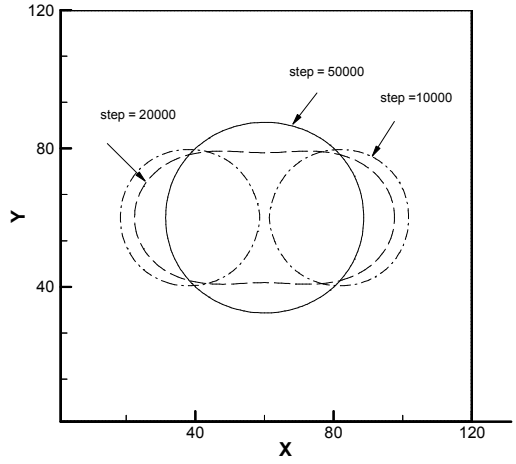
Where $\phi^* = \frac{\rho_H - \rho_L}{2}$.

2.2 Results

The computational domain is taken as 120x120. Initially, two circular bubbles with Radius *R* (=20) are located with a gap of *d* (=4). Fig. 1 shows numerical results for the two cases. At first, we set the width of the interface layer as *w*=2.0, and Γ=40. Next, the width of the interface layer was set as *w*=3.0, and Γ=400. It can be easily observed that the two bubbles do not merge at any time in the case of *w*=2.0. If the gap of the two bubbles is smaller than 2*w*, the two bubbles eventually merge without a collision [Fig. 1 (b)] .



(a) Results of two stationary bubbles without collision (*w*=2, *d*=4, Γ=40, density ratio=1000, $\sigma = 1$).



(b) Results of two stationary bubbles without collision (*w*=3, *d*=4, Γ=400, density ratio=1000, $\sigma = 1$).

Fig. 1. Interface contour of two stationary bubbles.

3. Conclusion

For the two stationary bubbles without a collision, it was found that the distance (gap) between the bubbles and the interface width are major factors to decide whether the two bubbles will merge together or not. When the gap of the two bubbles is larger than 2*w*, the two bubbles will not merge. Otherwise, they will merge together. In this last case, a larger mobility coefficient Γ makes the two bubbles merge together more quickly. A lattice Boltzmann method for multiphase flows with a high density ratio is developed in this paper. It can be easily implemented for the three dimensional case with a topology change.

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