Visualization of Two Phase Flow in a Horizontal Flow with Electrical Resistance Tomography based on Extended Kalman Filter

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1. Introduction

For the visualization of the phase distribution in two phase flows, the electrical resistance tomography (ERT) technique is introduced. In ERT, the internal resistivity distribution is reconstructed based on the known sets of the injected currents and measured voltages on the surface of the object. The physical relationship between the internal resistivity and the surface voltages is governed by a partial differential equation with appropriate boundary conditions.

This paper considers the estimation of the phase distribution with ERT in two phase flow in a horizontal flow using extended Kalman filter.

To evaluate the reconstruction performance of the proposed algorithm, the experiments simulated two phase flows in a horizontal flow were carried out.

The experiments with two phase flow phantom were done to suggest a practical implication of this research in detecting gas bubble in a feed water pipe of heat transfer systems.

2. Mathematical Method

2.1 Finite element formulation of the forward problem

The numerical model used in this paper is based on EIDORS[1]. The finite element model to solve the electrical resistance tomography problem can be found in Vauhkonen[2]. However, the finite element approximation should be described here briefly to formulate the inverse solution.

When electrical current I_ℓ is injected into the object $\Omega \in \mathbb{R}^2$ through the ℓ th electrode e_ℓ attached on the boundary $\partial\Omega$ and the conductivity distribution $\sigma(x,y)$ is known over Ω, the corresponding electrical potential $u(x,y)$ on the Ω can be determined uniquely from the so-called complete electrode model in the form

$$\nabla\cdot(\sigma\nabla u) = 0\,,\quad (x,y)\in\Omega \tag{1}$$

$$u + z_\ell \sigma\frac{\partial u}{\partial v} = U_\ell\,,\ (x,y)\in e_\ell\,,\ \ell = 1,2,...,L \tag{2}$$

$$\int_{e_\ell}\sigma\frac{\partial u}{\partial v}dS = I_\ell\,,\ (x,y)\in e_\ell\,,\ \ell = 1,2,...,L \tag{3}$$

$$\sigma\frac{\partial u}{\partial v} = 0\,,\ (x,y)\in\partial\Omega\setminus\bigcup_{\ell=1}^{L} e_\ell \tag{4}$$

where L is the number of electrodes, v stands for the outward normal unit vector on the surface $\partial\Omega$, z_ℓ is the contact impedance and U_ℓ is the measured boundary potential. In addition, the following two constraints for the injected currents and the measured voltages should be imposed to ensure the existence and uniqueness of the solution.

$$\sum_{\ell=1}^{L} I_\ell = 0 \text{ and } \sum_{\ell=1}^{L} U_\ell = 0 \tag{5}$$

In the context of finite element method(FEM), the object area is discretized into sufficiently small triangular elements having a node at each corner and it is assumed that the resistivity distribution is constant within each element mesh. The potential distribution u within the object is approximated as

$$u \approx u^h(x,y) = \sum_{i=1}^{N}\alpha_i\phi_i(x,y) \tag{6}$$

and the potential on the electrodes is represented as

$$U^h = \sum_{j=1}^{L-1}\beta_j n_j \tag{7}$$

where the function ϕ_i is two-dimensional first order basis function and the bases for the measurements are $n_1 = (1,-1,0,...,0)^T$, $n_2 = (1,0,-1,0,...,0)^T \in \mathbb{R}^{L\times 1}$, etc. in this, α_i and β_i are the coefficients to be determined.

2.2 Extended Kalman Filter

In order to enhance the temporal resolution of ERT, Kalman filter approaches have been widely accepted[1][3][4]. We consider the underlying inverse problem as a state estimation problem to estimate rapidly resistivity distribution. In the state estimation problem, we need the dynamic model that consists of the state equation, i.e., an equation for the temporal evolution of the resistivity and the observation equation, i.e., and equation for the relationship between the resistivity and boundary voltages. In general, the temporal evolution of the resistivity distribution ρ_k in the object Ω is related by the nonlinear mapping. Here, the state equation is assumed to be of the linear form, of which the modeling uncertainty is compensated by the process noise

$$\rho_{k+1} = F_k \rho_k + w_k \tag{8}$$

where $F_k \in R^{N \times N}$ is the state transition matrix at time k and N is the number of finite elements in the finite element mesh. In particular, we take $F_k = I_N$ where $I_N \in R^{N \times N}$ is an identity matrix, to obtain the so-called random-walk model. It is assumed that the process error w_k is white Gaussian noise with the covariance that determines the rate of changes in resistivity distribution:

$$\Gamma_k^w = E[w_k w_k^T]. \tag{9}$$

We obtain the recursive EKF algorithm that consists of the following two steps[5].

1) Measurement update

$$G_k = C_{k|k-1} H_k^T [H_k C_{k|k-1} H_k^T + \Gamma_k]^{-1} \tag{10}$$

$$C_{k|k} = (I - G_k H_k) C_{k|k-1} \tag{11}$$

$$\rho_{k|k} = \rho_{k|k-1} + G_k [\overline{y_k} - H_k \rho_{k|k-1}]. \tag{12}$$

2) Time update

$$C_{k|k-1} = F_k C_{k|k-1} F_k^T + \Gamma_k^w \tag{13}$$

$$\rho_{k+1|k} = F_k \rho_{k|k}. \tag{14}$$

3. Results and Reconstructed Images

To evaluate the reconstruction performance of the proposed algorithm, the experiments simulated two phase flows in a horizontal flow were carried out.

The experimental setup consists of a circular phantom with a radius of 150 mm and a height of 100 mm was considered around which 16 electrodes have each of length 26 mm were mounted. As for the current injection protocol, opposite current patterns are used.
In the experiment, 8 image frames are considered and each image frame comprises of 8 current patterns. It is assumed that different sized circular bubbles appear in the upper region and move fast.

Figure 1 can be seen that EKF is performing well in terms of reconstructed resistivity distribution. The experiment proved the positions and size of the objects simulated the gas bubbles.

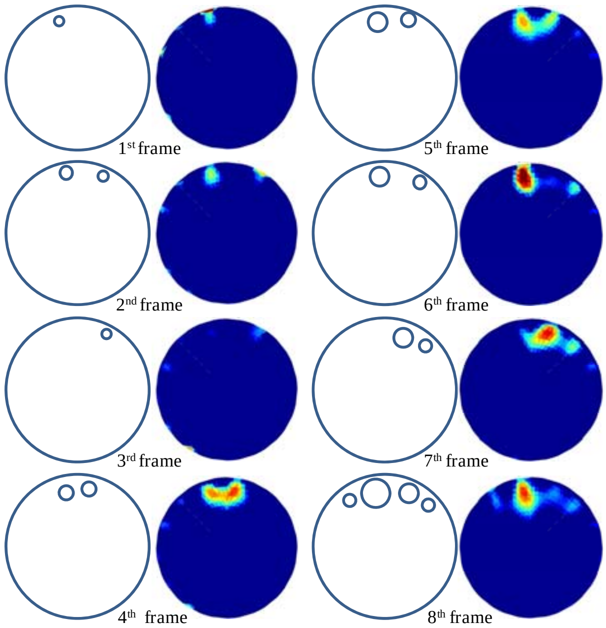

Fig. 1. Reconstructed images for the laboratory experiment.

4. Conclusions

In this paper, extended Kalman filter (EKF) is proposed as an image reconstruction algorithm in electrical impedance tomography to estimate the fast transient changes in resistivity distribution in horizontal two-phase flows. The experimental results proved the positions and size of the objects simulated the gas bubbles.

The experiments with two phase flow phantom were done to suggest a practical implication of this research in detecting gas bubble in a feed water pipe of heat transfer systems.

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