Improvement of Two-Dimensional Source Expansion Based Multigroup Pin Power Reconstruction Method

Joo Il Yoon and Han Gyu Joo
Department of Nuclear Engineering, Seoul National University, San 56-1, Sillim-dong, Seoul, 151-744

1. Introduction

As an effort to improve the accuracy of pin power reconstruction, two-dimensional analytic solution method was introduced by Boer[1] and a consistent formulation to preserve the nodal balance of the transverse-integrated nodal solution was proposed by Joo *et al.* [2] However, these methods are limited to two-group (2G) problems. With the increasing need for the multigroup (MG) core calculation, MG pin power calculation methods have renewed interest. Bahadir and Lindhal[3] developed a MG pin power calculation method employing two-dimensional (2D) submesh solutions consisting of exponential and polynomial functions. 5x5 submeshes per assembly are normalized in their method and the submeshes are used to generate rehomogenization parameters in the 3D nodal calculation. Thus it is expected that much longer computing time would be used with their method. On the contrary, we introduced a simpler method employing two-dimensional (2D) semi-analytic nodal formulation method[4] for the transverse-integrated multigroup nodal method. The accuracy of this method was, however, not verified extensively other than with the L336C5 2-D *two-group* pin power benchmark problem. It is also subject to further development for incorporating discontinuity factors and 3-D applications. In this work, we improve the original method for realistic multigroup applications and then evaluate the performance for the 2-D and 3-D problems of the C5G7 benchmark[5].

2. Methods

This method is based on a 2D polynomial expansion of the source distribution function which consists of the fission, scattering and axial leakage sources. The plane-wise pin-power reconstruction methods are described below and the incorporation of the assembly discontinuity factors in the pin power calculation is then presented.

2.1 Plane-wise Pin Power Reconstruction in 3D

On each plan in a 3D problem, the axially integrated neutron diffusion equation can be written as follows in terms of normalized variables:

$$-D_g \frac{4}{h^2}\nabla^2\phi_g(\xi,\eta)+\Sigma_{r,g}\phi_g(\xi,\eta)= \frac{\chi_g}{k_{eff}}\sum_{g'=1}^{G}\nu\Sigma_{f,g}\phi_g(\xi,\eta)+\sum_{\substack{g'=1\\ g'\neq g}}^{G}\Sigma_{g'g}\phi_{g'}(\xi,\eta)-L_g(\xi,\eta) \quad (1)$$

where $L_g(\xi,\eta)$ represents the axial leakage distribution. By source expansion, we represent the fission and scattering source term as a 13 term Legendre polynomial as:

$$S(\xi,\eta)=\sum_{i=0}^{i=2}\sum_{j=0}^{j=2}s_{i,j}P_i(\xi)P_j(\eta) +\sum_{i=3}^{4}s_{i,0}P_i(\xi)+\sum_{j=3}^{4}s_{0,j}P_j(\eta) \quad (2)$$

and the axial leakage with a 9 term polynomial as:

$$L_g(\xi,\eta)=\sum_{i=0}^{2}\sum_{j=0}^{2}l_{g,i,j}P_i(\xi)P_j(\eta) \quad (3)$$

The solution of Eq. (1) can be obtained in terms of the homogeneous solution given by:

$$\begin{aligned}\phi_h(\xi,\eta)&=a_1\sinh(B_t\xi)+a_2\cosh(B_t\xi)\\&+a_3\sinh(B_t\eta)+a_4\cosh(B_t\eta)\\&+a_5\sinh(\frac{B_t\xi}{\sqrt{2}})\cosh(\frac{B_t\eta}{\sqrt{2}})+a_6\sinh(\frac{B_t\xi}{\sqrt{2}})\sinh(\frac{B_t\eta}{\sqrt{2}})\\&+a_7\cosh(\frac{B_t\xi}{\sqrt{2}})\sinh(\frac{B_t\eta}{\sqrt{2}})+a_8\cosh(\frac{B_t\xi}{\sqrt{2}})\cosh(\frac{B_t\eta}{\sqrt{2}})\end{aligned} \quad (4)$$

and the particular solution determined by the source terms. The 4 surface currents and the corner point fluxes are used to determine the 8 coefficients of the homogeneous solution.

2.2 Incorporation of Assembly Discontinuity Factor

Due to the use of the surface currents, which is determined from 1D nodal calculation as the constraint in the 2D flux calculation, the nodal balance is automatically satisfied with the resulting 2D flux distribution regardless of using the assembly discontinuity factors in the 2D calculation. However, the ADF should be used to improve the flux distribution near the assembly periphery. This is done by requiring the continuity of heterogeneous flux at the corner point as:

$$\phi^h_{c,g}+\phi^p_{c,g}=\frac{1}{\zeta_g}\phi_{c,g} \quad (5)$$

where $\phi_{c,g}$, $\phi^p_{c,g}$, $\phi^h_{c,g}$, and ζ_g are predetermined heterogeneous corner flux, particular and homogeneous solution corner flux, and the discontinuity factor, respectively. The corner point fluxes are determined iteratively using the outgoing partial currents at the corner point as described in the previous work.[4]

3. Results

In order to examine the accuracy of the multigroup pin power reconstruction method presented in this work, the 2-D and 3-D models of C5G7 benchmark problem[5] were solved. In order to perform the nodal calculation, we need to obtain first assembly-wise homogenized

cross sections. The homogenization was done using the DeCART by single assembly calculations for the fuel assemblies and also by two-assembly (fuel and reflector) calculations to determine the reflector DFs.

The core calculation was done by the RENUS nodal code which is based on the source expansion nodal method[6]. The results reported below were obtained with 4 nodes/FA calculation. The reference solutions for this subpin level heterogeneous problem are the ones obtained by the DeCART whole core transport calculation

The results for the 2-D model shown in Table I indicate that excellent agreement between the nodal and whole core solution is possible with a proper use of ADFs. Especially, the large error of pin power at boundaries of fuel assemblies observed with no ADF is dramatically reduced with the fuel ADF as shown in Fig 1. The ADF for the reflector also slightly improves the results. It also is notable that errors at all corner points are negligible even though assembly discontinuity factor is employed where treating corner point's discontinuity effect as expressed in Eq. (5). This is due to that average of four corner point flux estimates is used and thus the discontinuities are flattened.

Table I. Results of 2-D C5G7 Problem

ERROR		ADF			REFERENCE*
		NONE[1]	FUEL[2]	BOTH[3]	
k-eff error (pcm)		-5.6	-18.4	22.7	1.18662
Assembly Power Error (%)	Center U	0.1	1.1	0.7	1.863
	Peri. U	-1.6	-0.7	-0.5	0.530
	MOX	0.8	-0.1	0.0	0.803
Pin Power Error (%)	RMS	1.6	0.9	0.7	-
	MAX	10.8	3.6	3.8	-

[1]NONE: without any of ADF
[2]FUEL : with ADF of fuel
[3]BOTH: with ADF of fuel and reflector

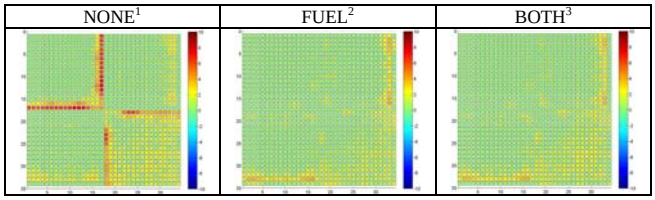

Fig 1. Pin Power Error with Different ADF options

The results for the 3D problem are shown in Tables II and III. The results are also considered excellent in that the maximum errors of planar pin power distribution and are about 2~5%, respectively. Plane 1 has the largest pin power error due to its adjacent reflector region. Note that Plane 9 is the mid plane. On the other hand, the axially averaged 2D pin power distribution has about 3.7% of the largest error as shown in Table 3 which is similar to the 2D result.

Table II. k-eff Error and Axially Averaged 2-D Pin Power Error for 3-D C5G7 Problem

ERROR		ADF BOTH	REFERENCE
k-eff error (pcm)		-70.6	1.18387
Assembly Power Error (%)	Center U	1.1	1.863
	Peri. U	-0.8	0.530
	MOX	-0.1	0.803
Integrated Pin Power Error (%)	RMS	0.8	-
	MAX	3.7	-

Table III. Planar Pin Error for 3-D C5G7 Problem (%)

Plane	RMS	MAX	Plane	RMS	MAX
1	1.0	4.9	6	0.8	3.5
2	1.0	4.8	7	0.8	2.9
3	0.9	4.6	8	0.9	2.4
4	0.9	4.3	9	1.0	2.6
5	0.9	3.9			

4. Conclusion

The pin power reconstruction method based on 2D source expansion has been improved to enable realistic multigroup core calculations by introducing assembly discontinuity factor and axial leakage terms in the plane-wise 2D flux calculations. In the test calculations for the 2D and 3D C5G7 benchmark problems, it was shown the pin power error is lower than 4 and 5% in the 2D can 3D cases, respectively, when compared against the DeCART whole core transport calculation results. The larger error noted near the plane near the bottom reflector is inevitable as long as the same homogenized cross sections are used for all planes. Nonetheless, with these good results, we can conclude that this method would be an efficient and accurate pin power calculation scheme applicable for real core problems.

References

[1] R. Boer and H. Finnemann, "Fast Analytical Flux Reconstruction Method for Nodal Space-Time Nuclear Reactor Analysis," *Annl. Nucl. Ener.*, **19**, 617, 1992.
[2] H. G. JOO *et al.*, "Consistent Analytic Pin Power Reconstruction Method for Static and Transient Reactor Safety Analysis," Proc. Mathematics and Computation, Reactor Physics and Environmental Analysis in Nuclear Applications, CD, Madrid, Spain, September 27-30, 1999.
[3] T. Bahadir and S.-Ö. Lindahl, "SIMULATE-4 Pin Power Calculations," Proc. of PHYSOR2006, CD, Vancouver, Canada, September 10-14, 2006.
[4] S. G. Baek, *et al.*., "Two-Dimensional Semi-Analytic Nodal Method for Multigroup Pin Power Reconstruction", ICAPP 2007, Nice, France, May 13-18, 2007.
[5] E. E. Lewis, et al., "Benchmark Specification for Deterministic 2-D/3-D MOX fuel Assembly Transport Calculations without Spatial Homogenization (C5G7MOX)," OECD Nuclear Energy Agency (2001).
[6] J. I. Yoon and H. G. Joo," Two-Level Coarse Mesh Finite Difference Formulation with Multigroup Source Expansion Nodal Kernels", *J. Nucl. Sci. Technol.*, **45**, No.7, 2008. (to appear).