# A Fuzzy Support Vector Regression Model for Estimation of Collapse Moment of Wall-Thinned Pipes

In Ho Bae, Man Gyun Na\*, and Jin Weon Kim

Department of Nuclear Engineering, Chosun University, 375 Seosuk-dong, Dong-gu, Gwangju 501-759 \*Corresponding author: magyna@chosun.ac.kr

## 1. Introduction

The pipe bends and elbows are regarded as critical components in piping systems of nuclear power plants because they are incorporated into piping systems to allow modification of the isometric routing and more importantly pipe bends are usually incorporated to reduce anchor reaction forces. Also, the pipe bends and elbows are capable of absorbing considerably large thermal expansion and seismic movement through the energy dissipation as a result of local plastic deformation so that they maintain the integrity of piping system under transiently loading conditions. Significant care must be taken to avoid their collapse moment. Therefore, it is important to accurately assess the safety margin for a collapse of pipe bends and elbows under various operating conditions.

The wall-thinned defect is mainly caused by flowaccelerated corrosion, and it reduces failure pressure, load-carrying capacity, deformation ability, and fatigue resistance of pipe bends and elbows. Therefore, it is necessary to investigate the effect of wall-thinned defects on the failure behavior of pipe bends and elbows and to accurately estimate the collapse loads of wallthinned bends and elbows under various loading conditions.

## 2. Development of FSVR Model

In this study, an FSVR model was used for accurate estimation of collapse moment of wall-thinned pipes. A regression problem approximates an unknown function that can be expressed as a linear expansion of basis functions. The regression problem is transformed to determine the coefficients of the basis function of linear expansion. The SVR nonlinearly maps the original input data **x** into higher dimensional feature space,  $\phi(\mathbf{x})$ . The SVR considers the following regression function:

$$y = f(\mathbf{x}) = \sum_{k=1}^{N} w_k \phi_k(\mathbf{x}) = \mathbf{w}^T \mathbf{\phi}(\mathbf{x}) + b$$
(1)

The function  $\phi_k(\mathbf{x})$  is called the feature and the parameters **w** and **b** are the support vector weight and bias. The first term of Eq. (2) characterizes the complexity of the SVR models. The FSVR is known as support vector regression (SVR) that is equipped with a fuzzy concept. The proposed FSVR enhances the SVR by reducing the effect of outliers and noise. By applying a fuzzy membership function to each data point of the SVR model, the regularized risk function can be

reformulated, such that different input data points can make different contributions to the learning of a regression function as follows[1]:

$$R(\mathbf{w},\boldsymbol{\xi},\boldsymbol{\xi}^*) = \frac{1}{2} \mathbf{w}^T \mathbf{w} + \lambda \sum_{k=1}^N \mu_k \left| y_k - f(\mathbf{x}) \right|_{\varepsilon}$$
  
$$= \frac{1}{2} \mathbf{w}^T \mathbf{w} + \lambda \sum_{k=1}^N \mu_k \left( \xi_k + \xi_k^* \right),$$
 (2)

subject to the constraints

$$\begin{cases} y_k - \mathbf{w}^T \mathbf{\phi}(\mathbf{x}) - b \le \varepsilon + \xi_k, & k = 1, 2, \mathbb{L}, N \\ \mathbf{w}^T \mathbf{\phi}(\mathbf{x}) + b - y_k \le \varepsilon + \xi_k^*, & k = 1, 2, \mathbb{L}, N \\ \xi_k, & \xi_k^* \ge 0, \quad k = 1, 2, \mathbb{L}, N \end{cases}$$
(3)

where

$$|y_k - f(\mathbf{x})|_{\varepsilon} = \begin{cases} 0 & |y_k - f(\mathbf{x})| < \varepsilon \\ |y_k - f(\mathbf{x})| - \varepsilon & \text{otherwise} \end{cases}$$
(4)

where  $\mu_k$  is a fuzzy membership grade. Commonly used SVR methods apply an equal weighting to all data points. However, FSVR uses different weightings according to their importance, which is specified by the fuzzy membership grade. The constrained optimization problem can be solved by applying the Lagrange multiplier technique to Eqs. (2) and (3).

The appropriate selection of training data is very important because it can affect the performance of the FSVR model. The input and output training data is expected to have many clusters in each group and the data at these cluster centers is more informative than the neighboring data. An FSVR model for each data set can be well trained using the informative data. The cluster centers were located using a subtractive clustering (SC) scheme and used as the training data set.

 $N_g$  input/output training data  $\mathbf{z}_k = (\mathbf{x}_k, y_k)$  in a group was assumed to be available and the data points were normalized in each dimension. The SC scheme begins by generating a number of clusters in  $m \times N$  dimensional input space. The SC scheme uses a measure of the potential of each data point, which is a function of the Euclidean distances to all other input data points [2]:

$$P_1(k) = \sum_{j=1}^{N_g} e^{-4\left\|\mathbf{x}_k - \mathbf{x}_j\right\|^2 / r_\alpha^2} , \ k = 1, 2, ..., N_g,$$
(5)

where  $\gamma_{\alpha}$  is a radius to define a particular neighborhood. It should be noted that the potential of a data point is high when it is surrounded by an abundance of neighboring data. After the potential of each data point was calculated, the data point with the highest potential was selected as the first cluster center.

In general, after determining the *i*-th cluster center  $\mathbf{c}_i$  and its potential value  $P_i^c$ , the potential of each data point is revised using the following equation:

$$P_{i+1}(k) = P_i(k) - P_i^c e^{-4\|\mathbf{x}_k - \mathbf{c}_i\|^2 / r_{\beta}^2}, k = 1, 2, ..., N_g$$
(6)

where  $\gamma_{\beta}$  is usually greater than  $\gamma_{\alpha}$  in order to limit the number of clusters generated. Equation (6) means that an amount of potential is subtracted from each data point as a function of its distance from the cluster center. The data points near the cluster center have greatly reduced potential and are unlikely to be selected as the next cluster center. When the potentials of all data points have been revised according to Eq. (6), the data point with the highest potential is selected as the  $(i+1)^{\text{th}}$ cluster center. These calculations stop if the inequality,  $P_i^c < \varepsilon P_1^c$ , is true, otherwise the calculations are repeated. If the calculations are stopped finally at an iterative step  $N_c$ , then there are  $N_c$  cluster centers in a data group. The input/output data (training data) positioned in the cluster centers of the data group will be selected to train the FSVR model for each group. In addition, every five time-steps, the test data is selected from the remaining sequential data where the training data has already been eliminated. Hence, the optimization data and test data comprise 80% and 20% of the remaining sequential data, respectively.

It is reasonable that the data points with high potential calculated by Eq. (5) are more important and weighted more highly than the other neighboring data points when training the FSAR models. Therefore, the potential of the cluster centers calculated by Eq. (5) was used as a fuzzy membership grade in Eq. (2) as follows:

$$\mu_k = 1 - \frac{1}{P_1(k)}, \ k = 1, L, N_c.$$
(7)

# 3. Application of FSVR to Estimation of Collapse Moment

The collapse moment of the bends subjected to inplane bending can be defined by various methods. The wall-thinning defects are located at the intrados and extrados centerlines of the pipe bend, and the axial and circumferential shapes of defects are circular. Fig. 1 shows wall-thinned pipes on extrados, but wall-thinning can be occurred on intrados or crown occasionally. In this study, FSVR model has been developed for more accurate estimation of collapse moment.

There is a little difference according to the defect location. But it can be verified that RMS errors decrease less than 0.3%. Therefore, their estimation performances are very accurate (refer to Table 1).

The collapse moment estimation was done in a previous work [3] by using SVR models that could accurately estimate the collapse moment. But the

estimation performance of the FSVR models has improved  $30 \sim 50\%$  more than the SVR model. It is known in this study that the FSVR models can estimate the collapse moment more accurately.



Fig. 1. Definition of dimensions of wall-thinned defects

Table 1. Estimation results of the FSVR models.

Defect location		Extrados	Intrados	Crown	Total
Fitness		0.9288	0.8995	0.9678	-
Number (No.) of SVs		947	952	205	-
Training Data	No.	1361	1361	250	2972
	RMS Error (%)	0.2098	0.2420	0.0849	0.2140
	Max error (%)	2.8201	2.6766	0.1022	2.8201
Optimization Data	No.	282	282	51	615
	RMS Error (%)	0.2155	0.2715	0.1367	0.2346
	Max error (%)	0.8279	0.8970	0.5927	0.8970
Test data	No.	57	57	11	125
	RMS Error (%)	0.2543	0.3171	0.1540	0.2741
	Max error (%)	0.6529	1.4957	0.4050	1.4957

### 4. Conclusion

In this paper, the FSVR method has been used to estimate the collapse moment due to the wall-thinned defects of pipes in piping systems. The FSVR models have been developed for three data sets divided into the three classes of extrados, intrados, and crown defects. As a result, it is known that the estimation performance of an FSVR method is superior to any other methods. Therefore, it is expected that the proposed method can be applied to assess the integrity of the wall-thinned pipes by estimating their collapse moments very fast and accurately.

### References

[1] M. G. Na, D. H. Lim, and H. Y. Yang, "Soft-Sensing Models for Feedwater Flowrate," Proc. of KNS Autumn Mtg, pp. 829-830, Pyeongchang, Korea, Oct. 25-26, 2007.

[2] S. L. Chiu, "Fuzzy model identification based on cluster estimation," *J. Intell. Fuzzy Systems*, vol. 2, pp. 267-278, 1994.
[3] Man Gyun Na, Jin Weon Kim, and In Joon Hwang, "Collapse Moment Estimation by Support Vector Machines for Wall-Thinned Pipe Bends and Elbows," *Nucl. Eng. Des.*, Vol. 237, No. 5, pp. 451-459, Mar. 2007.