Transactions of the Korean Nuclear Society Spring Meeting
Gyeongju, Korea, May 29-30, 2008

Development of an Implicit Scheme to Solve Three Dimensional Three Field Equations

M. T. Oh, C. E. Park, and S. Y. Lee
Korea Power Engineering Company, Inc., 150 Deokjin-dong, Yuseong-gu, Daejeon, 305-353

1. Introduction

In this study, the multi-fluid implicit scheme has been developed to eliminate the material Courant time step restriction. The multi-fluid implicit scheme in which the momentum convection terms are solved implicitly is provided as an alternative option for steady-state calculations and for slowly varying, quasi-steady transient calculations. The multi-fluid implicit method involves one predictor and two corrector steps and one stabilization step. After the series of predictor and corrector steps, the corrected velocities satisfy the continuity and momentum equations more closely, compared to semi-implicit scheme. The final step in the module is the solution of the stabilizer mass and energy equations. The stabilization step is used to stabilize the convective terms in the mass and energy balance equations.

2. General Description of Multi-Fluid Implicit Scheme

In the multi-fluid implicit scheme, because the momentum flux terms are implicit, the momentum equations cannot be directly solved to obtain a linear relationship between velocity and pressure gradient as done in the reference[1]. Therefore, the multi-fluid implicit method splits the solution procedure into a series of predictor-corrector steps and stabilization step.

In predictor step, discretized momentum equations are solved with the pressure field of previous time step to give intermediate velocities.

In first corrector step, the convection terms of momentum equations are discretized in explicit manner with the intermediate velocities. Then, the discretized momentum equation gives a linear relationship between velocity and pressure gradient as done in the semi-implicit scheme. A single pressure equation can be derived by substituting this relationship to the implicit velocity terms of mass and energy conservation equations, and inverting the cell matrix. This is done for each cell, giving rise to the system pressure matrix. Thus, only an N x N system of pressure equations is solved simultaneously at each time step, where N is the total number of cells used to simulate the fluid system. After the system pressure matrix is solved, the intermediate solutions for other primitive variables are obtained by the back substitution.

In second corrector step, the same procedure as the first corrector step is taken once more in order to obtain the final solutions for phasic velocities and pressures. Provisional new time values for phasic volume fractions and phasic temperatures can also be obtained.

The stabilization step is used to stabilize the convective terms in the mass and energy balance equations. The flux terms in mass and energy equations are evaluated implicitly at the new time, as compared to their explicit evaluation in the corrector steps. The stabilization step provides the final solutions for phasic volume fractions and phasic temperatures with the new variables known.

3. Steps of Multi-Fluid Implicit Scheme

The split-operator scheme, in which operations are split into a series of predictor-corrector steps, is applied for the solution of the discretized equations. Let the superscripts ~, *, and ** denote intermediate field values obtained during the splitting process

3.1 Predictor step

The equations for momentum are solved in this step implicitly, using the pressures of previous time step. To yield the $\mathring{\mathbf{U}}$ velocity field, the following matrix form is used.

$$\begin{bmatrix} \mathbf{M_P}11 & \mathbf{M_P}12 & \mathbf{M_P}13 & \mathbf{M}_{\mathbf{gPN_i}} & 0 & 0 \\ \mathbf{M_P}21 & \mathbf{M_P}22 & \mathbf{M_P}23 & 0 & \mathbf{M}_{\mathbf{lPN_i}} & 0 \\ \mathbf{M_P}31 & \mathbf{M_P}32 & \mathbf{M_P}33 & 0 & 0 & \mathbf{M}_{\mathbf{dPN_i}} \\ \mathbf{M}_{\mathbf{gPN_i}} & & & \mathbf{M_{N_i}}11 & \mathbf{M_{N_i}}12 & \mathbf{M_{N_i}}13 \\ & \mathbf{M}_{\mathbf{lPN_i}} & & \mathbf{M_{N_i}}21 & \mathbf{M_{N_i}}22 & \mathbf{M_{N_i}}23 \\ & & \mathbf{M}_{\mathbf{dPN_i}} & \mathbf{M_{N_i}}31 & \mathbf{M_{N_i}}32 & \mathbf{M_{N_i}}33 \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \tilde{\mathbf{U}}_g \\ \tilde{\mathbf{U}}_l \\ \tilde{\mathbf{U}}_d \end{bmatrix}_P \\ \begin{bmatrix} \tilde{\mathbf{U}}_g \\ \tilde{\mathbf{U}}_l \\ \tilde{\mathbf{U}}_d \end{bmatrix}_{N_i} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \mathbf{p}_g \\ \mathbf{p}_l \\ \mathbf{p}_d \end{bmatrix}_P \\ \begin{bmatrix} \mathbf{p}_g \\ \mathbf{p}_l \\ \mathbf{p}_d \end{bmatrix}_{Ni} \end{bmatrix} + \begin{bmatrix} \begin{bmatrix} \mathbf{s}_g \\ \mathbf{s}_l \\ \mathbf{s}_d \end{bmatrix}_P \\ \begin{bmatrix} \mathbf{s}_g \\ \mathbf{s}_l \\ \mathbf{s}_d \end{bmatrix}_{Ni} \end{bmatrix}$$

3.2 First corrector step

The first corrector step is introduced to give a new velocity field (U^*) together with a corresponding new pressure field (P^*). The convection terms of the momentum equations are taken as the known velocities ($\mathring{\mathbf{U}}$) instead of velocities discretized implicitly. As in semi-implicit algorithm, the momentum equations can be expressed in the following matrix form:

$$\begin{bmatrix} m11 & m12 & m13 \\ m21 & m22 & m23 \\ m31 & m32 & m33 \end{bmatrix} \begin{bmatrix} \mathbf{U}_g^* \\ \mathbf{U}_l^* \\ \mathbf{U}_d^* \end{bmatrix} = \begin{bmatrix} \mathbf{p}_g^* \\ \mathbf{p}_l^* \\ \mathbf{p}_d^* \end{bmatrix} + \begin{bmatrix} \mathbf{s}_g \\ \mathbf{s}_l \\ \mathbf{s}_d \end{bmatrix} + \begin{bmatrix} mflux_{gPN_{id}} \\ mflux_{lPN_{id}} \\ mflux_{dPN_{id}} \end{bmatrix}$$

3.3 Second corrector step

A new velocity field, U^{**}, together with its corresponding new pressure field, P^{**}, is formulated as the explicit type equation. The momentum equations can be expressed in the following matrix form:

$$\begin{bmatrix} m11 & m12 & m13 \\ m21 & m22 & m23 \\ m31 & m32 & m33 \end{bmatrix}\begin{bmatrix} \mathbf{U}_g^{**} \\ \mathbf{U}_l^{**} \\ \mathbf{U}_d^{**} \end{bmatrix} = \begin{bmatrix} \mathbf{p}_g^{**} \\ \mathbf{p}_l^{**} \\ \mathbf{p}_d^{**} \end{bmatrix} + \begin{bmatrix} \mathbf{s}_g \\ \mathbf{s}_l \\ \mathbf{s}_d \end{bmatrix} + \begin{bmatrix} mflux_{gPN_{id}}^{*} \\ mflux_{lPN_{id}}^{*} \\ mflux_{dPN_{id}}^{*} \end{bmatrix}$$

Once the second pressure corrections are known, the velocity components can also be obtained through a linear relationship between velocity and pressure gradient. The U^{**} field and the P^{**} field are approximations of the exact solution $\mathbf{U}^n$ and P^n . As in the paper of Issa [2], the accuracy with which U^{**} and P^{**} approximate the exact solutions is sufficient for most practical purposes, which makes further corrector steps superfluous. Therefore, only two corrector steps are taken in order to obtain the velocity and pressure fields at every time step

3.4 Stabilizer step

This step is used to stabilize the convective terms in the mass and energy balance equations. This step uses the velocities from the second corrector step.

4. Test results

4.1 Single phase liquid injection test (1D)

This test used a straight vertical pipe of 10 volumes (each 1m in length). The subcooled liquid was injected at the velocity, 1.0 m/s, through the bottom junction. This simulation was carried out with time step 5s and 10s, which are 5 to 10 times the material Courant limit. The results show that the multi-fluid implicit scheme module is able to achieve stability for time steps that exceed the material Courant limit and give sufficiently accurate predictions when a numerical simulation is for a basically quasi-steady process.

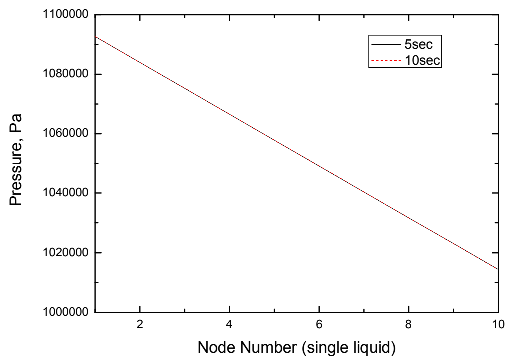

Figure 1 Pressure variation during the liquid injection test

4.2 Single phase liquid injection test (2D)

This problem was a two-dimensional single phase liquid cavity flow problem. The test domain consisted of 25 uniform regular hexahedrons. The inlet boundary condition was given at the lower left corner of the rectangular cavity, and the outlet boundary condition was given at the upper right corner. This simulation was carried out with time step 5s, which is 5 times the material Courant limit.

Fig. 2 shows the velocity vectors at the cells. The cavity flow is mainly formed along the bottom and the right walls. The remaining part of the cavity shows a flow pattern of swirl. This indicates that the material Courant time step can be violated using the multi-fluid implicit scheme.

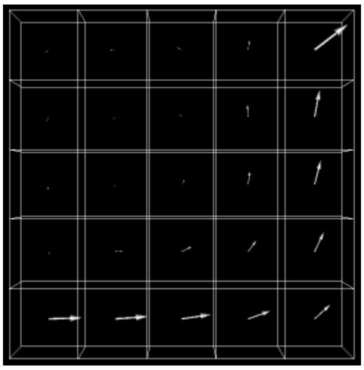

Figure 2 Velocity vector distribution

5. Conclusion

The multi-fluid implicit scheme module has been developed to eliminate the material Courant time step restriction. The multi-dimensional three-field pilot code using the multi-fluid implicit scheme can be used for steady-state calculations and for slowly varying, quasi-steady transient calculations. The predictions of transient behaviors using the scheme show stability for time steps that exceed the material Courant limit and accurate results as in the semi-implicit scheme. Consequently, it is expected that the computation time is much reduced compared to the semi-implicit scheme.

Acknowledgment

This study was performed under the project, "Development of safety analysis codes for nuclear power plants" sponsored by the Ministry of Knowledge Economy.

REFERENCES

[1] M. T. Oh, A Collocated Grid Semi-Implicit Scheme to Solve One Dimensional Three Field Equations, Korean Nuclear Society Autumn Meeting, 2007.

[2] R. I. Issa, "Solution of the Implicitly Discretised Fluid Flow Equations by Operator-Splitting," Journal of Computational Physics 62, pp. 40-65, 1986.