Development of a Staggered Mesh Semi-implicit Scheme and its Application to the Multi-D Two Phase Flow

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1. Introduction

KOPEC has been developing a two-phase thermal hydraulic solver for the safety analysis of nuclear power plants [1]. The pilot code adopts the multi-dimensional, three field governing equations, which are spatially discretized using a general unstructured polyhedral mesh system. Several numerical schemes, such as collocated, staggered, semi-implicit, and implicit schemes, have been tried so far.

Among the various numerical solution schemes implemented in the pilot code, the staggered mesh semi-implicit scheme will be described in detail in this paper. In addition, some of its application results, such as manometric oscillation and simplified direct vessel injection tests, will be presented.

2. Discretized Field Equations

2. 1 Staggered mesh system

Orthogonal hexahedral mesh system is used for the staggered semi-implicit scheme. All the geometric quantities of mesh are described in terms of cell volume, centroid, face area, and face center, so that it can naturally represent not only the 1-D or 3-D Cartesian, but also the cylindrical or spherical mesh system.

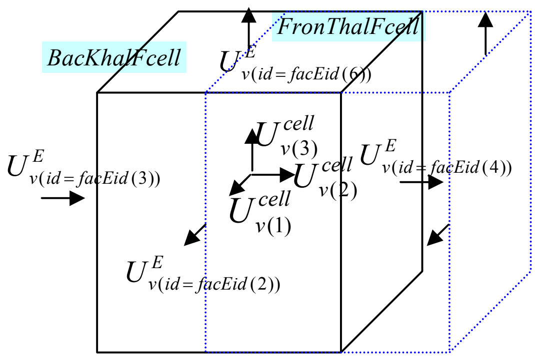

Fig 1. Staggered mesh

Each momentum cell is shifted by the half size of scalar cell so that it consists of the front half part of the owner scalar cell and the back half part of the neighbor scalar cell.

2.2 Discretized equations

Using the definition of flux terms at faces, phasic mass and energy conservation equations can be expressed as follows.

- Continuity equation

$$\frac{\varepsilon V}{\Delta t}\left(\alpha_l^{n+1}\rho_l^{(n+1)}-\alpha_l\rho_l\right) = -\sum_{i=1,id=facEid(i)}^{facEcount} \varepsilon_{(id)}^{E}\,{}^{d}\alpha_{l(id)}^{E}\,{}^{d}\rho_{l(id)}^{E}\left(\iota(i)Flux_{l(id)}\right)+\gamma_l+\theta_l \tag{1}$$

- Energy equation

$$\frac{\varepsilon V}{\Delta t}\left[\left(\alpha_l^{n+1}\rho_l^{(n+1)}e_l^{(n+1)}-\alpha_l\rho_l e_l\right)+P\left(\alpha_l^{(n+1)}-\alpha_l\right)\right] = -\sum_{i=1,id=facEid(i)}^{facEcount} \iota(i)\varepsilon_{(id)}^{E}\,{}^{d}\alpha_{l(id)}^{E}\left({}^{d}\rho_{l(id)}^{E}\,{}^{d}e_{l(id)}^{E}+P_{(id)}^{E}\right)Flux_{l(id)}+E_l+\Phi_l \tag{2}$$

In the hexahedral shape of a scalar cell, each orthogonal face can be categorized into three different types, depending on the direction of the face vector. The components of the cell velocity vector, $U_{(k)}$, are calculated by averaging the same type face velocities. If the transverse direction velocity is denoted by $XU_{(k)}$ at each face, the momentum equation can be expressed as follows.

- Momentum equation

$$\begin{aligned}
&\frac{\varepsilon V}{\Delta t}\left(U_l^{E(n+1)}-U_l^{E}\right)+\varepsilon_{(k)}^{E,Neigh}\,{}^{d}U_{l(k)}^{Neighbor}\left(A_{(k)}U_{l(k)}^{Neighbor}\right)-\varepsilon_{(k)}^{E,Own}\,{}^{d}U_{l(k)}^{Owner}\left(A_{(k)}U_{l(k)}^{Owner}\right)\\
&+\sum_{E,typ\neq k}^{Owner.cell}\varepsilon_{(id)}^{E}\,{}^{d}XU_{l(id)(k)}^{Fhcell}(\iota(i)\frac{1}{2}Flux_{l(id)})+\sum_{E,typ\neq k}^{Neighbor.cell}\varepsilon_{(id)}^{E}\,{}^{d}XU_{l(id)(k)}^{Bhcell}(\iota(i)\frac{1}{2}Flux_{l(id)})\\
&+F_{l(k)}V-\ U_l^{E}\varepsilon_{(k)}^{E,Neighbor}\left(A_{(k)}^{Neighbor}U_{l(k)}^{Neighbor}\right)+U_l^{E}\varepsilon_{(k)}^{E,Owner}\left(A_{(k)}^{Owner}U_{l(k)}^{Owner}\right)\\
&-U_l^{E}\sum_{E,typ\neq k}^{Owner.cell}\varepsilon_{(id)}^{E}(\iota(i)\frac{1}{2}Flux_{l(id)})-U_l^{E}\sum_{E,typ\neq k}^{Neighbor.cell}\varepsilon_{(id)}^{E}(\iota(i)\frac{1}{2}Flux_{l(id)})\\
&=\frac{1}{\rho_l}\varepsilon_{(id)}^{E}A_{(id)}^{E}(P_{owner}^{n+1}-P_{neighbor}^{n+1})+\varepsilon V\frac{1}{\alpha_l\rho_l}\left(-F_{wl}U_l^{E(n+1)}\right)+\varepsilon V\mathbf{B}\cdot\mathbf{n}^{E}+M_l+\Theta_l
\end{aligned} \tag{3}$$

3. Time Advancement

In the semi-Implicit scheme, the convection terms of momentum equations are discretized in explicit manner, while the pressure gradient terms are implicitly treated. Then, the discretized momentum equation gives a linear relationship between velocity and pressure gradient. A single pressure equation can be derived by substituting this relationship to the implicit velocity terms of mass and energy conservation equations, and inverting the cell matrix. This is done for each cell, giving rise to the system pressure matrix. Thus, only an N x N system of pressure equations is solved simultaneously at each time step, where N is the total number of cells used to simulate the fluid system. After the system pressure matrix is solved, the solutions for other primitive variables are obtained by the back substitution

4. Test results

4.1 Manometric oscillation

As shown in Fig. 2, a 21-cell nitrogen-water manometer test problem is set up to check the validity of the non-condensable gas model and the momentum formulation. The bottom four cells on the left hand side and the bottom eight cells on the right hand side of the manometer are initially filled with water. The remaining cells are initialized with dry nitrogen. After the test starts, the head difference drives the liquid to oscillate back and forth between the two vertical columns. The test result demonstrates that the staggered semi-implicit numerical scheme works properly for this periodic flow.

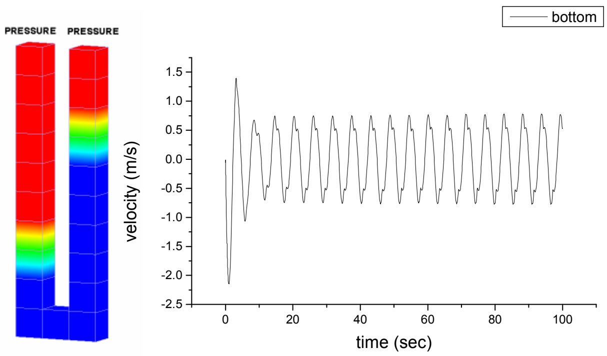

Figure 2 Manometer problem and velocity at the bottom

4.2 Direct vessel injection test

A simplified direct vessel injection test is performed to check the code capability to analyze two-phase flow in the cylindrical geometry. The annulus shown in Fig. 3 is initialized with pure vapor and then the liquid starts to fill from the two injection nozzle. Each of the liquid injection nozzles is located at the opposite side of the medium level outer wall. A broken nozzle and a vapor injection nozzle are also aligned at the same level, 90 degree apart from the liquid injection nozzles in the azimuthal direction.

The test result shows that liquid film penetrates to the bottom of annulus, in spite of the vapor flow crossing the liquid flow. Once the liquid fills up to the level of broken nozzle, the remaining liquid spills out except the near region of vapor injection nozzle, where liquid droplet or liquid film is driven to the top of the annulus due to the strong vapor flow.

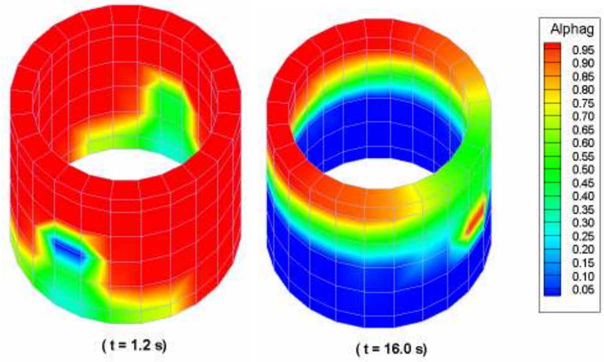

Figure 3 Test problem and distribution of void fraction

5. Conclusion

The staggered semi-implicit scheme is developed as a hydraulic solver for SPACE code. The numerical solution scheme covers not only the 1-D or 3-D Cartesian, but also the cylindrical or spherical mesh system. The manometric oscillation test shows that the non-condensable gas state calculation and the momentum formulation work properly for periodic flow. The direct vessel injection test shows that the numerical scheme is applicable to more complex two-phase flow, like the hydraulic interference between the liquid and vapor flow in a cylindrical geometry.

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[1] S. Y Lee, Development of a Hydraulic Solver for the Safety Analysis Codes for Nuclear Power Plants(I). Korean Nuclear Society Spring Meeting, 2007.

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