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Determination of Unidentified Leakage Using a Kalman Smoother

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1. Introduction

Since the safety significance of leaks from the RCS can widely vary depending on the source of the leak as well as the leak rate, the detection of the leakage is an important issue. The leakage is classified into 1) identified leakage which is defined as leakage into closed systems such as pump seal or valve packing leaks that can be captured, and 2) unidentified leakage which is all other leakage. The unidentified leakage is typically determined by the RCS inventory balance method which is based on NUREG-1107. [1] Since the accuracy of leak rate calculation is dependent of the plant operating condition, the change in the RCS temperature, inventory, and the transient operating condition should be avoided during the measurement period. Nevertheless, the operation of the makeup of the borated water into the RCS and the diversion of the inventory to the outside of the RCS boundary makes it difficult to maintain the plant stable over an hour. Due to the large fluctuation of the calculated leak rate, it is sometimes hard to know the trend of the leakage as well as the instantaneous leak rate. Any fluctuation of operating conditions can results in unreliable leak rate.

This study proposes a new way of determining the unidentified leak rate using a Kalman filter and smoother technique. The proposed algorithm enhances the accuracy of the leak rate calculation not only for the steady state operations but also for transients in a well-timed manner.

2. Methods and Results

2.1 Fundamentals of Kalman Filter/Smoother

The Kalman filter or Kalman smoother was originally developed for spacecraft navigation. Later it turned out that the Kalman filter is very useful for estimating the states in a system that can only be observed indirectly. In mathematical terms, a Kalman filter estimates the states of a linear system at discrete points in time. The Kalman filter not only works well in practice, but it is theoretically attractive because it can be shown that it minimizes variance in the estimation error. Recently, an extended Kalman filter has been developed which has a robust noise distribution assumption and is available for non-linear systems. [2, 3]

Since the Kalman filter is very popular in, particularly control area, so the fundamental equations to represent a Kalman filter will be mentioned in this paper. Let's assume that the state of the system is represented as a vector of real numbers such as the matrices $\mathbf{F}_k$, $\mathbf{H}_k$, $\mathbf{Q}_k$, $\mathbf{R}_k$, and, if necessary, $\mathbf{B}_k$ for each time-step, k, as in Equations (1) through (4). When we determine these matrices, we should need the results of system modeling using first principles. The Kalman filter assumes the true state at time, k, is evolved from the state at $(k-1)$ according to

$$\mathbf{x}_k = \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{B}_k \mathbf{u}_k + \mathbf{w}_k , \tag{1}$$

where $\mathbf{F}_k$ is the state transition model which is applied to the previous state, $\mathbf{x}_{k-1}$, $\mathbf{B}_k$ is the control-input model which is applied to the control vector, $\mathbf{u}_k$, $\mathbf{w}_k$ is process noise which is assumed to be drawn from a zero mean and multivariate normal distribution with covariance, $\mathbf{Q}_k$,

$$\mathbf{w}_k \sim N(0, \mathbf{Q}_k) . \tag{2}$$

At time k, an observation, $\mathbf{z}_k$, in a visible or observable state, $\mathbf{x}_k$, in a hidden or true state is made according to

$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k , \tag{3}$$

where $\mathbf{H}_k$ is the observational model which maps the true state space into the observed space, and $\mathbf{v}_k$ is the observational noise which is assumed to be a multivariate normal distribution with zero mean and covariance $\mathbf{R}_k$,

$$\mathbf{v}_k \sim N(0, \mathbf{R}_k) . \tag{4}$$

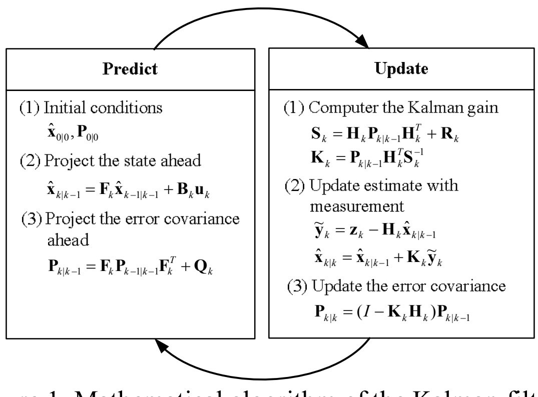

Figure 1. Mathematical algorithm of the Kalman filter

Figure 1 summarizes the mathematical algorithm of the Kalman filter. The prediction step uses the state estimate from a previous time step to produce an estimate of the state at the current time step. In the update step, observations at the current time step are used to refine the estimate to get a more accurate state estimate. In order to smooth the data we take initial conditions from the Kalman filter $\hat{\mathbf{x}}_{k|k}$ and $\mathbf{P}_{k|k}$ and run a backwards algorithm in Equation (5) – (7).

$$x_{k-1|k} = x_{k-1|k-1} + \mathbf{K}_{k-1}(x_{k|k} - x_{k|k-1}) \qquad (5)$$

$$\mathbf{P}_{k-1|k} = \mathbf{P}_{k-1|k-1} + \mathbf{K}_{k-1}(\mathbf{P}_{k|k} - \mathbf{P}_{k|k-1})\mathbf{K}'_{k-1} \qquad (6)$$

$$\mathbf{K}_{k-1} = \mathbf{P}_{k|k}\mathbf{F}'\mathbf{P}_{k|k-1} \qquad (7)$$

2.2 Unidentified Leak Rate using Kalman Smoother

The algorithm of determining unidentified leakage is generalized in order to apply it to any kinds of reactors regardless of steady states or transients. The governing equations are as follows:

$$m_k = c_l l_k \rho_k (T_k, P_k) \qquad (8)$$

$$m_k = m_{k-1} \pm \sum_{pipe} \int_{k_{in}}^{k_{fi}} \dot{w}\, dt = m_{k-1} \pm \sum_{pipe} v_{0/1} \dot{w}_{k-1} \Delta k \qquad (9)$$

$$L_{tot} = \sum_{\text{tank}} \left(m_{\text{tank},k_{fi}} - m_{\text{tank},k_{in}} \right) \Big/ \left(k_{fi} - k_{in} \right) \rho_{STP} \qquad (10)$$

$$L_{iden} = \sum_{\text{tank}} \left(m_{\text{tank},k_{fi}} - m_{\text{tank},k_{in}} \right) \Big/ \left(k_{fi} - k_{in} \right) \rho_{STP} \qquad (11)$$

$$\therefore L_{uniden} = L_{tot} - L_{iden} (volume/time) \qquad (12)$$

where m is the total mass of a tank, l is a tank level,
c_l is the conversion factor of a tank level to a volume at level l,
ρ is the density of the fluid inside a tank which is the function of temperature T and pressure P,
L is the total leakage,
$v_{0/1}$ is the coefficient for valve close(=0)/open(=1).

Subscript, *k*: a specific time,
in: initial condition, *fi*: final condition,
tot: total, *iden*: identified, *uniden*: unidentified,
tank: tanks or boundaries involved in the calculation of total (or identified) leakage,
pipe: pipes or boundary connections involved in the calculation of total (or identified) leakage,
STP: standard temperature and pressure condition.

We established the discretized time model of the RCS inventory balance method using the Kalman smoother algorithms. The final results are summarized in Equation (13) and (14).

$$m_k = \mathbf{F}_k m_{k-1} + \mathbf{B}_k (v_{0/1} \dot{w}_{k-1}) + \mathbf{w}_k \qquad (13)$$

$$z_k = \mathbf{H}_k m_k + \mathbf{v}_k \qquad (14)$$

where $\mathbf{F}_k = [1]$, $\mathbf{B}_k = [\Delta k]$, $\mathbf{H}_k = [1]$, $\mathbf{w}_k$ is determined by the uncertainty of all of the in/out flowrate, and $\mathbf{v}_k$ is determined by the uncertainty of all of the measurement channels.

As an example, in case of YGN1&2, the tanks involved in estimating total leakage are reactor coolant loop, pressurizer, volume control tank, and the tanks involved in calculating identified leakage are pressurizer relief tank, reactor coolant drain tank, equipment drain tank, and safety injection tank. The pipe involved in the calculation belongs to steam generator leakage.

2.3 Results

Figure 2 shows the results of calculating the unidentified leakage using the conventional snapshot technique and the Kalman smoother for YGN1&2. Most of fluctuation was eliminated and the identity of the leak rate result was apparently provided in the case of the Kalman smoother. The wide range of validation in applying the Kalman smoother is being investigated from the viewpoint of accuracy and response time in detail for field applications.

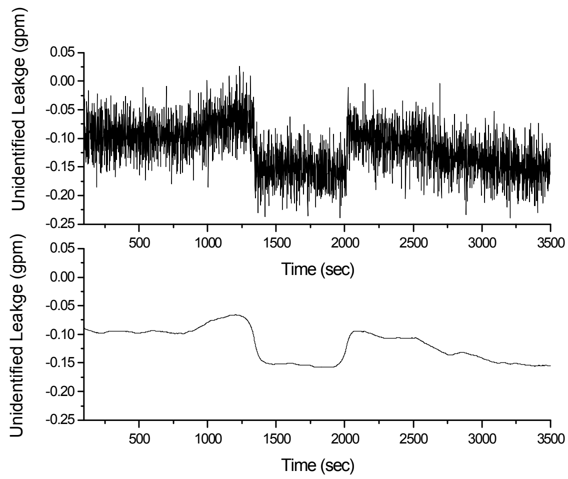

Figure 2. Unidentified leak rate using a conventional technique (upper) and a Kalman smoother (lower)

3. Conclusions

This study attempted to enhance the numerical algorithm of determining a RCS leak rate using a Kalman filter and smoother technique. The leak rate calculation algorithm requires accuracy as well as a suitable response time not only for the steady state operations but also for transients. It is expected that the proposed algorithm enable to achieve such requirements, which is superior to the conventional snapshot based calculation.

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