Development and Verification of a 2-D Semi-implicit Three-Field Solver

Moonkyu Hwang*, Jae Jun Jung
*Korea Atomic Energy Research Institute, P.O. box 105, Yusong-gu, Deajeon, Korea, mkhwang@kaeri.re.kr**

1. Introduction

The thermal-hydraulic behavior of a multiphase flow is predicted by solving the related governing equations. The equations are devised based on the mass, momentum, and energy conservation laws and physical models defining the thermal and mechanical interactions between the phases involved. A great deal of effort has been paid, during the past two or three decades, to solve the equations in an efficient and stable manner. A reliable methodology to obtain the solution of the equation set is essential, especially in a nuclear thermal/ hydraulic safety analysis where the consequences of the various hypothetical incidents are required to be within a predefined range of acceptance.

In this study, a semi-implicit solver is developed for a two-dimensional flow as three-fields. This work is an extension of the previous study where one-dimensional solver was developed [1]. The three-field modeling of water is similar to a method employed in codes such as COBRA-TF [2]. As in the previous study, the three fields are comprised of a gas, continuous liquid and entrained liquid field. All the three fields are allowed to have their own velocities. The continuous liquid and the entrained liquid are, however, assumed to be in a thermal equilibrium.

2. Methods

2.1 Governing equations

The governing equation set for the three-field modeling of a two phase flow is based on the time-space average equations of a single-pressure two-fluid model [3]. The equations are:

$$\frac{\partial}{\partial t}(\alpha_v\rho_v)+\frac{1}{A}\frac{\partial}{\partial x}(\alpha_v\rho_v v_v A)=\Gamma_v \tag{1}$$

$$\frac{\partial}{\partial t}(\alpha_l\rho_l)+\frac{1}{A}\frac{\partial}{\partial x}(\alpha_l\rho_l v_l A)=-(1-\eta)\Gamma_v - S \tag{2}$$

$$\frac{\partial}{\partial t}(\alpha_e\rho_e)+\frac{1}{A}\frac{\partial}{\partial x}(\alpha_e\rho_e v_e A)=-\eta\Gamma_v + S \tag{3}$$

where η stands for the evaporation fraction from the entrained liquid, and the subscriptions v, l and d are for the vapor, continuous liquid and entrained liquid, respectively. Similarly, the momentum equation (in x-direction) and energy equations for the gas phase are, respectively,

$$\alpha_v\rho_v\frac{\partial u_v}{\partial t}+\alpha_v\rho_v u_v\frac{\partial u_v}{\partial x}+\alpha_v\rho_v w_v\frac{\partial u_v}{\partial z}=-\alpha_v\frac{\partial P}{\partial x}+\alpha_v\rho_v B_x - F_{xwv}(u_v)$$
$$-\Gamma_C u_v+(1-\eta)\Gamma_E u_l+\eta\Gamma_E u_d-\Gamma_v u_v-F_{xvd}(u_v-u_d)-F_{xvl}(u_v-u_l)$$
$$-C_{v,vd}\alpha_v\alpha_d\rho_{m,vd}\frac{\partial(v_v-v_d)}{\partial t}-C_{v,vl}\alpha_v\alpha_l\rho_{m,vl}\frac{\partial(v_v-v_l)}{\partial t}, \tag{4}$$

$$\frac{\partial(\alpha_v\rho_v u_v)}{\partial t}+\left(\frac{1}{A}\right)\frac{\partial(A\alpha_v\rho_v u_v v_v)}{\partial x}=-P\frac{\partial\alpha v}{\partial t}-\left(\frac{P}{A}\right)\frac{\partial(A\alpha_v v_v)}{\partial x}$$
$$+Q_{wv}+Q_{iv}+\Gamma_{vi}h_g^*+\Gamma_w h_g^{'}+DISS_v+\&\& \tag{5}$$

The momentum and energy equations for the continuous liquid and entrained liquid can be similarly obtained.

2.2 Finite Difference Equations

The differential equation set is integrated over the 2-diminentional node depicted in Figure 1.

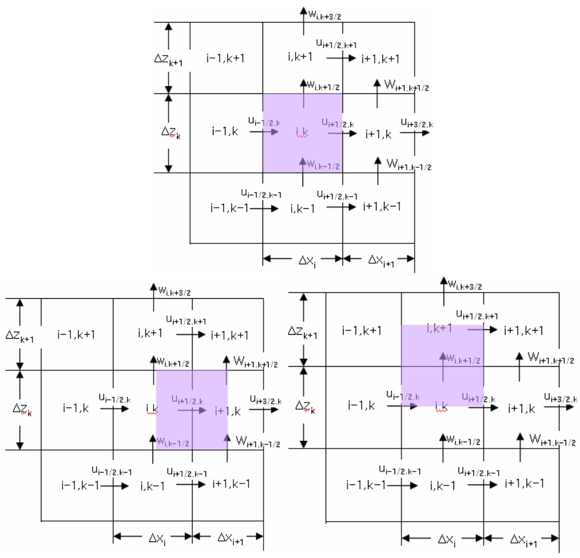

Figure 1. Two-dimensional Nodes for the Pilot Code

The momentum equation for the vapor in the x-direction is in the form of

$$(\alpha_v\rho_v)^n_{i+1/2,k}(u_v^{n+1}-u_v^n)_{i+1/2,k}/\Delta t$$
$$+(\&\&u_v)^n_{i+1/2,k}\left[(\bar{u}_v)_{i+1,k}-(\bar{u}_v)_{i,k}\right]/\Delta x_{i+1/2}$$
$$+(\&\&w_v)^n_{i+1/2,k}\left[(\bar{u}_v)_{i+1/2,k+1/2}-(\bar{u}_v)_{i+1/2,k-1/2}\right]/\Delta z_k$$
$$=-(\alpha_v)^n_{i+1/2,k}(P^{n+1}_{i+1,k}-P^{n+1}_{i,k})/\Delta x_{i+1/2}$$
$$+(\alpha_v\rho_v)^n_{i+1/2,k}B_x-(F_{xwv})^n_{i+1/2,k}(u_v)^{n+1}_{i+1/2,k}$$
$$-(\Gamma_E)^n_{i+1/2,k}(u_v)^{n+1}_{i+1/2,k}+(1-\eta^n_{i+1/2,k})(\Gamma_E)^n_{i+1/2,k}(u_l)^{n+1}_{i+1/2,k}$$
$$+\eta^n_{i+1/2,k}(\Gamma_E)^n_{i+1/2,k}(u_d)^{n+1}_{i+1/2,k}$$

$$
\begin{aligned}
&-(F_{xvd})^{n}_{i+1/2,k}\left\{(u_v)^{n+1}_{i+1/2,k}-(u_d)^{n+1}_{i+1/2,k}\right\}\\
&-(F_{xvl})^{n}_{i+1/2,k}\left\{(u_v)^{n+1}_{i+1/2,k}-(u_l)^{n+1}_{i+1/2,k}\right\}\\
&-(\alpha_v\alpha_d)^{n}_{i+1/2,k}(C_{xv,vd}\rho_{m,vd})^{n}_{i+1/2,k}\\
&\times\left\{(u_v)^{n+1}_{i+1/2,k}-(u_v)^{n}_{i+1/2,k}-(u_d)^{n+1}_{i+1/2,k}+(u_d)^{n}_{i+1/2,k}\right\}\ /\Delta t\\
&-(\alpha_v\alpha_l)^{n}_{i+1/2,k}(C_{xv,vl}\rho_{m,vl})^{n}_{i+1/2,k}\\
&\times\left\{(u_v)^{n+1}_{i+1/2,k}-(u_v)^{n}_{i+1/2,k}-(u_l)^{n+1}_{i+1/2,k}+(u_l)^{n}_{i+1/2,k}\right\}\ /\Delta t\\
&+\frac{1}{2}(VISV_x)^{n}_{i+1/2,k}/\Delta x_{i+1/2} \qquad (6)
\end{aligned}
$$

It is noted that the old time step is used for the advection term, whereas the implicitness of the velocity is maintained for the source terms originating from both the phase change and the surface forces. The correlations for determining the phase change and the surface force, however, are based on the old time step.

The finite difference equations for the mass and energy equations are derived in a similar fashion. The equations are expanded for the time and space derivative terms before applying the finite difference method. This is merely for a convenience during the process of finite differencing. The *unexpanded* original forms are again used at the end the time step to minimize the mass error introduced by the linearization process of the densities and temperatures for a time step (n+1). The procedure for solving the finite difference equations are very similar to that used by the RELAP5/MOD3 code [4].

3. Verification Tests

The integrity of the numerical scheme implemented in the pilot code is verified against single and two-phase calculations, for the geometry depicted in Figure 2.

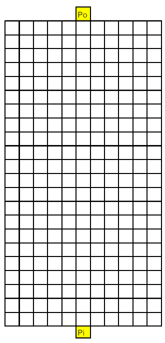

Figure 2. Input geometry for the verification tests (11x21)

In case of the single-phase flow test, the outlet pressure has been maintained as 1.0 MPa, whereas the inlet pressures are either 1.03MPa or 1.06MPa. Heat source is not considered in this particular test. For the two-phase flow test, the inlet and outlet pressures were set to 1.03 MPa and 1.0 MPa, respectively. The heat source was initially zero, and then increased to 20MW/m^3 during the initial period of 10 seconds. The increased value was maintained thereafter. The test results are shown in Figure 3. The 2-D contour plots are for the velocities in x and z directions, temperature, pressure and void fraction. The symmetrical behavior of each parameter confirms the soundness of the numerical solver. The time-behaviors of the inlet and outlet pressures as well as temperature and void fraction are depicted in the lower right corner of the plot. As seen in the figure, the outlet pressure oscillates during the interval between around 5 to 7 seconds. The behavior appears to be caused by the gas phase appearance.

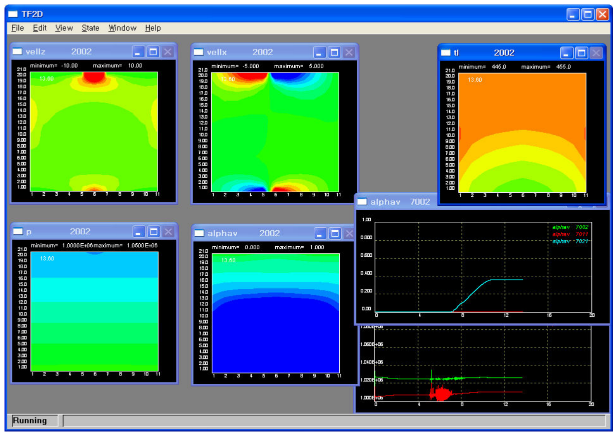

Figure 3. Two-phase flow test results
(In vel. plot, the red and blue designate + and – directions)

Detailed descriptions for the governing equations, numerical schemes/solution method as well as verification tests are found in reference [5].

4. Summary

In this study, a two-dimensional semi-implicit pilot code for a two phase flow is developed and verified. The two phase flow of water is modeled by a three field mass, momentum and energy equation set. The verification tests confirm the sound integrity of the numerical scheme implemented.

REFERENCES

[1] Hwang, M., et al., Development and Verification of a Pilot Code based on Two-fluid Three-field Model, KAERI/TR-3239/2006.
[2] Paik, C.Y., et. al., Analysis of FLECHT-SEASET 163-Rod blocked Bundle Data using COBRA-TR, NUREG/CR-4166
[3] Ishii, M. and Mishima, Two-fluid model and hydrodynamic constitutive relations. Nucl. Eng. Des. 82. 107-126 (1982)
[4] Scientech, Inc., RELAP5/MOD3 Code Manual, Volume 1: Code Structure, System Model, and Solution Methods (1998)
[5] Hwang, M., et al., A semi-implicit numerical scheme for a transient, two-dimensional, three-field thermo-hydraulic modeling," KAERI/TR-3436/2007.