Reduction of the spurious currents in the multiphase lattice Boltzmann method

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1. Introduction

Recently, the lattice Boltzmann method(LBM) has gained much attention for its ability to simulate fluid flows, and for its potential advantages over a conventional CFD method. The key advantages of LBM are, (1) suitability for parallel computations, (2) absence of the need to solve the time-consuming Poisson equation for a pressure, and (3) an ease with multiphase flows, complex geometries and interfacial dynamics may be treated[1].

A phenomenon known as spurious currents was first discovered in an early LB multiphase model as a small but finite amplitude circulating flow near the interface of a stationary bubble. The flow pattern was found to possess the same symmetry as the underlying lattice and the magnitude of the velocity scales with the surface tension and the inversion of the viscosity. Similar flow patterns have been found in the subsequent VOF, level set models for a multiphase flow although both the magnitude and the spatial extent of the spurious current are much reduced[2]. The presence of the anomalous circulation has caused some skepticism about the applicability of the LB method for the simulation of a multiphase flow. For engineering applications, the small velocities also compromise the accuracy of the numerical results, especially in situations where the detailed dynamics of the interface is of critical importance. Several attempts have been made to understand the origin and to reduce the magnitude of the spurious velocities[3,4]. Shan[3] showed that the origin of the spurious currents in the non-ideal gas LB model could be understood as caused by the lack of a sufficient isotropy when the gradient of the density is calculated. Lee et al.[4] concluded that discretization errors in the computation of the intermolecular force cause spurious currents. They showed that these currents can be eliminated if the potential form of the intermolecular force is used with a compact isotropic discretization.

In the present work, the lattice Boltzmann model for multiphase flows proposed by Zheng et al. [5] is used for simulating a stationary droplet in a quiescent environment. By adopting a finite difference gradient operator of a sufficient isotropy, the spurious currents can be made to be small. The main objective of the present work is to establish the lattice Boltzmann method as a viable tool for the simulation of multiphase or multi-component flows.

2. Methods and Results

2.1 Methodology

Here, we consider a flow with two phases which have different densities. The low density and high density are noted as ρ_L and ρ_H respectively. The flow can be described by the Navier-Stokes equations and an interface evolution equation as [5]

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \vec{u}) = \theta_M \nabla^2 \mu_\phi \quad (1)$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \vec{u}) = 0 \quad (2)$$

$$\frac{\partial n \vec{u}}{\partial t} + \nabla \cdot (n \vec{u} \vec{u}) = -\nabla \cdot P + \mu \nabla^2 \vec{u} + \vec{F}_b \quad (3)$$

where θ_M is called mobility, μ_ϕ is the chemical potential, *P* is the pressure tensor, $\vec{F}_b$ is the body force, and *n*, ϕ are defined as

$$n = \frac{\rho_A + \rho_B}{2}, \phi = \frac{\rho_A - \rho_B}{2}$$

where ρ_A and ρ_B are the density of fluid A and fluid B respectively.

Under the lattice Boltzmann framework, Eq. (1) can be solved by iterating the evolution equation for a set of distribution functions. These distribution functions evolve with a modified lattice Boltzmann equation and BGK approximation,

$$g_i(x + e_i \delta t, t + \delta t) = g_i(x,t) + \Omega_i + (1-q)\delta g_i$$

with

$$\Omega_i = \frac{g_i^0(x,t) - g_i(x,t)}{\tau_\phi}$$

$$\delta g_i = g_i(x + e_i \delta t, t) - g_i(x,t)$$

where g_i is the distribution function, Ω_i is the collision term, τ_ϕ is the dimensionless single relaxation time, e_i is the lattice velocity, and *q* is a constant coefficient.

In Eq. (3), the term $\nabla \cdot P$ is related to the surface tension force. This force can be rewritten as a potential term,

$$\vec{F}_s = -\nabla \cdot P = -\phi \nabla \mu_\phi - \nabla p_0$$

where $p_0 = nc_s^2$, c_s is the speed of sound.

The potential form for the surface tension force is adopted to keep the energy conservation. Mathematically, the potential form and stress form are identical. However, numerically, the discretization error is different[3,4,5]. Thus, it is useful to eliminate spurious currents.

The lattice Boltzmann implementation of Eqs. (2) and (3) can be described as

$$f_i(x+e_i\delta t, t+\delta t) = f_i(x,t) + \Omega_i$$

with

$$\Omega_i = \frac{f_i^0(x,t) - f_i(x,t)}{\tau_n} +$$

$$(1-\frac{1}{2\tau_n})\frac{w_i}{c_s^2}[(\vec{c}_i - \vec{u}) + \frac{(\vec{c}_i \cdot \vec{u})}{c_s^2}\vec{c}_i](-\phi\nabla\mu_\phi + \vec{F}_b)\delta t$$

The equilibrium distributions satisfy the conservation laws as

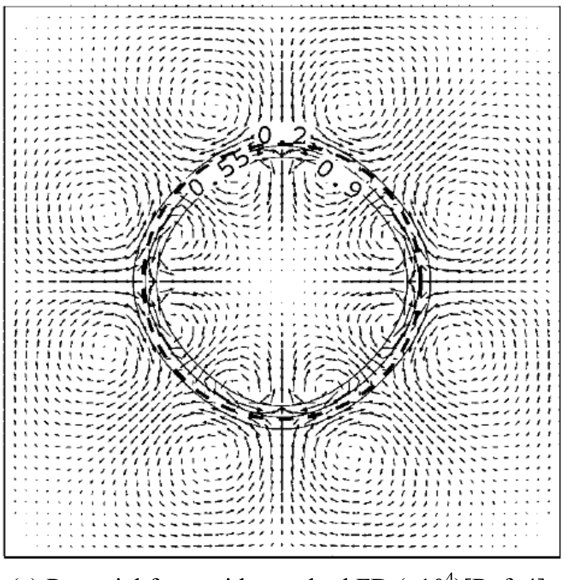
$$\phi = \sum_i g_i, n = \sum_i f_i$$

$$\vec{u} = [\sum_i f_i \vec{c}_i + \frac{1}{2}(-\phi\nabla\mu_\phi + \vec{F}_b)]/n$$

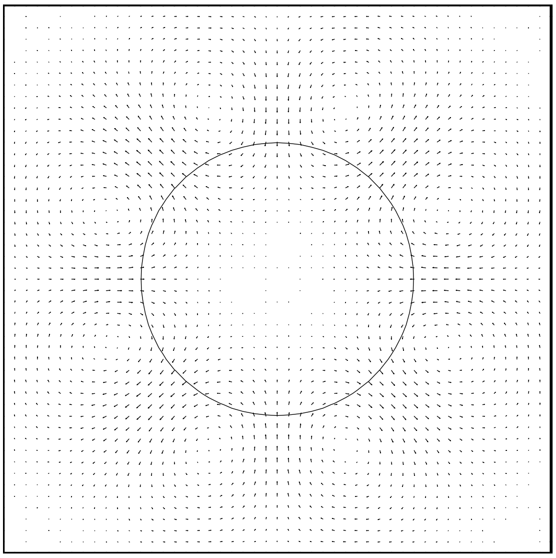
The details are Ref. [5].

2.2 Results

Fig. 1 shows the steady state velocity fields for the two cases. To illustrate the structure of the velocity field clearly, the lengths of the velocity vectors are multiplied by 10^4 and 10^{13} respectively. It can be seen that spurious currents of a eightfold symmetry are present in all the cases in the vicinity of the liquid vapor interface. However, both the magnitude and the spatial extent of the spurious currents are significantly reduced as a higher order isotropy is enforced in the calculation of the gradient.



(a) Potential form with standard FD (x10^4)[Ref. 4].



(b) Potential form with 4th order isotropic discretization (x 10^{13}).

Fig. 1. Velocity fields after 100 000 time steps.

3. Conclusion

In summary, two different sources of error in the computation of a surface tension force lead to the development of spurious currents. A slight imbalance between the pressure gradient and the stresses initiates these spurious currents. If an isotropy of the numerical scheme is not maintained, the spurious currents eventually develop into organized flow patterns. The LB method with an isotropic discretization can reduce the magnitude and the spatial extent of the spurious currents. Furthermore, the use of the potential form of a surface tension related term is helpful to reduce these spurious currents.

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