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The AFEN Method in Cylindrical (r,Θ ,z) Geometry for Pebble Bed Reactors – Incorporation of Acceleration and Discontinuity Factor –

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1. Introduction

Most existing methods of nuclear design analysis for pebble bed reactors (PBRs) are based on old finite-difference solvers or on statistical methods [1]. These methods require very long computer times. Therefore, there is strong desire of making available high fidelity coarse-mesh nodal computer codes.

Recently, we extended the analytic function expansion nodal (AFEN) method developed quite extensively in Cartesian (x,y,z) geometry and in hexagonal-z geometry to the treatment of the full three-dimensional cylindrical (r,θ,z) geometry for pebble bed reactors(PBRs).[2] The AFEN methodology in this geometry as in hexagonal geometry is "robust", due to the unique feature of the AFEN method that it does not use the transverse integration.

This paper presents an acceleration scheme based on the coarse-group rebalance (CGR) concept and provides test results verifying the method and its implementation in the TOPS code. Also, we implemented discontinuity factors in the TOPS code and tested on benchmark problems. The TOPS results are in excellent agreement with those of the VENTURE code, using significantly less computer time.

2. Basic Theory and Method

The AFEN formulation in the (r,θ,z) coordinates system starts from the following multi-group diffusion equations in a homogenized node:

$$-\nabla^2 \vec{\phi}(r,\theta,z) + [\Lambda]\vec{\phi}(r,\theta,z) = 0, \qquad (1)$$

where

$$[\Lambda] = [D]^{-1}\left([\Sigma] - \frac{1}{k_{eff}}[\chi][\nu\Sigma_f] \right).$$

All the notations are standard. The equations can be rewritten in cylindrical geometry as follows:

$$\frac{\partial^2 \vec{\phi}}{\partial r^2} + \frac{1}{r}\frac{\partial \vec{\phi}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \vec{\phi}}{\partial \theta^2} + \frac{\partial^2 \vec{\phi}}{\partial z^2} - [\Lambda]\vec{\phi} = 0. \qquad (2)$$

Since the general solution method is described in detail elsewhere [2], we focus on acceleration and discontinuity factors in this paper.

The coarse group rebalance (CGR) acceleration method was developed and applied to the AFEN method in rectangular and hexagonal geometries [3]. To apply the CGR acceleration to cylindrical geometry, first the net current variables in node balance and coupling equations are reformulated in terms of partial currents :

$$\vec{J}_s = \vec{J}_s^{outgoing} - \vec{J}_s^{incoming}, \qquad (3)$$

$$\vec{\phi}_s = 2(\vec{J}_s^{outgoing} + \vec{J}_s^{incoming}), \qquad (4)$$

where s is surface index.

Then, node balance equation can be written after group summation as

$$\begin{aligned} &\frac{S_z}{V}\sum_{s=0,1}\sum_{u=0,1}\sum_{g=1}^{G}(J_{z,s,u,g}^{outgoing} - J_{z,s,u,g}^{incoming}) \\ &+\frac{S_\theta}{V}\sum_{s=0,1}\sum_{u=0,1}\sum_{g=1}^{G}(J_{\theta,s,u,g}^{outgoing} - J_{\theta,s,u,g}^{incoming}) \\ &+\frac{S_r}{V}\sum_{s=0,1}\sum_{u=0,1}\sum_{g=1}^{G}(J_{r,s,u,g}^{outgoing} - J_{r,s,u,g}^{incoming}) \\ &+\sum_{g=1}^{G}\Sigma_{ag}\phi_{av,g} = \frac{1}{k_{eff}}\sum_{g=1}^{G}\nu\Sigma_{fg}\phi_{av,g}. \end{aligned} \qquad (5)$$

Let us define a rebalance factor per node as follows :

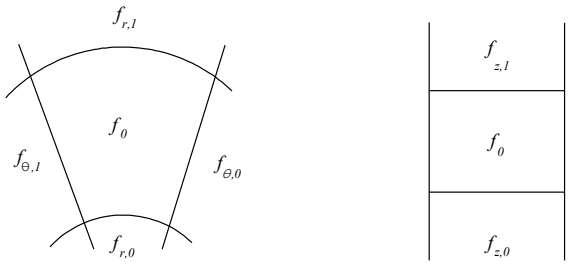
$$\phi_{av,g}^{new} = f_0 \phi_{av,g}^{old}, \qquad (6a)$$

$$J_{d,s,u,g}^{outgoing,new} = f_0 J_{d,s,u,g}^{outgoing,old}, \qquad (6b)$$

$$J_{d,s,u,g}^{incoming,new} = f_{d,s} J_{d,s,u,g}^{incoming,old}, \qquad (6c)$$

where $d = z,r,\theta,\ s = 0,1,\ u = 0,1,\ g = 1,2,...,G.$

The rebalance factors are depicted in Fig. 1.



(a) in r-Θ plane b) along z axis

Fig. 1 Rebalance factors

Substituting Eq. (6) in Eq. (5) leads to

$$R_0 f_0 - \sum_{d=z,r,\theta}\frac{S_d}{V}\sum_{s=0,1}\sum_{u=0,1}\sum_{g=1}^{G} J_{d,s,u,g}^{incoming} f_{d,s} = \frac{P_0}{k_{eff}} f_0, \qquad (7)$$

where

$$R_0 = \sum_{d=z,r,\theta} \frac{S_d}{V} \sum_{s=0,1} \sum_{u=0,1} \sum_{g=1}^{G} J_{d,s,u,g}^{outgoing} + \sum_{g=1}^{G} \Sigma_{ag} \phi_{av,g}, \quad (8)$$

$$P_0 = \sum_{g=1}^{G} \nu\Sigma_{fg} \phi_{avg,g}. \quad (9)$$

Eq. (7) is the CGR acceleration equation with eigenvalue k_{eff} and eigenvector $\vec{f}$ composed of rebalance factors. Once $\vec{f}$ is found, new nodal unknowns are obtained via Eq. (6).

3. Numerical Results

To verify the CGR acceleration, discontinuity factor and its implementation in the TOPS code, we solved two test problems. The PBMR-400 benchmark problem [4] is tested to examine the CGR acceleration effect.

The result for CGR testing is shown in Table I. As additional results, Fig. 2 shows radial node-average flux distributions obtained by TOPS and compared with those of VENTURE.

TABLE I. Results of the PBMR-400 benchmark problem (void problem) [a]

	k_{eff}	diff. (pcm)	CPU time [d]
VENTURE[b]	1.00461	reference	3030 (sec)
TOPS[c]	1.00464	3	1856 (sec)
TOPS[c] (CGR)	1.00464	3	134 (sec)

[a] D_{top_void} = 22.8055 cm, D_{side_void}= 0.268625 cm
[b] r-z (580x2900) calculation ; Chebyshev and LSOR acceleration
[c] r-θ-z (20x4x29) calculation
[d] Intel Core 2 2.40 GHz, 3.37GB RAM

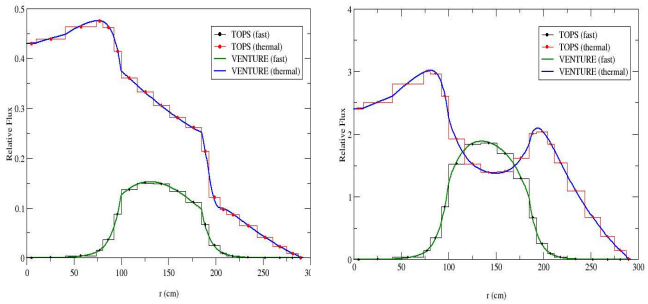


Fig 2. Radial node-average flux distributions

We note that the speedup due to CGR acceleration exceeds 13 relative to no acceleration. With regard to comparison with finite-difference methods, the computer time reduction of the TOPS nodal code in three-dimensional problems will be spectacular against the fine-mesh finite-difference calculations, if we note that the VENTURE run is a two-dimensional calculation in Table I.

To test performance of the TOPS code for problems with discontinuity factors in the homogenized diffusion parameters, we consider a variant of the PBMR-400 benchmark problem, in which non-unity discontinuity factors are assumed given around the top void region as in Fig. 3. The results of with and without CGR acceleration in TOPS using AFEN-consistent void treatment are shown in Table II. The speedup is 10.

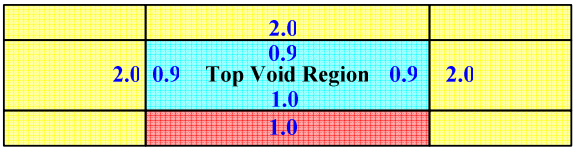


Fig 3. Discontinuity factors assumed around the top void region

TABLE II. Results of the PBMR-400 benchmark problem with discontinuity factors

	k_{eff}	#of terations	CPU time[b]
TOPS[a]	1.00514	842	1652 (sec)
TOPS[a] (CGR)	1.00514	75	163 (sec)

[a] r-z (580x2900) calculation ; Chebyshev and LSOR acceleration
[b] Intel Core 2 2.40 GHz, 3.37GB RAM

4. Conclusions

The results show that the CGR acceleration and discontinuity factors implemented in the TOPS code work correctly. The TOPS results are in excellent agreement with those of the VENTURE code, using significantly less computer time.

Acknowledgment

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