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A collocated coupled solver for multi-dimensional two-phase flows

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1. Introduction

A staggered-grid scheme is efficient and stable for a structured grid and used in many two-phase system analysis codes such as RELAP5-3D and MARS. However, in a multi-dimensional component analysis, the geometry is complex and a structured grid is hard to use. An unstructured grid is useful in this case. And the collocated scheme, where all the system variables are defined at cell center, is easy to apply to the unstructured grid. The purpose of this work is to extend the semi-implicit coupled scheme to the collocated grid system for the analysis of transient two-phase fluids in a complex geometry. A 3D pilot code is developed to verify the numerical scheme [1].

2. Governing Equations

The two-phase three-field (i.e. vapor, liquid, and drop) governing equations are employed for the transient two-phase analysis. The continuity, momentum, and energy equations are;

$$\frac{\partial}{\partial t}(\alpha_k\rho_k) + \nabla\cdot(\alpha_k\rho_k\underline{u}_k) = \Gamma_k \qquad (1)$$

$$\frac{\partial}{\partial t}(\alpha_k\rho_k\underline{u}_k) + \nabla\cdot(\alpha_k\rho_k\underline{u}_k\underline{u}_k) = -\alpha_k\nabla P + \nabla\cdot[\alpha_k\tau_k] \qquad (2)$$
$$+\alpha_k\rho_k\underline{g} + P\nabla\alpha_k + M_k^{mass} + M_k^{drag} + M_k^{VM}$$

$$\frac{\partial}{\partial t}[\alpha_k\rho_k e_k] + \nabla\cdot(\alpha_k\rho_k e_k\underline{u}_k) = -\nabla\cdot(\alpha_k q_k) \qquad (3)$$
$$+\nabla\alpha_k\tau_k : \nabla\underline{u}_k - P\frac{\partial}{\partial t}\alpha_k - P\nabla\cdot(\alpha_k\underline{u}_k) + I_k + Q'''_k$$

where α_k, ρ_k, $\underline{u}_k$, P_k, and Γ_k are the k-phase volume fraction, density, velocity, pressure, and interface mass transfer rate, respectively. M_k represents the interfacial momentum transfer due to the mass exchange, the drag, and the virtual mass. Thermal equilibrium is assumed between the liquid and droplet fields and the energy equation is omitted for the droplet field.

3. Numerical Methods

A semi-implicit scheme is applied for the collocated grid. At first, the phasic momentum equations are solved explicitly to obtain temporal velocities ($\underline{u}_{k,i}^{*}$) at a cell center. The interfacial terms are calculated implicitly. Then the new time velocities are found by a pressure correction ($p'_i = p^{n+1} - p^n$) which satisfies the mass and energy equations.

$$\underline{u}_{k,i}^{n+1} = \underline{u}_{k,i}^{*} + \gamma_i\nabla p'_i \qquad (4)$$

Since equation (4) should be integrated on the cell face, the cell face values are interpolated by using the neighbor cell values.

$$\underline{u}_{k,j}^{n+1} = \underline{u}_{k,j}^{*} + \gamma_j\frac{p'_{i1} - p'_i}{d_{ij}}, \qquad (5)$$

where, subscript j is the cell face between cell i and $i1$. The velocity at the cell face has to be interpolated carefully to prevent the well-known checker-board oscillations in the pressure and velocity [2]. Equation (5) is substituted into the scalar equation to obtain the pressure system equation.

4. Verifications

A set of test calculations was carried out to verify the pilot code. For the verification of the 3D unstructured numerical scheme, a boiling in a vertical pipe of a 1 m diameter and 2 m height was simulated. Figure 1(a) shows the unstructured grid using the Voronoi polygons in the X-Y plane. For the axial direction, 10 equal-length meshes were used. The total number of the cells was 1110. A constant volumetric heating of 23 MW/m^3 was provided with an inlet subcooled liquid flow of 0.1 m/s. The exit pressure was at 1.0 MPa. A transient calculation was conducted to reach a steady state. Figure 2 (b) shows the steady state void fraction due to the boiling. The conservation of the mass was also confirmed as shown in figure 2. At a steady state the mass error between the inlet and outlet flows was below 10^{-4}%.

To assess the code's capability to predict a flashing, a conceptual problem was established. A 2-dimensional plane of 0.1 m x 2 m was considered as shown in Figure 3(a). At the left, the inlet velocity was kept constant at 4.0 m/s with a water temperature of 450.0 K. At the right, the exit pressure was linearly decreased as a function of the time during the first 10 seconds from 1.0 MPa to 0.854 MPa and, then, kept constant. The saturation temperatures at 1.0 MPa and 0.854 MPa are 453.034 K and 446.270 K, respectively. Therefore the flow is initially subcooled, but a two-phase flow was expected later due to a flashing. Figures 3(b) and 3(c) show the pressure and void respectively. The inlet pressure was predicted to be 0.88 MPa when the pressure at the exit was given as 0.854 MPa. This means the superheated water is injected into the inlet. In all the computational cells, the liquid was slightly superheated and the steam was nearly saturated.

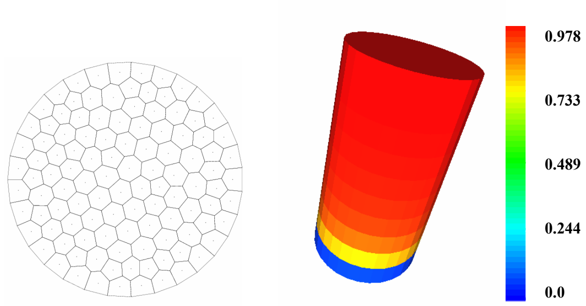
A cavitation in a sudden contraction flow was simulated by using a 2D structured grid. Figure 4(a) shows the 2D grid for this problem. The water velocity at the inlet was 3.0 m/s and the pressure at the exit was 1.0 MPa. The inlet water temperature was 453.0 K, which is slightly lower than the saturation temperature at 1.0 MPa (i.e., 453.034 K). Figures 4(b) and 9(c) show the steady-state results. The pressure near the walls close to the throat decreased to 0.96 MPa due to an acceleration, which is lower than the exit pressure. The temperature in these areas was higher than the saturation temperature. This results in a local cavitation and, thus, the steam exists near the walls only. The maximum void fraction was predicted to be about 0.01.

5. Conclusions

A 3-dimensional coupled scheme using collocated meshes has been proposed for a transient two-phase flow. To assess the scheme, a pilot code was developed and verified against several conceptual problems. The test calculations were carried out for both structured and unstructured grids and the calculation domains included 2D, and 3D ones. The results show that the numerical scheme is robust and efficient for the prediction of phase change and flow transitions due to a boiling and a flashing. The coupling between the momentum and energy balance was predicted well.

REFERENCES

[1] J. J. JEONG et al., "An unstructured hydrodynamic solver for a two-fluid three-field model," to be presented at the IAEA TOPICAL MEETING on Advanced Safety Assessment Methods for Nuclear Reactors, Korea Institute of Nuclear Safety, Daejon, Republic of Korea, 30 Oct. – 2 Nov. 2007.
[2] C. M. RHIE, W. L. CHOW, "Numerical study of the turbulent flow past an airfoil with trailing edge separation," *AIAA Journal,* Vol. 21. No. 11, 1525-1532 (1983).



(a) unstructured grids (b) void fraction

Figure 1. Boiling in 3D unstructured grid

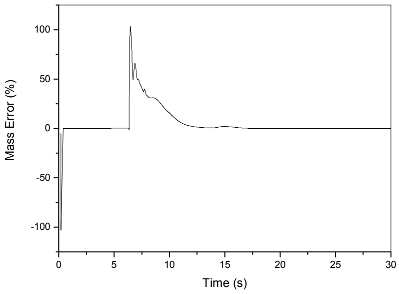
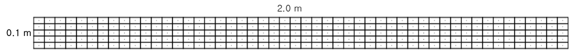
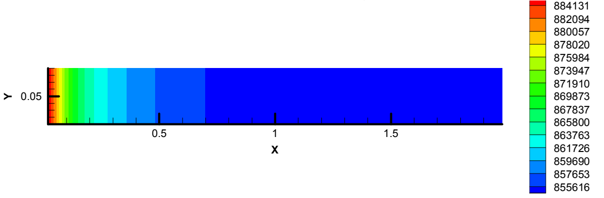


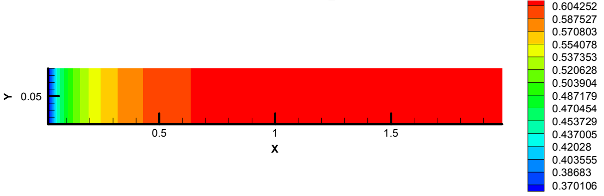
Figure 2. (mass_in – mass_out) / mass_tot *100.



(a) Calculation grid

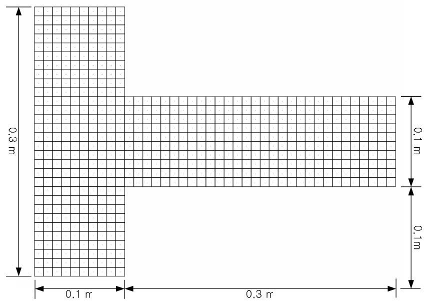


(b) Pressure profile

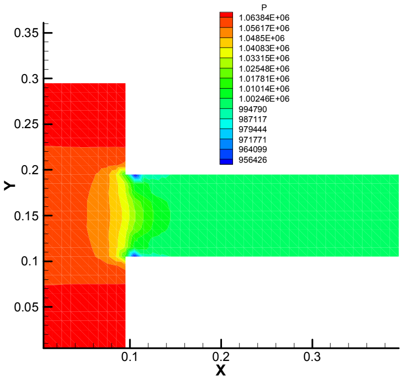


(c) Void fraction profile

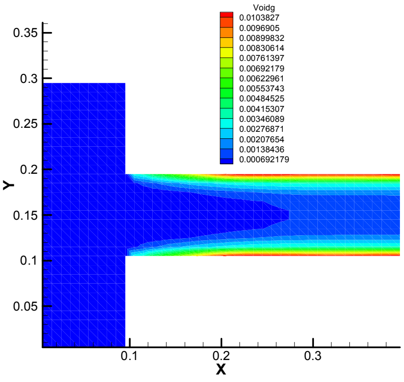
Figure 3. Flashing in a horizontal pipe



(a) Calculation grid



(b) Pressure profile



(c) Void profile

Figure 4. Cavitations in sudden contraction flow