Preconditioners for Krylov Iterative Method for Two-Dimensional MOC Problems

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1. Introduction

An appropriate selection of preconditioner for Krylov subspace method improves convergence of the iterative solutions of transport equations. The recent work, e.g. [1,2], indicate that the consistent discretization requirement can be relaxed in diffusion synthetic acceleration (DSA) if it is used as a preconditioner for the Krylov iterative method.

Coarse mesh rebalance (CMR) acceleration method also has been tested as a preconditioner for Krylov method. [3] It is known that the CMR method is unstable or ineffective with scattering ratio c close to unity for optically thin or thick cells. In 1-D problems, this deficiency, however, disappeared and even improved when the CMR is used as a preconditioner. But in 2-D problems, CMR preconditioner sometimes had worsening effect on Krylov method. In other words, the Krylov method without preconditoiner preformed better than the preconditioned Krylov method.

This paper, as an extension of previous work for finding good preconditioners for Krylov method, tested several preconditioners such as coarse mesh finite difference (CMFD), partial current coarse mesh finite difference (p-CMFD), and Jacobi preconditioners. The results are presented with discussions.

2. Methods and Results

2.1. Linearized Form of Acceleration Methods

In usual iterative methods, scarlar flux is updated by the following equation:

$$\boldsymbol{\phi}^{l+1/2} = \boldsymbol{T}\boldsymbol{\phi}^{l} + \boldsymbol{b}, \tag{1}$$

where $\boldsymbol{T}$ represents transport sweep process, $\boldsymbol{\phi}$ is scalar flux, $\boldsymbol{b}$ is a fixed source, l is iteration index. In acceleration methods of linear form, low order equation is solved additionally. The linear form of low order equation is

$$\boldsymbol{L}\boldsymbol{f} = \boldsymbol{\phi}^{l+1/2} - \boldsymbol{\phi}^{l}, \tag{2}$$

where $\boldsymbol{L}$ means the matrix of low order equation, $\boldsymbol{f}$ is updating factor of the linear acceleration method.
Using this updating factor, scalar flux is updated by

$$\boldsymbol{\phi}^{l+1} = \boldsymbol{\phi}^{l+1/2} + \boldsymbol{f}. \tag{3}$$

Substitution of Eq. (1) into Eq. (2) results in

$$\boldsymbol{L}\boldsymbol{f} = (\boldsymbol{T}-\boldsymbol{I})\boldsymbol{\phi}^{l} + \boldsymbol{b} \tag{4}$$

and if Eq.(4) is substituted into Eq.(3), Eq. (3) becomes

$$\boldsymbol{\phi}^{l+1} = (\boldsymbol{T}\boldsymbol{\phi}^{l} + \boldsymbol{b}) + \boldsymbol{L}^{-1}[(\boldsymbol{T}-\boldsymbol{I})\boldsymbol{\phi}^{l} + \boldsymbol{b}]. \tag{5}$$

After full convergence of the scalar flux, Eq. (1) and Eq.(5) become, respectively,

$$(\boldsymbol{I}-\boldsymbol{T})\boldsymbol{\phi} = \boldsymbol{b}, \tag{6}$$

$$(\boldsymbol{I}+\boldsymbol{L}^{-1})(\boldsymbol{I}-\boldsymbol{T})\boldsymbol{\phi} = (\boldsymbol{I}+\boldsymbol{L}^{-1})\boldsymbol{b}. \tag{7}$$

Eq. (6) is a matrix equation we solve when there is no acceleration. When we apply an acceleration method, the matrix equation changes into Eq. (7). Comparing these two equations, it is obvious that $\boldsymbol{I}+\boldsymbol{L}^{-1}$ works as a preconditioner. Instead of the traditional source iteration and acceleration framework, the Krylov iteration method can be applied to Eq. (6) or Eq. (7). A special feature of the Krylov method is that $\boldsymbol{T}$ and $\boldsymbol{L}$ need not be defined explicitly with matrix elements and suffice to have only their "actions".

2.2. Numerical Tests

Using several sample problems, numerical tests of preconditioned Krylov method have been done. For comparisons with well known acceleration methods, test problems chosen are the same with those Ref. 4.

2.2.1. Test Problem I

The first test problem is heterogeneous and based on a modified Kavenoky's problem with vacuum boundaries as shown in Fig. 1. Material properties are given in Table I.

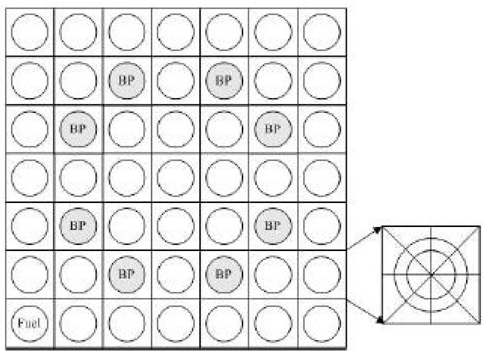

Fig. 1. Geometry of Test Problem I

Table I. Material Properties of Test Problem I

	Moderator	Fuel	BP
Source density ($cm^{-3}sec^{-1}$)	1.000	0.000	0.000
$\sigma(cm^{-1})$	1.250	0.625	14.000
$\sigma_s(cm^{-1})$	1.242	0.355	0.000

The problem has 7×7 coarse-mesh cells and a coarse-mesh cell contains 24 fine-mesh cells. The size of a coarse-mesh cell is 1.25cm×1.25cm and radii of circles are 0.45cm and 0.35cm. (8,4) angles and 50 rays per coarse-mesh cell per direction are used to solve the problem. Convergence criteria are 10^{-6}. Table II shows the results of the calculation.

Table II. Results of Test Problem I

	Number of iterations	Computing time (sec)[a]
CRX (No Acc.)	58	25.93
CMADR	7	5.12
Krylov	9	8.47
CMFD_Krylov	10	9.37
pCMFD_KRylov	8	8.27
CMR_Krylov	9	8.29
Jacobi_Krylov	9	9.35

[a] on Intel Xeon 3.0GHz Linux Machine

2.2.2. Test Problem II

In Test Problem II, we consider various material heterogeneities. The problem geometry of Test Problem II is identical with that of Test Problem I, but the material properties are heterogeneous with varying degrees (as shown in Table III as Cases 1,2, and 3). The scattering ratio is very high in all cases. Table IV shows the results.

Table III. Material Properties of Test Problem II

		Moderator	Fuel	BP
Source density ($cm^{-3}sec^{-1}$)		1.000	0.000	0.000
$\sigma(cm^{-1})$	Case 1	1.250	0.625	14.000
	Case 2	1.250	0.625	140.000
	Case 3	1.250	0.00625	140.000
$c(=\sigma_s/\sigma)$		0.999	0.999	0.999

3. Conclusions

In this paper, various preconditioners have been tested for Krylov iterative method and compared with source iteration (no acceleration) and CMADR acceleration. Preliminary conclusions based on tests performed so far are as follows. When the problem is not highly heterogeneous, optically thick, or thin, the preconditioned Krylov method showed almost similar performance (in the aspect of both number of iterations and computing time). However, if the problem is heterogeneous, pure Krylov method was better than preconditioned ones (Test Problem II, Case 1,2,3). This means that the preconditioners tested in this kind of problems does not improve but deteriorate the pure Krylov method.

Table IV. Results of Test Problem II

		Number of iterations	Computing time (sec)[a]
Case 1	CRX (No Acc.)	288	114.87
	CMADR	10	13.61
	Krylov	19	17.04
	CMFD_Krylov	19	17.12
	pCMFD_Kryov	18	17.96
	CMR_Krylov	35	31.14
	Jacobi_Krylov	19	18.41
Case 2	CRX (No Acc.)	2155	859.21
	CMADR	10	45.16
	Krylov	31	27.48
	CMFD_Krylov	51	48.08
	pCMFD_Kryov	60	58.65
	CMR_Krylov	190	166.83
	Jacobi_Krylov	37	35.04
Case 3	CRX (No Acc.)	2155	883.45
	CMADR	10	40.50
	Krylov	34	30.96
	CMFD_Krylov	43	39.24
	pCMFD_Kryov	37	37.31
	CMR_Krylov	178	165.55
	Jacobi_Krylov	41	39.83

[a] on Intel Xeon 3.0GHz Linux Machine

References

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