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Direct and Iterative Two-Node Multi-group Semi-Analytic Nodal Methods

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1. Introduction

As a means of efficient multigroup nodal calculation, a one-node multigroup Semi-Analytic Nodal Method (SANM) kernel based two-group CMFD formulation had been developed.[1] In the SANM formulation of that work, a quartic Legendre expansion of the source was used unlike others[2,3] employing a quadratic form. Since the one-node nodal kernel provides the node-wise multi-group neutron spectrum as well as the multigroup interface currents, a two-group CMFD formulation, which uses dynamically condensed cross sections and nodal coupling parameters, was possible for the acceleration of the multi-group nodal calculation. Some drawbacks, however, were noted in using the one-node based method. One is the need for multiple one- node sweeps resulting from the slower convergence of the one-node formulation and the other is a stability problem for large optical length conditions [4].

While the two-node nodal method doesn't have such drawbacks, there are other issues in the implementation of the multi-group two-node nodal kernel. The first is that two-node kernel cannot update the neutron spectrum by itself so that a multigroup CMFD solver is needed. The other is that a more complicated nodal problem is to be solved for the direct coupled solution involving both nodes [2,3]. This work is to investigate an efficient two-node solution method employing the SANM with a quartic source expansion and the source iteration scheme.

2. Two-Node Semi-Analytic Nodal Method

In this section, the direct SANM that is to solve the group-coupled SANM equations directly is introduced first, and the iterative SANM is introduced as a more efficient way.

2.1 Direct SANM (DSANM)

The SANM solves transverse integrated 1-D neutron diffusion equation (given in a normalized form below):

$$-D\frac{4}{h_x^2}\frac{\partial^2}{\partial\xi^2}+\Sigma_r\phi(\xi)=\lambda\psi(\xi)+S(\xi)-L(\xi) \qquad (1)$$

with the right hand side source term expanded into a 2-nd or 4-th order polynomial. Here a 4^{th} order polynomials is used to express more accurately spatial distributions [1] as in Kim's work.[4] The solution then consists of sinh, cosh and a 4^{th} order Legendre polynomial as:

$$\phi_g(\xi)=\bar{\phi}_g+\sum_{i=1}^{6}c_{i,g}P_i(\xi) \qquad (2)$$

where

P_i : i-th order Legendre Polynomial (i=0~4)

$$P_5(\xi)=\frac{1}{\mathrm{Sinh}(\kappa)+M_1+M_3}\left(\mathrm{Sinh}(\kappa\xi)+M_1P_1(\xi)+M_3P_3(\xi)\right)$$

$$P_6(\xi)=\frac{1}{\mathrm{Cosh}(\kappa)+M_0+M_2+M_4}\begin{pmatrix}\mathrm{Cosh}(\kappa\xi)+M_0\\+M_2P_2(\xi)+M_4P_4(\xi)\end{pmatrix}$$

Since the number of unknown coefficients is 6G for each node, 12G unknown coefficients for the two node problem have to be determined. Among them even term coefficients; c_2, c_4, c_6 for each node and group can be solved independently from the other node because group and node coupling between the coefficients can be removed by weighting the equation with P_0, P_2, P_4. But the remaining 6G odd coefficients; c_1, c_3, c_5 of both nodes are coupled by interface conditions: flux and current continuity at the surface between two nodes. It can't be decoupled without several times of inversion of G x G full matrices. It is thus unavoidable to solve a 6G linear system to obtain the coefficients which requires nontrivial computing time except for the 2 group cases in which the inversion of a 2x2 matrix is trivial. This makes DSANM potentially slow as more groups are used.

2.2 Iterative SANM (ISANM)

ISANM is to use a predetermined 4^{th} order polynomial for the RHS source term whose coefficients are obtained from the previous solution using the least square method which is equivalent to the orthogonal expansion with the Legendre polynomial. Namely, the source is represented as the following:

$$\lambda\psi(\xi)+S(\xi)-L(\xi)=\sum_{i=0}^{4}q_iP_i(\xi) \qquad (3)$$

where

$$q_i=\int_{-1}^{1}\phi(\xi)P_i(\xi)d\xi \Big/ \int_{-1}^{1}P_i(\xi)P_i(\xi)d\xi \qquad (4)$$

Since the RHS is a 4^{th} order polynomial, we can obtain a particular solution of the 1-D diffusion equation using undetermined coefficients method. And the coefficient of cosh is obtained using average flux that is obtained by MG CMFD.

$$\int_{-1}^{1}\phi(\xi)d\xi=\phi_{avg} \qquad (5)$$

The remaining coefficients of the sinh term of the two nodes are coupled each other and they can be obtained using the two interface conditions: flux and current continuity.

ISANM doesn't need to any complex operation like 6G matrix operation used in DSANM but needs group iterations to converge solutions due to approximation of RHS. Therefore, it's important to decide whether a sufficient convergence is reached to get the solution efficiently. Here we use the relative l2-norm of a changed shape between current and previous fission sources to check convergence, namely,

$$\varepsilon = \left\| \int_{-1}^{1} (\psi^{n}(\xi) - \psi^{n-1}(\xi))^2 d\xi \right\|_2 / \int_{-1}^{1} \psi_{avg} d\xi \,. \qquad (6)$$

2.3 Two-Level CMFD Acceleration

The multi-group CMFD need to be performed when the two-node multigroup solver used. In that case, the 2G CMFD can also be used to accelerate MG CMFD. A two level CMFD consisting of the M-G CMFD and the 2-G CMFD was thus implemented. In the two-level structure, the eigenvalue acceleration is mostly achieved by the Wielandt shift method.

3. Result

In order to compare the performance of the iterative and direct two-node SANM formulation, well known 2-group benchmark problem, IAEA 3D, was first solved with the two methods. As shown in Table 1, ISANM and DSANM require the same numbers of nodal updates and CMFD sweeps. But DSANM gave better performance than ISANM because in the DSANM the 3G x 3G linear system could be easily reduced with the inverses of 2x2 matrices instead of solving 3G x 3G linear system directly. This makes DSANM is better for the 2 group case. On the other hand, ISANM shows much better performance than DSANM in case of 7-group problems as shown in Table 2. In this case, it is more efficient to solve 3Gx3G linear system than performing the reduction using the inverse of 7x7 matrices. In the iterative solution, the shape convergence criterion of ISANM, Eq. (6), was set to 0.01 and ISANM gives almost the same results as DSANM in terms of the number of nodal updates.

In addition, we compared the performance and running time of two-node and one-node ISANM and the results are shown in Table 3. It indicates that two-node ISANM gives better performance, more than 2 times, than one-node ISANM. And the number of nodal updates does not increase much as the number of meshes per assembly grows up when using two-node ISANM, This result verifies two-node kernel guarantees the stability of convergence regardless of optical length.

4. Conclusion

We have implemented the two-node SANM in two ways: iterative and direct solution. For 2-group problems, DSANM shows better performance than ISANM but ISANM is better for multi-group problems. Thus it is better to use ISANM for multigroup problems. It was also verified that two-node SANM provides better convergence characteristics than the one-node formulation.

Table 1. IAEA 3D, 2-Group Calculation Results

Node[1]	Method[2]	NodalUpd.[3]	CMFDSwp.[4]	Time(sec)[5]	k-eff
1X1	Iterative	7(16)	41	0.356	1.02915
	Direct	7	41	0.281	1.02915
2X2	Iterative	7(15)	41	0.984	1.02909
	Direct	7	41	0.891	1.02909
4X4	Iterative	8(16)	47	4.656	1.02907
	Direct	8	47	3.891	1.02907

[1] Node: the number of meshes per an assembly
[2] Method: Iterative-ISANM, Direct-DSANM
[3] NodalUpd. : # of nodal updates
[4] CMFDSwp.: # of CMFD Iterations
[5] Time : Intel XEON 2.0Ghz, 2GB Memory

Table 2. MOX 2D, 7-Group

Node[1]	Method[2]	NodalUpd.[3]	CMFDSwp.[4]	Time(sec)[5]	k-eff
1X1	Iterative	9(20)	37(MG) 48(2G)	0.063	1.21064
	Direct	9	37(MG) 48(2G)	0.212	1.21064
2X2	Iterative	7(15)	28(MG) 36(2G)	0.121	1.21043
	Direct	7	27(MG) 36(2G)	0.375	1.21043
4X4	Iterative	6(12)	25(MG) 30(2G)	0.484	1.21038
	Direct	7	27(MG) 36(2G)	1.211	1.21038

Table 3. one- and two-node SANM for MOX 2D, 7-Group.

Node[1]	Method[2]	NodalUpd.[3]	CMFDSwp.[4]	Time(sec)[5]	k-eff
1X1	1NSANM	9(74)	36(MG) 48(2G)	0.219	1.21064
	2NSANM	9(20)	37(MG) 48(2G)	0.063	1.21064
2X2	1NSANM	6(27)	25(MG) 30(2G)	0.255	1.02909
	2NSANM	7(15)	28(MG) 36(2G)	0.121	1.21043
4X4	1NSANM	5(26)	21(MG) 24(2G)	1.094	1.21038
	2NSANM	6(12)	25(MG) 30(2G)	0.484	1.21038

[2] Method: 1NSANM / 2NSANM - one-node / two-node

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