A Collocated Grid Semi-Implicit Scheme to Solve One Dimensional Three Field Equations

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1. Introduction

In this study, a three-field pilot code is developed for a one-dimensional channel flow. Unlike the study in the reference[1], the collocated grid system is used instead of the staggered one. Using staggered grid typically results in stable and robust solutions, as shown in the reference[1]. For general unstructured meshes, however, the staggered methods tend to become rather complex. To avoid this difficulty, an alternative approach is to use collocated pressure and velocity nodes. However, the collocated grid leads to indeterminate oscillation in the pressure field due to decoupling of odd node values from even node values. To provide linkage between adjacent pressure nodes, Rhie and Chow interpolation scheme [2] imitating the staggered grid solution is used. Then, comparative simulations using the staggered grid and the collocated grid codes have been performed for several test cases.

2. Governing equations

The present one-dimensional thermal hydraulic pilot code solves the following ten governing equations.

2.1 Continuity equations

$$\frac{\partial}{\partial t}(\alpha_v\rho_v)+\frac{1}{A}\frac{\partial}{\partial x}(\alpha_v\rho_v u_v A)=\Gamma_l+\Gamma_d$$

$$\frac{\partial}{\partial t}(\alpha_l\rho_l)+\frac{1}{A}\frac{\partial}{\partial x}(\alpha_l\rho_l u_l A)=-\Gamma_l-S_E+S_D$$

$$\frac{\partial}{\partial t}(\alpha_d\rho_d)+\frac{1}{A}\frac{\partial}{\partial x}(\alpha_d\rho_d u_d A)=-\Gamma_d+S_E-S_D$$

$$\frac{\partial}{\partial t}(\alpha_n\rho_n)+\frac{1}{A}\frac{\partial}{\partial x}(\alpha_n\rho_n u_v A)=0$$

The interfacial mass transfer is divided into the mass transfer (Γ_l) at the vapor/liquid interface and the mass transfer (Γ_d) at the vapor/droplet interface.

2.2 Momentum equations

$$\begin{aligned}&\alpha_g(\rho_v+\rho_n)\frac{\partial u_v}{\partial t}+\alpha_g(\rho_v+\rho_n)u_v\frac{\partial u_v}{\partial x}=-\alpha_g\frac{\partial P}{\partial x}\\&+\alpha_g(\rho_v+\rho_n)B_x-F_{wv}(u_v)-F_{vd}(u_v-u_d)-F_{vl}(u_v-u_l)\\&+\Gamma_{l,E}u_l+\Gamma_{d,E}u_d-\Gamma_{l,E}u_v-\Gamma_{d,E}u_v\\&-C_{v,vd}\alpha_v\alpha_d\rho_{m,vd}\frac{\partial(u_v-u_d)}{\partial t}-C_{v,vl}\alpha_v\alpha_l\rho_{m,vl}\frac{\partial(u_v-u_l)}{\partial t}\end{aligned}$$

$$\begin{aligned}&\alpha_l\rho_l\frac{\partial u_l}{\partial t}+\alpha_l\rho_l u_l\frac{\partial u_l}{\partial x}=-\alpha_l\frac{\partial P}{\partial x}+\alpha_l\rho_l B_x\\&-F_{wl}(u_l)-F_{lv}(u_l-u_v)+S_D u_d\\&-\Gamma_{l,C}u_l+\Gamma_{l,C}u_v-u_l S_D-C_{v,lv}\alpha_l\alpha_v\rho_{m,lv}\frac{\partial(u_l-u_v)}{\partial t}\end{aligned}$$

$$\begin{aligned}&\alpha_d\rho_d\frac{\partial u_d}{\partial t}+\alpha_d\rho_d u_d\frac{\partial u_d}{\partial x}=-\alpha_d\frac{\partial P}{\partial x}+\alpha_d\rho_d B_x\\&-F_{wd}(u_d)-F_{dv}(u_d-u_v)+S_E u_l\\&-\Gamma_{d,C}u_d+\Gamma_{d,C}u_v-u_d S_E-C_{v,dv}\alpha_d\alpha_v\rho_{m,dv}\frac{\partial(u_d-u_v)}{\partial t}\end{aligned}$$

The phasic momentum equations are used in an expanded form. These equations have the pressure gradient, the body force, interface frictional drag, wall friction, interfacial mass transfer, and virtual mass term.

2.3 Energy equations

$$\begin{aligned}&\frac{\partial(\alpha_g(\rho_v e_v+\rho_n e_n))}{\partial t}+\left(\frac{1}{A}\right)\frac{\partial(A\alpha_v u_v(\rho_v e_v+\rho_n e_n))}{\partial x}=-P\frac{\partial\alpha_g}{\partial t}\\&-\left(\frac{P}{A}\right)\frac{\partial(A\alpha_g u_v)}{\partial x}+Q_{iv-l}+\Gamma_l h_{vl}^*+\Gamma_d h_{vd}^*+Q_{iv-d}+Q_{l-n}+Q_{d-n}\end{aligned}$$

$$\begin{aligned}&\frac{\partial(\alpha_l\rho_l e_l)}{\partial t}+\left(\frac{1}{A}\right)\frac{\partial(A\alpha_l\rho_l e_l u_l)}{\partial x}=-P\frac{\partial\alpha_l}{\partial t}-\left(\frac{P}{A}\right)\frac{\partial(A\alpha_l u_l)}{\partial x}\\&+Q_{il}-\Gamma_l h_l^*-S_E h_l+S_D h_d-Q_{l-n}\end{aligned}$$

$$\begin{aligned}&\frac{\partial(\alpha_d\rho_d e_d)}{\partial t}+\left(\frac{1}{A}\right)\frac{\partial(A\alpha_d\rho_d e_d u_d)}{\partial x}=-P\frac{\partial\alpha_d}{\partial t}-\left(\frac{P}{A}\right)\frac{\partial(A\alpha_d u_d)}{\partial x}\\&+Q_{id}-\Gamma_d h_d^*+S_E h_l-S_D h_d-Q_{d-n}\end{aligned}$$

The vapor/non-condensable gas is assumed to be at the thermal equilibrium state but separate energy equations are set up both for liquid and droplet fields.

3. Solution Methodology

A semi-implicit scheme is used to solve the two-phase flow field. The momentum equations are discretized first in an explicit manner except for the pressure gradient terms, which are treated implicitly. The discretized momentum equation gives a link between velocity and pressure. This relationship yields an equation for pressure through the continuity equation. On a collocated grid, however, the pressure gradient term in cell contains only the adjacent cell pressure values and not the pressure in cell. This pressure field can cause decoupling of the velocity and pressure fields giving a checkerboard effect. This can be overcome by using the Rhie-Chow interpolation algorithm.

The following is the Rhie-Chow procedure adapted for multiphase flows. At first, the discretized momentum equation at cell can be written in the following relation between the cell-center velocity and the face pressures:

$$u_k^{n+1} = \xi_k \nabla P^{n+1} + {}^{\exp}u_k$$

To find the phasic velocities at the faces, the first part ξ and the quantity ${}^{\exp}u$ are interpolated linearly between the adjacent nodes. The pressure difference across the face may be written directly as the difference between cell-center pressures:

$$u_k^{f(n+1)} = \xi_k^f \nabla P^{f(n+1)} + {}^{\exp}u_k^f$$

These face velocities are substituted into continuity and energy equation. Then, the cell matrix inversion leads to a single equation involving only pressures. This is done for each cell, giving rise to the system pressure matrix. After the system pressure matrix is solved, the solutions for independent variables are obtained by the back substitution

4. Test results

4.1 Single phase liquid injection test

In this test, the subcooled liquid was injected at the flowrate, 1.0 kg/s, through the bottom junction. The results show that the gravity head and the frictional pressure drop calculated by the collocated grid agree well with those by the staggered grid.

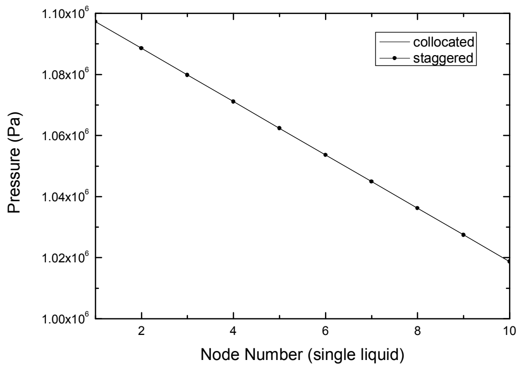

Figure 1 Pressure variation during the liquid injection test

4.2 Boil-off test

Simulation of boil-off test was performed to assess whether the pilot code using collocated grid predicts properly the transition of flow regime. In this test, 30 vertical volumes were initially filled with subcooled liquid at 1000 kPa. Heat source is continuously added from 4th node to the last node from 10 seconds of the test initiation.

As shown in Figure 2, the void appears almost immediately from the last node after the initiation of the transient in the calculation. With heat transfer added, the flow field undergoes a transition to the bubbly, slug, annular mist, mist and single phase vapor.

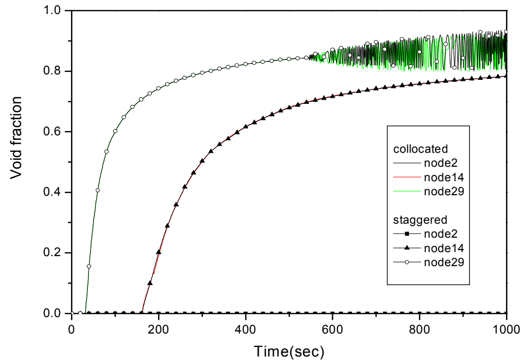

Figure 2 Void fraction behaviors during the boil-off test

5. Conclusion

Before the development of muti-dimensional code, three-field pilot code using collocated grid has been tested for a one-dimensional channel flow. The predictions of transient behaviors using collocated grid code show a good agreement with the results by staggered grid code. Consequently, it is expected to be promising to utilize the potential advantages of a collocated variable arrangement for the development of muti-dimensional two phase flow analysis code.

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[2] C. M. Rhie and W. L. Chow, A Numerical Study of the Turbulent Flow Past an Isolated Airfoil with Trailing Edge Separation, AIAA Journal, vol. 21, pp. 1525-1532, 1983.