

# Reconstruction of Fuel Rod Powers for CANDU Reactors Using a Support Vector Regression Method

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## 1. Introduction

CANDU (CANada Deuterium Uranium) reactors are a pressurized heavy water reactor that uses heavy water for moderator and coolant and uses natural uranium fuel with an on-power refueling scheme. Because of the on-power refueling, fuel elements in CANDU reactors experience a linear power change during their residence in a fuel channel. The linear power change and the ramped power of the fuel element itself are essential to determining the fuel integrity parameters such as the stress corrosion cracking (SCC) failure probability. In this paper, a support vector regression (SVR) model optimized by a genetic algorithm is applied to estimating detailed fuel rod power distributions in CANDU reactor cores and is developed by referring to the previous work [1]. This method reconstructs rod-wise power distributions of a CANDU fuel bundle based on full-core diffusion calculations performed on a coarse-mesh finite-difference scheme of the RFSP (Reactor Fueling Simulation Program) code system. The inputs for the SVR model are the assembly powers determined by the coarse-mesh full-core calculations and the form functions obtained from single-assembly lattice calculations.

## 2. Support Vector Regression Model

The SVR model is developed to predict the detailed fuel rod powers from the results of the full-core diffusion calculation performed on a coarse-mesh finite-difference scheme.

### A. Support Vector Regression (SVR)

The SVR model considers a regression function of the following form:

$$y = f(\mathbf{x}, \mathbf{w}) = \sum_{i=1}^N w_i \phi_i(\mathbf{x}) + b = \mathbf{w}^T \boldsymbol{\phi}(\mathbf{x}) + b. \quad (1)$$

The parameters  $\mathbf{w}$  and  $b$  are a support vector weight and a bias that are calculated by minimizing the following regularized risk function:

$$R(\mathbf{w}) = \frac{1}{2} \mathbf{w}^T \mathbf{w} + \lambda \sum_{i=1}^N |y_i - f(\mathbf{x}, \mathbf{w})|_{\varepsilon}, \quad (2)$$

where

$$|y_i - f(\mathbf{x}, \mathbf{w})|_{\varepsilon} = \begin{cases} 0 & |y_i - f(\mathbf{x}, \mathbf{w})| < \varepsilon \\ |y_i - f(\mathbf{x}, \mathbf{w})| - \varepsilon & \text{otherwise} \end{cases} \quad (3)$$

The constants  $\lambda$  and  $\varepsilon$  are user-specified parameters and  $|y_i - f(\mathbf{x}, \mathbf{w})|_{\varepsilon}$  is called the  $\varepsilon$ -insensitive loss function [2].

### B. Genetic Optimization of the SVR Model

The genetic algorithm is used to optimize the insensitivity zone  $\varepsilon$ , the regularization parameter  $\lambda$ , and the sharpness  $\sigma$  of the radial basis kernel function used in this paper that is expressed as follow:

$$K(\mathbf{x}, \mathbf{x}_i) = \exp\left(-\frac{(\mathbf{x} - \mathbf{x}_i)^T (\mathbf{x} - \mathbf{x}_i)}{2\sigma^2}\right). \quad (4)$$

The SVR model is trained using exemplary situations (training data) for which the desired output is already known. It is assumed that the model will also be able to predict the correct output for other examples, thus generalizing to situations not presented during training. The regularization is accomplished by making the SVR models have smaller support vector weights, which causes the SVR model to respond smoother and less likely to overfit. As shown in Eq. (1), it is possible to reduce the sensitivity of the estimated output according to the change of each input variable by decreasing the magnitude of the weight vector, which prevents an overfitting problem. Also, although the regularization was already considered by minimizing the risk function of (2), the regularization is additionally taken into account by minimizing the support vector weights during optimizing the design parameters of SVR models. Therefore, the following objective for optimizing the SVR models is suggested:

$$F = \exp(-\mu_1 E_1 - \mu_2 E_2 - \mu_3 E_3) \quad (5)$$

where  $\mu_1$ ,  $\mu_2$  and  $\mu_3$  are the weighting coefficients, and  $E_1$ ,  $E_2$  and  $E_3$  are defined as follows:

$$E_1 = \sqrt{\frac{1}{N} \sum_{k=1}^N ((y(k) - \hat{y}(k))/y(k))^2} \quad (6)$$

$$E_2 = \max_k \{(y(k) - \hat{y}(k))/y(k)\} \quad (7)$$

$$E_3 = \mathbf{w}^T \mathbf{w}. \quad (8)$$

The SVR models is optimized through multiple objectives which minimize the root mean squared error

$E_1$ , the maximum error  $E_2$ , and the square of the magnitude of the weight vector  $E_3$ .

### 3. Application Results

The proposed SVR models were applied for reconstructing the rod-wise power distributions of CANDU fuel bundles from the coarse-mesh finite difference solutions obtained using the RFSP code. A total of 37 SVR models are used for power prediction of thirty seven fuel rods in fuel bundles. Each SVR model is trained using the assembly-average powers determined by core calculations using the HELIOS code and also, the form function obtained from single-assembly lattice calculations using the HELIOS code. The SVR model has nine inputs, which are represented as follows:

$$x_1 = \phi_1 H_p, x_2 = \phi_2 H_p, L, x_9 = \phi_9 H_p$$

where  $\phi$  denotes the assembly-average power,  $H_p$  is a form function value at the corresponding rod location, each subscript denotes the location of the relevant assembly and those of assemblies surrounding it.

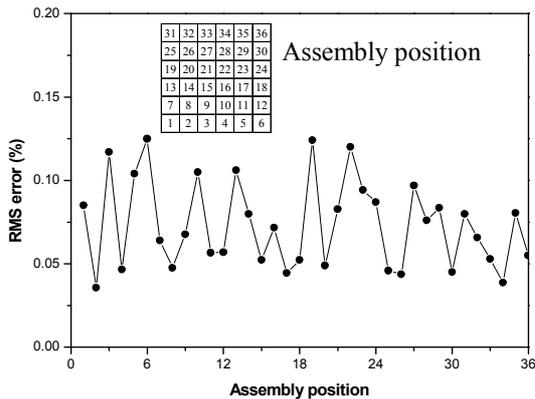


Fig. 1. Reconstruction RMS errors of fuel rod powers versus assembly position (test data).

Figure 1 shows the reconstruction errors of fuel rod powers versus assembly position to examine the effect of the assembly position to the SVR model. The assembly position does not influence the performance of the SVR model.

Figure 2 shows the reconstruction errors of fuel rod powers versus fuel rod position in a fuel assembly to investigate the effect of the fuel rod position to the SVR model. The fuel rod position number increases from the inner side (center) to the outer one.

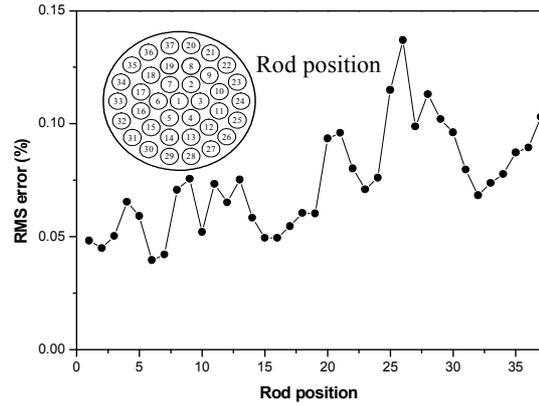


Fig. 2. Reconstruction RMS errors of fuel rod powers versus 37-fuel rod positions in fuel assemblies (test data).

In order to estimate the total reconstruction error including the errors due to the coarse-mesh finite-difference diffusion calculation, the fuel rod powers were reconstructed using the assembly-average powers obtained from the DIF3D calculations. The RMS error averaged over 36 fuel assemblies is 1.10% for the  $6 \times 6$  fuel bundle problem and 1.54% for the  $3 \times 3$  one. Also, the maximum rod power errors of 1332 ( $=36 \times 37 = \text{no. of assemblies multiplied by no. of fuel rods per assembly}$ ) fuel rods is 3.20% for the  $6 \times 6$  fuel bundle problem and the maximum rod power errors of 333 ( $=9 \times 37$ ) fuel rods is 2.26% for the  $3 \times 3$  one.

### 4. Conclusion

In order to assess the performance of the reconstruction schemes, benchmark calculations have been performed for partial core representations of a natural uranium CANDU reactor. It was known from the benchmark calculations that the reconstruction schemes are quite accurate, yielding RMS rod power errors of less than 1.54%. The main contribution to the reconstruction error is made by the errors in the assembly-average powers obtained from the coarse-mesh finite-difference diffusion calculation and the errors due to the reconstruction scheme itself are less than 0.14%. In particular, since the rod power errors for the test data are similar to those for the training data, an SVR model trained for a specified data set can be successfully used for another data set.

### References

1. M. G. Na, W. S. Yang and H. Choi, "Pin power reconstruction for CANDU reactors using a neuro-fuzzy inference system," IEEE Trans. Nucl. Sci., vol. 48, no. 2, pp. 194-201, Apr. 2001.
2. V. Vapnik, The Nature of Statistical Learning Theory. New York: Springer, 1995.