Transactions of the Korean Nuclear Society Autumn Meeting
PyeongChang, Korea, October 25-26, 2007

Development of a Neutron Diffusion Equation Solver based on the Finite Element Method for the CAPP Code

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1. Introduction

Neutron diffusion equation solvers based on the finite difference method (FDM) have been widely used in the analysis of pebble bed reactors [1,2,3]. These FDM-based solvers have a drawback of a long computation time and huge memory requirement, especially in three dimensional (3-D) applications. Recently, neutron diffusion equation solvers based on the nodal methods were developed for the analysis of pebble bed reactors to overcome these drawbacks [4,5,6].

In this paper, we present a neutron diffusion equation solver based on the finite element method (FEM). FEM has several advantages over the nodal methods. It has a flexibility in geometry, firm mathematical basis, and high computational efficiency.

2. Methods and Results

2.1. Implementation of the FEM Solver

Triangular and rectangular finite elements were implemented for 2-D application. There are three types of triangular finite elements depending on the order of the shape functions. They are linear(3-node), quadratic (6-node), and cubic(10-node) triangular finite elements. Five types of rectangular finite elements were implemented. They are bi-linear (4-node), bi-quadratic (9-node), bi-cubic (16-node), incomplete-quadratic (8-node), and incomplete-cubic (12-node) rectangular finite elements.

Triangular prismatic and rectangular prismatic finite elements were implemented for 3-D application. There are ten types of triangular prismatic finite elements. Nine of them are the combinations of the three triangular finite elements and the three (linear, quadratic, and cubic) axial 1-D finite elements. The last one is the incomplete-quadratic (15-node) triangular prismatic finite elements. There are seventeen types of rectangular prismatic finite elements. Fifteen of them are the combinations of the five rectangular finite elements and the three axial 1-D finite elements. The last two of them are the incomplete-quadratic (20-node), and incomplete-cubic (32-node) rectangular prismatic finite elements. Figure 1 shows some of the 3-D finite elements.

A polynomial mapping from the master finite element to a real finite element was adopted for flexibility in dealing with complex geometry. Two types of mapping were implemented. They are linear mapping and iso-parametric mapping. In linear mapping, only the vertex nodes are used as the mapping points. In iso-parametric mapping, all the nodal points in the finite element are used as the mapping points, which enables the real finite element with curved surfaces.

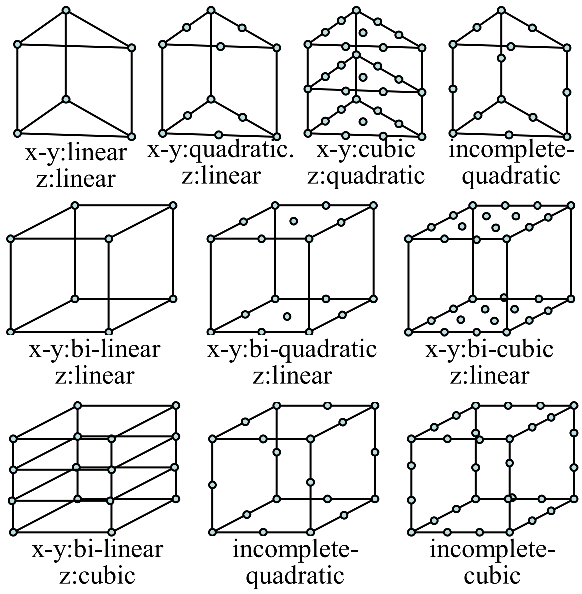

Figure 1. Several types of 3-D finite elements.

For the treatment of spatial dependency of cross-sections in the finite elements, polynomial expansion of the cross-sections in the finite elements is allowed. There are three options for the expansion. They are constant, linear, and iso-parametric cross-section expansions. In the linear cross-section expansion, the cross-section values at the vertex nodes are used for the linear expansion of the cross-sections. In the iso-parametric cross-section expansion, the cross-sections at all the nodes are used for the expansion of the cross-sections.

The power method with the Wielandt acceleration technique[7] was adopted as the outer iteration algorithm. The BiCGSTAB algorithm with the ILU (Incomplete LU) decomposition preconditioner[8] was used as the linear equation solver in the inner iteration.

2.2 Verification of the Solver

The FEM neutron diffusion equation solver developed in this study was verified against two well known benchmark problems. One is the IAEA PWR benchmark problem [9] and the other is OECD/NEA PBMR400 benchmark problem [10].

Table 1 and Table 2 compare several FEM results for the IAEA 2-D and 3-D benchmark problem, respectively. The solutions converge as the order of the element shape function increase and as the number of elements increase.

Figure 2 shows the material map of the OECD/NEA PBMR400 benchmark problem in r-z plane. Though the problem is defined as a 2-D problem, 3-D finite elements were used to model this problem because it is defined on a cylindrical geometry. The triangular prismatic finite elements were used for the inner most column of the problem while rectangular prismatic finite elements were used for the other part of the problem.

Table 3 compares the FEM and FDM solutions to the OECD/NEA PBMR400 benchmark problem. The first three solutions have similar accuracy – around 1 % of maximum power error and around 0.3% of RMS error. However, the computation time of the incomplete cubic finite element case is about 6 times and about 18 times shorter than those of the bi-linear finite element case and FDM case, respectively. Similar trend is observed with the last three cases.

3. Conclusion

In this paper, we developed a FEM based neutron diffusion equation solver and we compared the FEM and FDM solutions to benchmark problems. The results showed that the solutions converge as the order of the shape functions and as the number of finite elements increases. The results also showed that a higher order FEM is much more efficient than a low order FEM or a fine mesh FDM.

Table 1. The FEM solutions to the IAEA 2-D problem

Element Type	Elements / FA	k_{eff} Error (pcm)	Power Error (%)	
			Max.	RMS
bi-linear	2x2	+151	32.61	14.88
	4x4	+27	8.29	3.78
incomplete quadratic	2x2	+14	4.26	1.77
	4x4	+1	0.47	0.20
incomplete cubic	2x2	+4	1.77	0.54
	4x4	0	0.14	0.04

Ref. solution : incomplete cubic 8x8, k_{eff}=1.02958

Table 2 The FEM solutions to the IAEA 3-D problem

Element	incomplete quadratic			incomplete cubic	
Elmt./FA	2x2x19	4x4x38	6x6x57	2x2x19	4x4x38
k_{eff}	1.02957	1.02940	1.02943	1.02939	1.02939

Table 3. The FEM and the FDM solutions to the OECD/NEA PBMR400 benchmark problem

Method	k_{eff} Error (pcm)	Max. Pow. Error (%)	RMS Pow. Error (%)	CPU Time (s)
FEM 1x1 C[1)]	-5	0.92	0.35	1.67
FEM 6x6 L[2)]	9	1.03	0.27	9.97
FDM 12x12	-7	0.99	0.30	30.3
FEM 1x1 Q[3)]	-4	4.09	1.08	0.58
FEM 3x3 L[2)]	37	3.95	1.02	1.26
FDM 5x5	-40	3.77	0.99	1.83

Ref. Sol : FEM 15x15 r-z : incomplete cubic, θ : linear
1) : r-z : incomplete cubic, θ : linear
2) : r-z : bi-linear, θ : linear
3) : r-z : incomplete quadratic, θ : linear

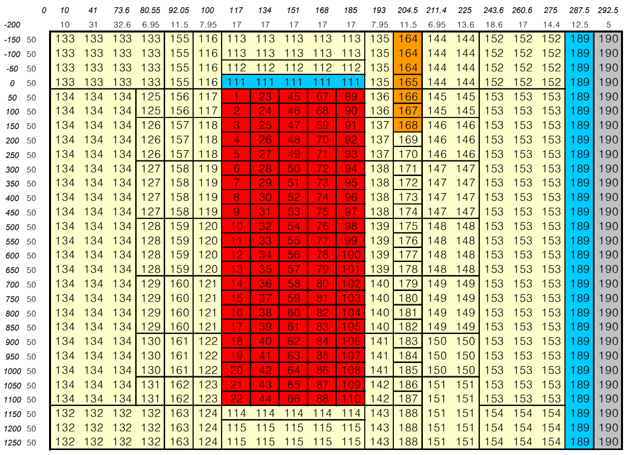

Figure 2. Material map of the OECD/NEA PBMR400 benchmark problem

ACKNOWLEDGEMENTS

This work has been performed as a part of the Nuclear R&D Program supported by the Ministry of Science and Technology (MOST) of the Republic of Korea.

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