Dynamic Equations of Motion of a Flexible Single Block on a Rigid Surface

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1. Introduction

Prismatic graphite blocks are major components of the reactor core structure of a prismatic type HTGR (High Temperature Gas Cooled Reactor). The vertically stacked prismatic graphite fuel blocks and graphite reflector blocks form a group of graphite block columns. And each column has gaps between neighboring columns and stands on the core bottom structure by itself. An earthquake loading on stacked blocks causes rocking responses and solid impacts between them, and may lead to structural integrity problems. The seismic analysis of a HTGR core structure composed of stacked prismatic graphite blocks has been an important design issue and has a long history.

The dynamics of a block as well as stacked blocks are quite complex, and they are not fully understood yet. A basic systematic understanding of the rocking response of a rigid block resting on a rigid floor had not been well established until when G.W. Housner first presented it in 1963 [1]. In 1975, T.H. Lee presented a methodology for analyzing the nonlinear response of a column of stacked prismatic fuel blocks [2]. In 1979 T. Ikushima and T. Nakazawa presented their work results on a seismic analysis of a column of stacked prismatic fuel blocks and compared them to the seismic test results of a half scale model [3]. Their numerical model is similar to that of Lee's study, but the parameters in their analysis were from their scaled model tests and the results were verified. A stochastic analysis methodology for a rocking block was introduced by Pol D. Spanos and Aik-Sion Koh; they considered a rocking block on the Winkler foundation [4]. After their works some other researchers have studied new techniques to solve the nonlinearity problems of block impacts on the base. S. J. Hogan considered the dynamics of a slender rigid block mounted on a vibrating rigid table with side walls [5]. The governing equation is quite simple, but it shows complex nonlinear dynamics and gives many types of solutions nevertheless.

Although, as summarized above, diverse articles of study have been presented, less attention has been paid on the effect of a gravitational force on the vibratory motion of a column of higher stacked blocks until when D.O. Kim reported the significance of the effect on it [6]. He studied the gravity effect on the free bending vibration of a column of higher stacked graphite fuel blocks and concluded that while the column height shortening and the vibrational mode shape alteration by the gravitational force are negligible, the downward shifting of natural frequency is quite drastic in a taller one. He modeled the stacked graphite blocks as a single column with an assumption that the stacked blocks may behave like a continuous column when the amplitude of a lateral motion is small. There is no proper model for studying the gravity effect on a stacked block structure yet. This paper presents an analytical model of a flexible block resting on a rigid floor and with a gravity effect.

2. Classical Dynamic Models of Rigid Blocks

2.1 Housner's Model of a Single Block

G.W. Housner considered a linearized equation of motion of a single rigid block on a rigid floor, see Eq. (1) where $p^2=WR/I_0$, and showed that the rocking period is a function of the angular displacement of the block, see Fig. 1.

$$\ddot{\theta} - p^2\theta = -p^2\alpha \tag{1}$$

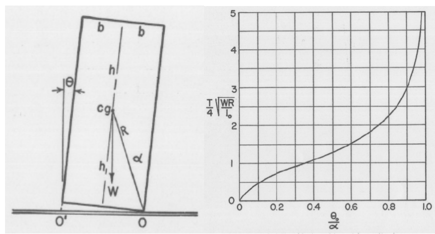

Figure 1. Housner's block model and the period of rocking motion as a function of initial angular displacement

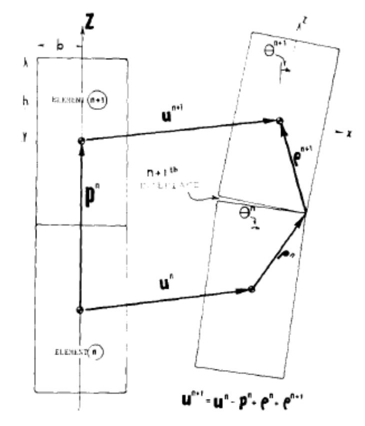

Figure 2. Kinematic relation between blocks for Lee's model

2.2 Lee's Model of Multi-Stacked Blocks

He developed two different models; first one is a rigid block model with rigid dowels and the other one with flexible dowels. The Lagrange's method with the kinematic relation between blocks, Fig. 2, gives the equations of motion for stacked rigid blocks with rigid dowels as follows;

$$\mathbf{J}\ddot{\boldsymbol{\theta}} + \mathbf{D}\dot{\boldsymbol{\theta}}^2 + \mathbf{m}_Y - \mathbf{A}^T\mathbf{f}_X - \mathbf{A}^T\mathbf{M}\{\mathbf{1}\}\ddot{u}_0(t) = 0 \qquad (2)$$

$$\mathbf{M}\ddot{\mathbf{w}} + \mathbf{f}_Z + g\mathbf{M}\{\mathbf{1}\} = 0 \qquad (3)$$

Where $\boldsymbol{\theta}$, and $\mathbf{w}$ are the vectors of degrees of freedom of the blocks, and $\mathbf{J}$, $\mathbf{D}$, $\mathbf{A}$, and $\mathbf{M}$ are system matrices, and $\mathbf{m}_Y$, $\mathbf{f}_X$, and $\mathbf{f}_z$ are boundary force vectors, and g and $u_0(t)$ are the gravitational acceleration and the ground motion, respectively.

2. Proposed Dynamic Model of a Flexible Block

A nuclear power plant of HTGR system in the near future will have a taller core structure to guarantee its passive safety in an accident condition. In this section, a dynamic model of a flexible block resting on a rigid moving floor is proposed. The proposed model can be used for the analysis and design of the reactor core structure of the HTGR with high stacked graphite blocks in it.

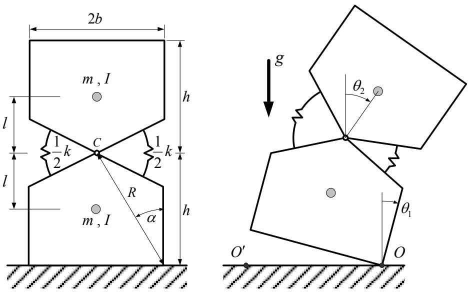

Figure 3. Geometry and degrees of freedom of the proposed flexible block model

The kinetic energy and the potential energy of the flexible block model in a rocking motion and a constraint equation are as follows;

$$T = \frac{1}{2}(I + ml^2)\begin{Bmatrix}\dot{\theta}_1 \\ \dot{\theta}_2\end{Bmatrix}^T \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}\begin{Bmatrix}\dot{\theta}_1 \\ \dot{\theta}_2\end{Bmatrix} + m\begin{Bmatrix}\dot{x}_c \\ \dot{y}_c\end{Bmatrix}^T\begin{Bmatrix}\dot{x}_c \\ \dot{y}_c\end{Bmatrix} + ml\begin{Bmatrix}\dot{\theta}_1 \\ \dot{\theta}_2\end{Bmatrix}^T\begin{bmatrix}-\cos\theta_1 & \sin\theta_1 \\ \cos\theta_2 & -\sin\theta_2\end{bmatrix}\begin{Bmatrix}\dot{x}_c \\ \dot{y}_c\end{Bmatrix} \qquad (4)$$

$$V = \frac{1}{2}k\begin{Bmatrix}\theta_1 \\ \theta_2\end{Bmatrix}^T\begin{bmatrix}1 & -1 \\ -1 & 1\end{bmatrix}\begin{Bmatrix}\theta_1 \\ \theta_2\end{Bmatrix} + mg[2y_c + l(\cos\theta_2 - \cos\theta_1)] \qquad (5)$$

$$\begin{Bmatrix}x_c \\ y_c\end{Bmatrix} = \begin{Bmatrix}x_g \\ y_g\end{Bmatrix} + \begin{Bmatrix}R\sin(\theta_1 - \alpha) \\ R\cos(\theta_1 - \alpha)\end{Bmatrix} \qquad (6)$$

where θ_1, θ_2 are degrees of freedom of the block, and x_c, y_c are displacements of the block center, and x_g, y_g are displacements of the ground.

Substituting the energy equations and the constraint equation above into the Lagrange's method gives the following matrix equation of motion of a flexible block on a moving rigid floor.

$$\mathbf{J}\ddot{\boldsymbol{\theta}} + \mathbf{D}\dot{\boldsymbol{\theta}}^2 + \mathbf{K}\boldsymbol{\theta} + \mathbf{f}_{gv} = -\mathbf{G}\ddot{\mathbf{u}}_g \qquad (7)$$

where the system matrices and vectors are as follows;

$$\mathbf{J} = \begin{bmatrix} I + ml^2 + 2mR(R - l\cos\alpha) & mlR\cos(\theta_2 - \theta_1 \pm \alpha) \\ mlR\cos(\theta_2 - \theta_1 \pm \alpha) & I + ml^2\end{bmatrix},$$

$$\mathbf{D} = mlR\sin(\theta_2 - \theta_1 \pm \alpha)\begin{bmatrix}0 & -1 \\ 1 & 0\end{bmatrix}, \quad \mathbf{K} = k\begin{bmatrix}1 & -1 \\ -1 & 1\end{bmatrix},$$

$$\mathbf{f}_{gv} = mg\begin{Bmatrix}l\sin\theta_1 + 2R\sin(-\theta_1 \pm \alpha) \\ -l\sin\theta_2\end{Bmatrix}, \quad \mathbf{u}_g = \begin{Bmatrix}\ddot{x}_g(t) \\ \ddot{y}_g(t)\end{Bmatrix},$$

$$\mathbf{G} = m\begin{bmatrix}-l\cos\theta_1 + 2R\cos(-\theta_1 \pm \alpha) & l\cos\theta_2 \\ l\sin\theta_1 + 2R\sin(-\theta_1 \pm \alpha) & -l\sin\theta_2\end{bmatrix}^T, \quad \boldsymbol{\theta} = \begin{Bmatrix}\theta_1 \\ \theta_2\end{Bmatrix},$$

$+\alpha$ *when* $\theta_1 \geq 0$, *and* $-\alpha$ *when* $\theta_1 < 0$

3. Conclusion

A dynamic model of flexible block resting on a rigid moving floor is proposed after reviewing classical dynamic models for the analyses of rigid block structures. The proposed model is for a single block analysis only, but it can be extended to a model for multi stacked blocks by introducing several terms for contact forces between blocks. A study on the characteristics and the extension of the proposed model will be presented sooner or later. The results of this study will be used for NHDD project [7].

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