Electrical Resistance Imaging of Bubble Boundary in Annular Two-Phase Flows Using Unscented Kalman Filter

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1. Introduction

For the visualization of the phase boundary in annular two-phase flows, the electrical resistance tomography (ERT) technique is introduced. In ERT, a set of predetermined electrical currents is injected trough the electrodes placed on the boundary of the flow passage and the induced electrical potentials are measured on the electrode. With the relationship between the injected currents and the induced voltages, the electrical conductivity distribution across the flow domain is estimated through the image reconstruction algorithm. In this, the conductivity distribution corresponds to the phase distribution.

In the application of ERT to two-phase flows where there are only two conductivity values, the conductivity distribution estimation problem can be transformed into the boundary estimation problem.[1],[2],[3]

This paper considers a bubble boundary estimation with ERT in annular two-phase flows. As the image reconstruction algorithm, the unscented Kalman filter (UKF) is adopted since from the control theory it is reported that the UKF shows better performance than the extended Kalman filter (EKF) that has been commonly used. [4],[5] We formulated the UKF algorithm to be incorporate into the image reconstruction algorithm for the present problem. Also, phantom experiments have been conducted to evaluate the improvement by UKF.

2. Mathematical Method

2.1 Boundary expression

In this paper, we assume that the outer boundaries of objects are sufficiently smooth and they can be approximated in the form [1], [2]

$$C_l(s) = \begin{pmatrix} x_l(s) \\ y_l(s) \end{pmatrix} = \sum_{n=1}^{N_\theta} \begin{pmatrix} \gamma_n^{x_l} \theta_n^x(s) \\ \gamma_n^{y_l} \theta_n^y(s) \end{pmatrix} \quad (1)$$

where $C_l(s)(l = 1,2,...,S)$ is the boundary of the l th object to be detected, S is the number of objects, $\theta_n(s)$ are periodic and differentiable basis function and N_θ is the number of basis functions. As the basis function, we use the form of

$$\theta_1^\alpha(s) = 1$$
$$\theta_n^x(s) = \sin\left(2\pi \frac{n}{2} s\right), \text{ n=2, 4, 6,…, even}$$
$$\theta_n^y(s) = \cos\left(2\pi \frac{(n-1)}{2} s\right), \text{ n=3, 5, 7,…, odd} \quad (2)$$

since the bubble boundary is nearly circular and it may be approximated with only a few terms. In this, $s \in [0,1]$ and α denotes either x or y. The boundaries are identified with the vector $\boldsymbol{\gamma}$ of the shape coefficients, that is,

$$\boldsymbol{\gamma} = (\gamma_1^{x_1},...,\gamma_{N_\theta}^{x_1},\gamma_1^{y_1},...,\gamma_{N_\theta}^{y_1},...,...,\gamma_1^{x_s},...,\gamma_{N_\theta}^{x_s},\gamma_1^{y_s},...,\gamma_{N_\theta}^{y_s})^T \quad (3)$$

where $\boldsymbol{\gamma} \in \mathbb{R}^{2SN_\theta \times 1}$.

2.3 Unscented Kalman filter Model

The extended Kalman filter (EKF) has become a standard technique used in the ERT as well as in a number of nonlinear estimation and machine learning applications [3].

In EKF, the state distribution is approximated by a Gaussian random variable (GRV), which is then propagated analytically through the first-order linearization of the nonlinear system. This can introduce large errors in the true posterior mean and covariance of the transformed GRV [4].

The UKF addresses this problem by carefully choosing sample points instead of GRV, and which when propagated through the true nonlinear system, captures the posterior mean and covariance accurately to the 3rd order (Taylor series expansion) for any nonlinearity. The EKF, in contrast, only achieves first-order accuracy [5].

3. Experimental results

In order to evaluate the performance of UKF, numerical and experimental studies were performed and the performance was assessed in comparison to EKF which is most often used as an ERT image reconstruction algorithm.

The experimental setup consists of a circular phantom with a radius of 40 mm and a height of 200 mm was considered around which L=36 electrodes (each of length 6 mm, 32 electrodes on surface of the phantom, 4 electrodes on an internal rod) were mounted. As for the current injection protocol, opposite current patterns are used. In the experiment, 8 image frames are considered and each image frame comprises of 8 current patterns. It is assumed that a circular small bubble appears in the right region and grows while moving clockwise.

Figure 1 can be seen that UKF is performing far better than EKF in terms of reconstructed boundary, especially when the bubble is small. In fact, the reconstruction of a smaller target is more difficult than the bigger one.

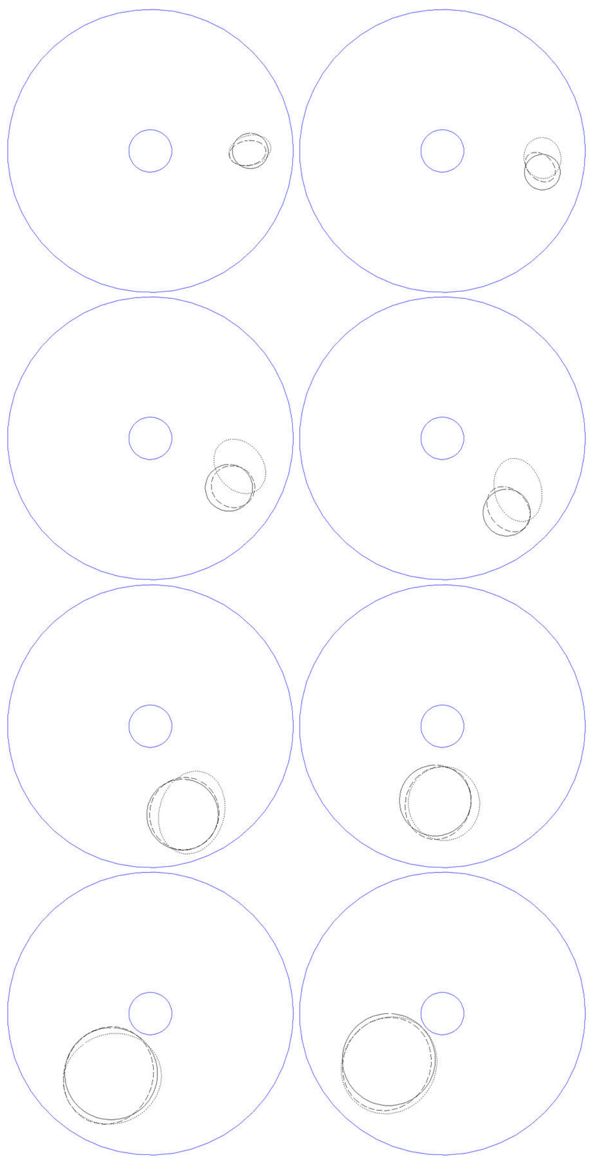

Figure 1. Reconstructed boundaries for the laboratory experiment. Solid line, dotted line and dashed line represent the true boundary, boundary estimated by EKF, and boundary estimated by UKF, respectively.

As a performance metric, root mean square error (RMSE) is defined as,

$$RMSE_{\gamma_k} = \frac{\| \gamma_{estimated,k} - \gamma_{true,k} \|}{\| \gamma_{true,k} \|} \tag{4}$$

In the RMSE comparisons (figure 2.), it can be seen that mostly the RMSE for UKF is less than EKF.

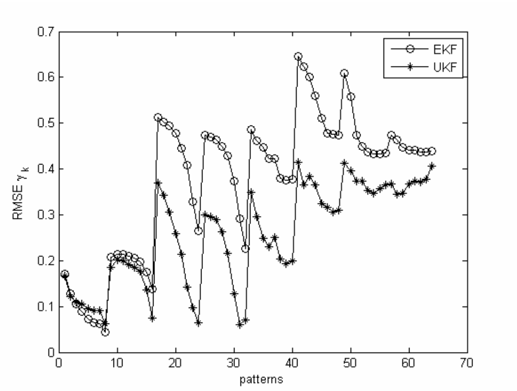

Figure 2. RMSE comparison for laboratory experiment.

4. Conclusion

In this paper, unscented Kalman filter (UKF) is proposed as an image reconstruction algorithm in electrical resistance imaging to estimate the fast transient changes in phase boundary in annular two-phase flows. Also, the UKF-based algorithm is successfully compared with the extended Kalman filter-based algorithm which has been commonly used.

The experiments with two-phase flow phantom were done to suggest a practical implication of this research in estimating the boundary of a gas bubble in heat transfer systems.

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